Introduction to Interferometry

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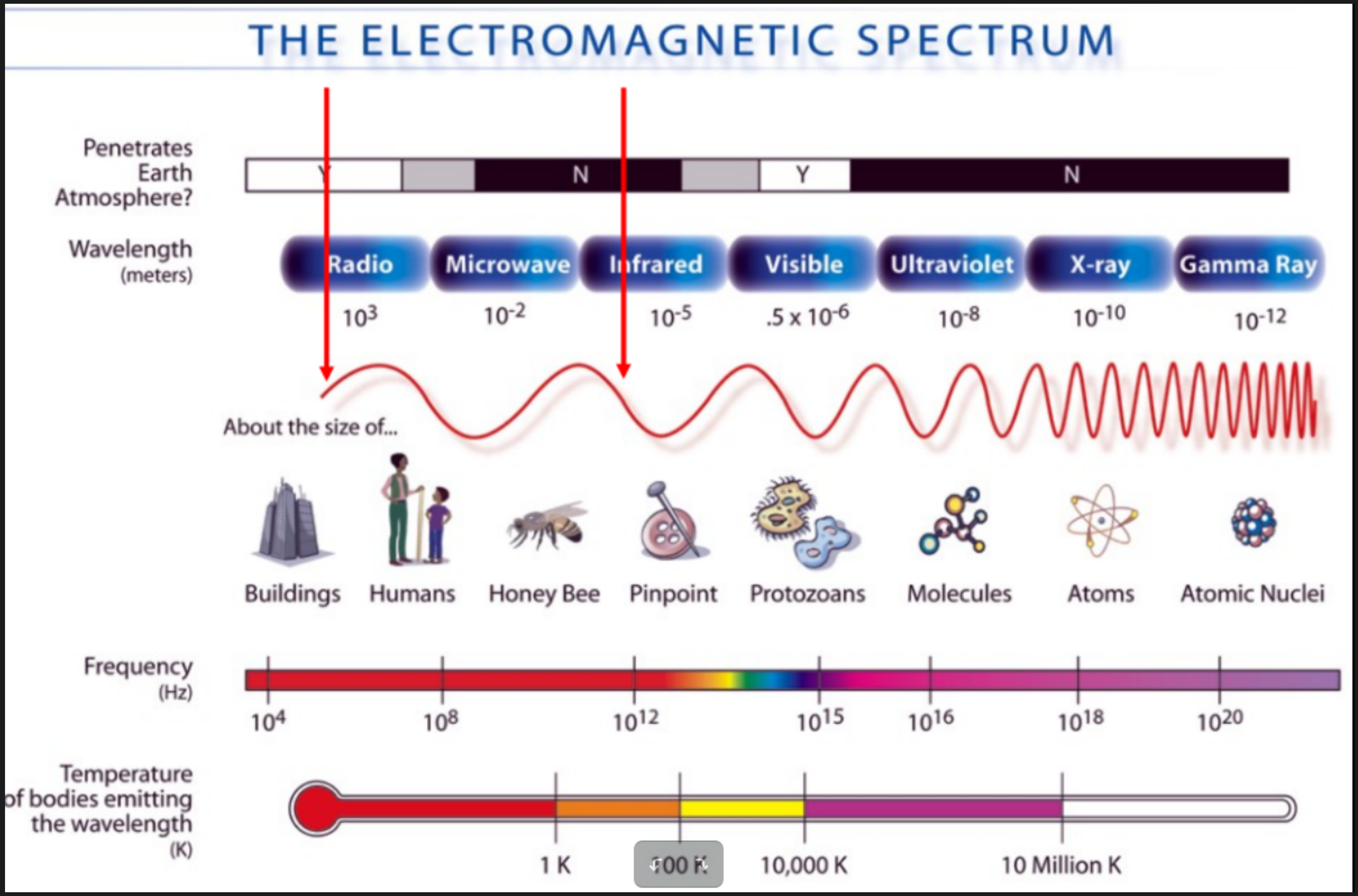
RadioNet has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 730562

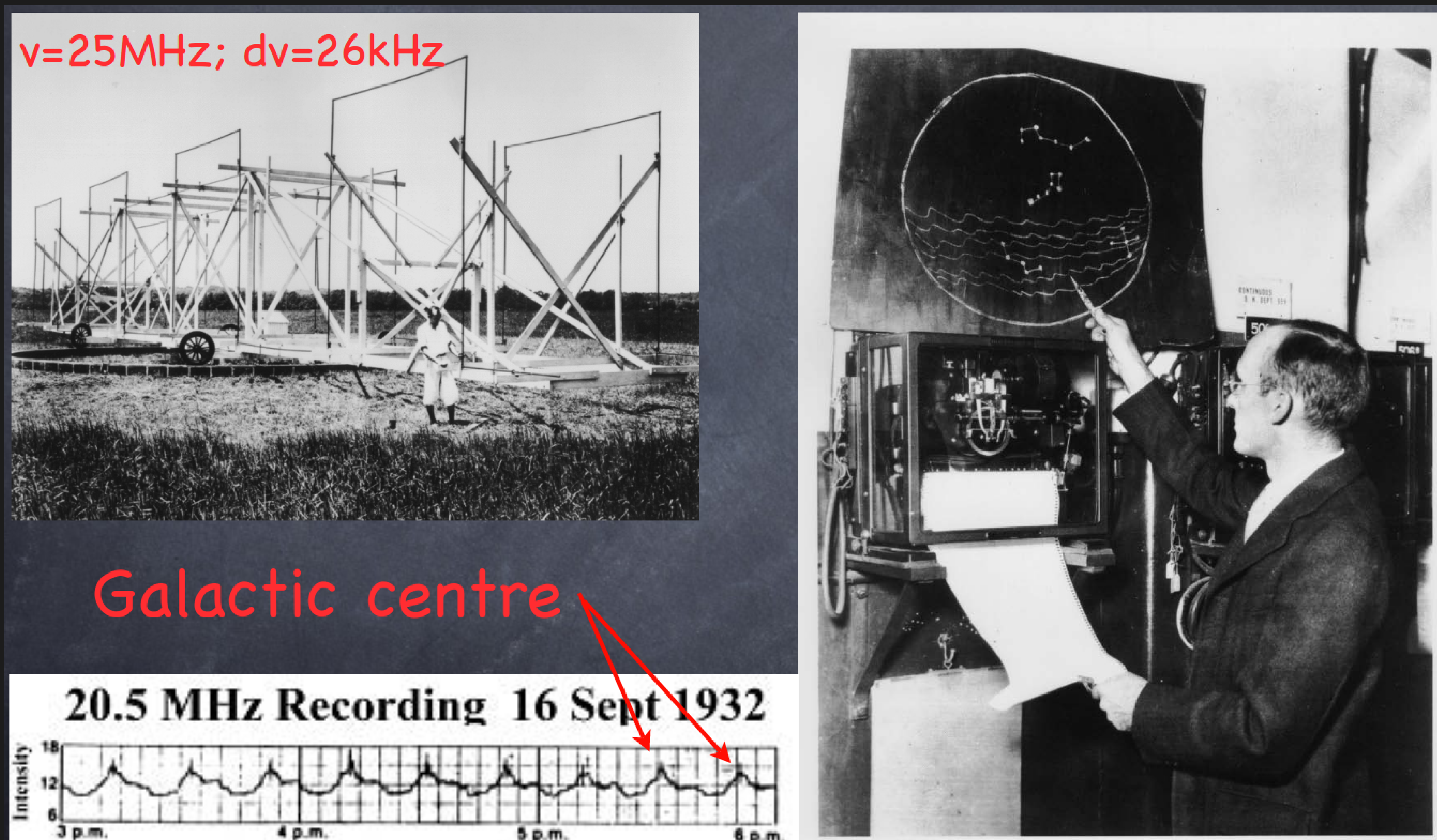
Radioastronomy

H.Hertz (1888)
- Hertz Oscillator: first radio wave transmitter
- Existence of electromagnetic waves (Maxwell's theory)
G. Marconi (1901)
- first transatlantic radio communication (820 Khz)



Radioastronomy

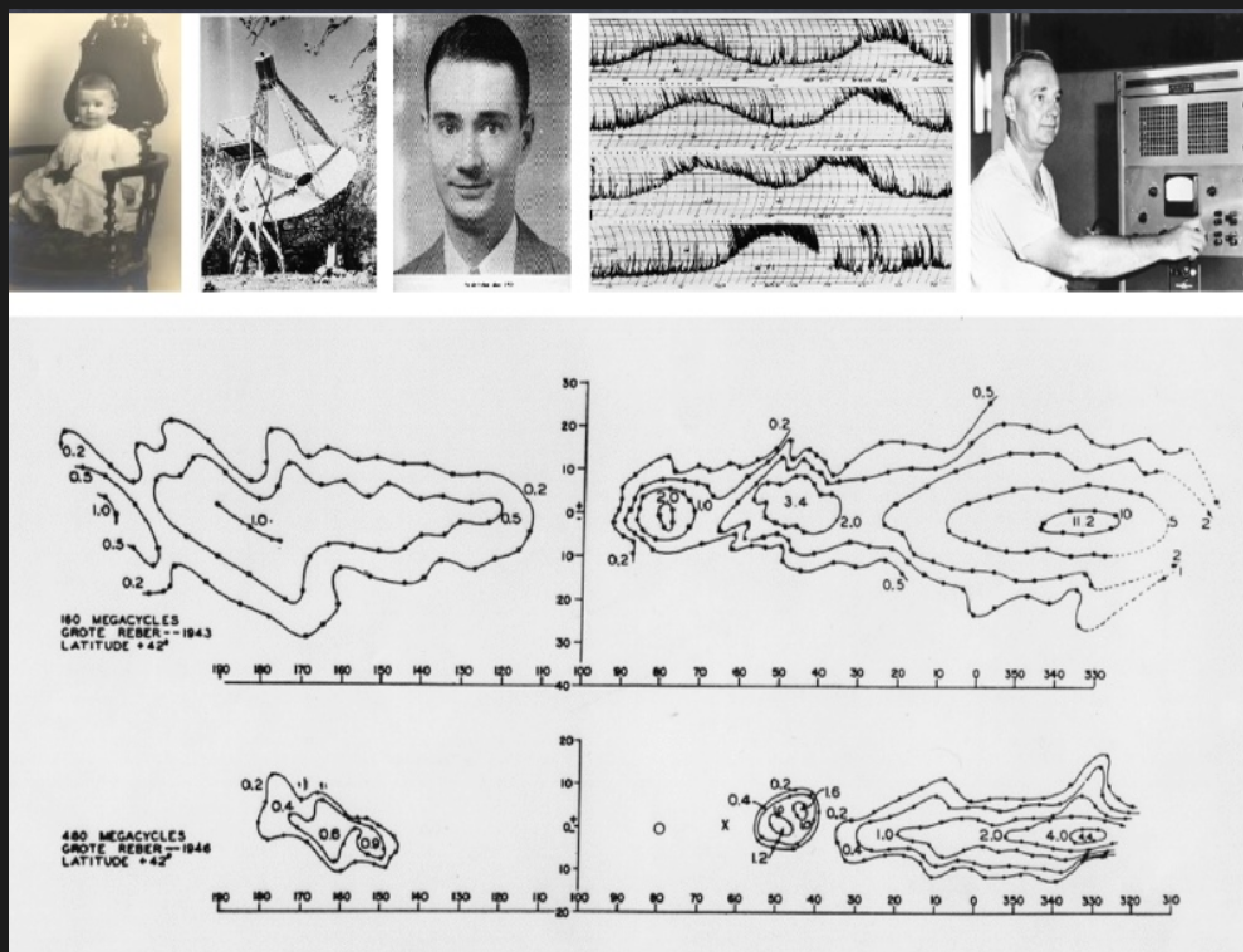
K. Jansky (1932)
- Build a rotating antenna (20.5 MHz)
- discovery of cosmic radio emission
- discover of galactic center



Radioastronomy

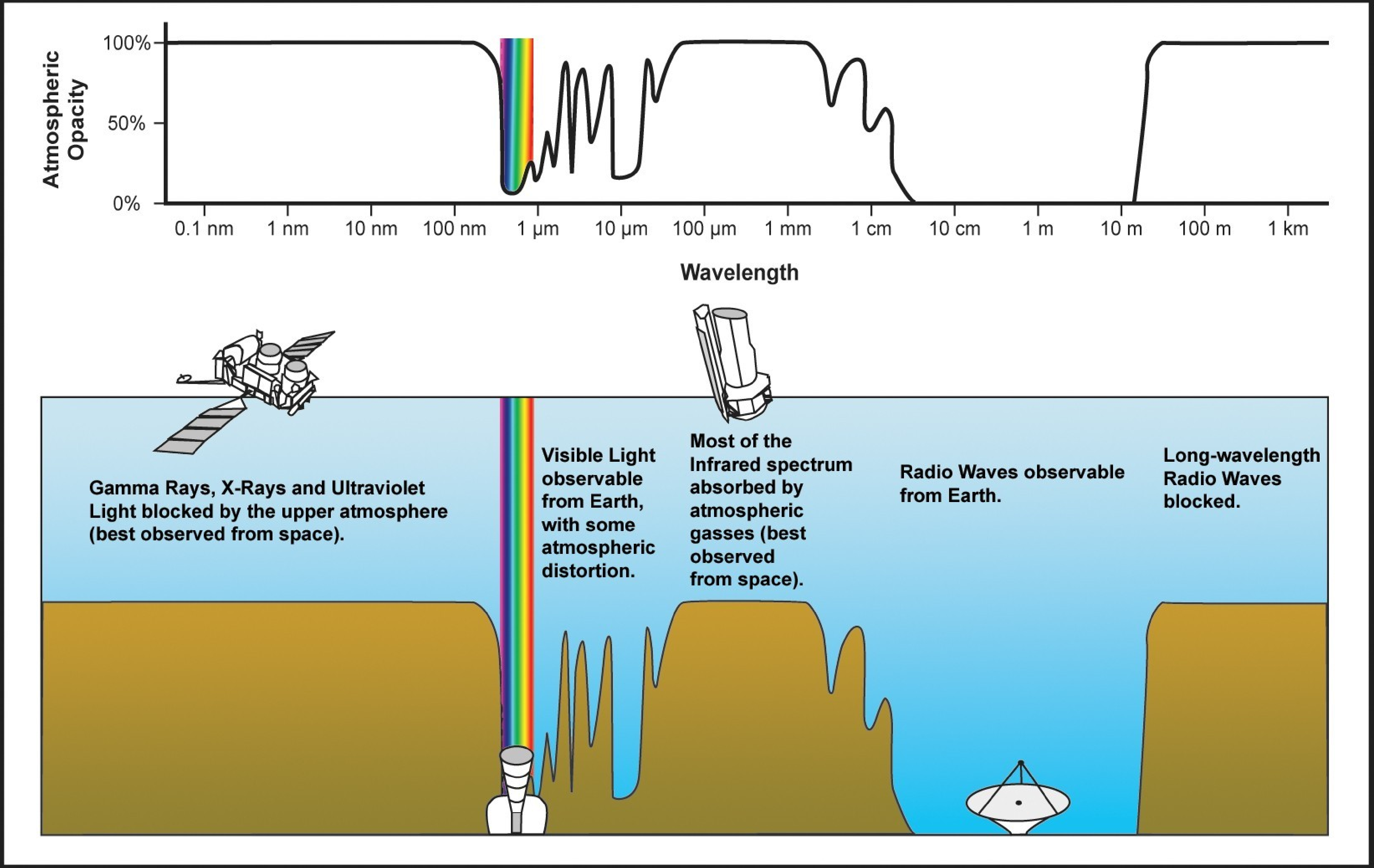
G.Reber (1944)
- Parabolic Radio Dish (160 MHz)

A. Penzias and R. Wilson (1965)
- discovery of the CMB (41 GHz)



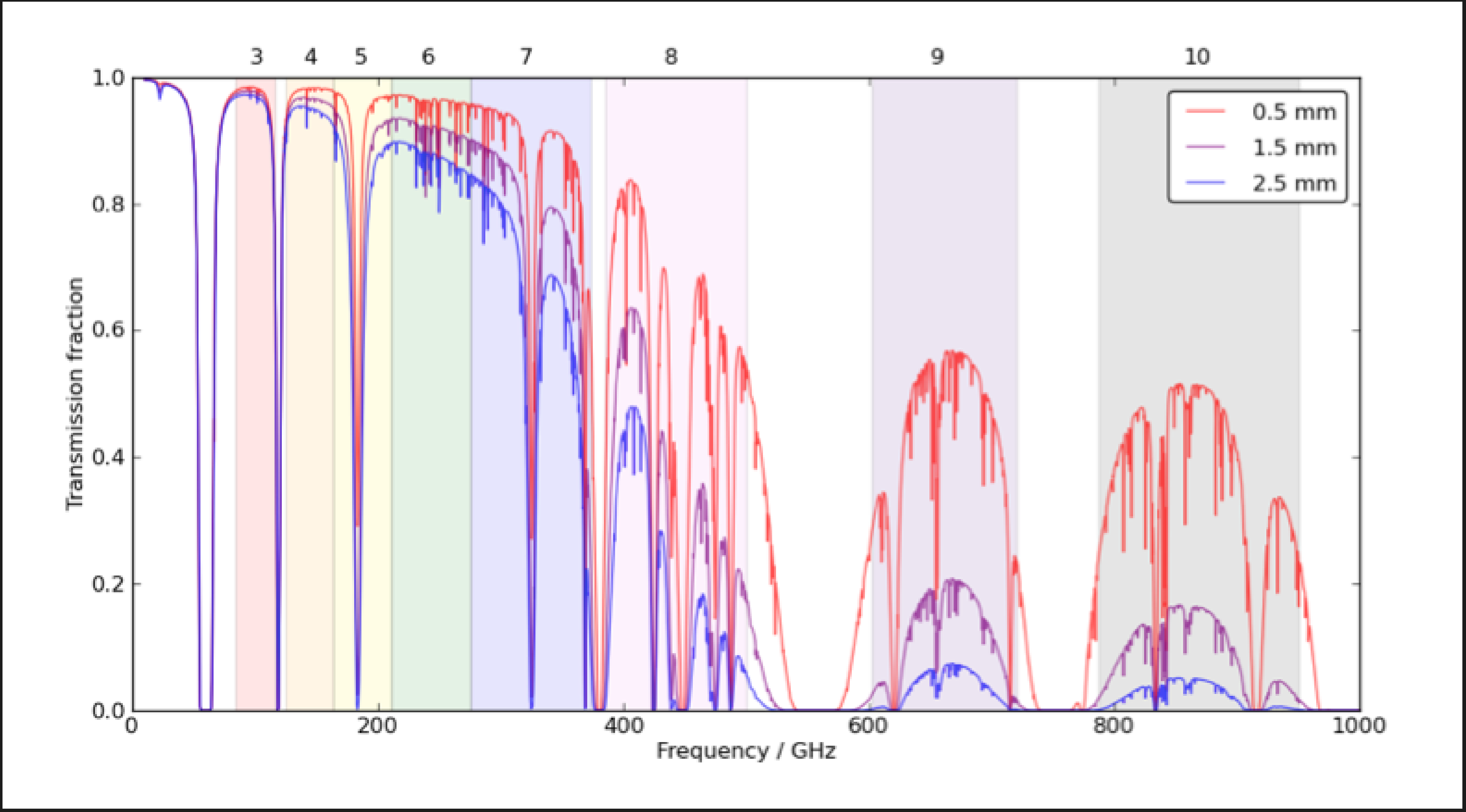
Why Radio

- Radio Waves reach the ground



Why Radio

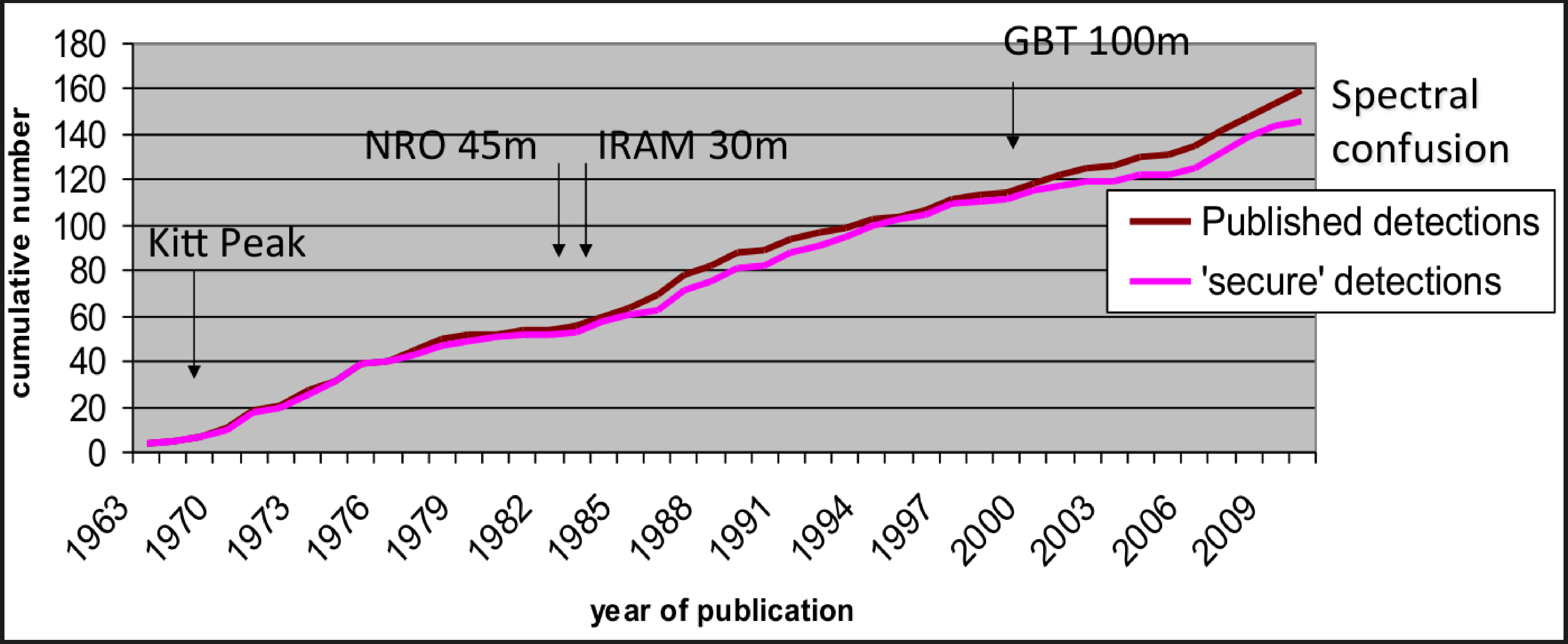
- Transmission of ALMA Band for three column densities of percipitable water vapour



Why Radio

Radio waves are produced in various processes:
- Molecular spectral lines (rotation - torsion)
- Synchroton
- Inverse Compton
- Continuum emission: free-free
Who Emits?
- Inter- stellar/galactic medium in various phases
- Matter in ionized, atomic, molecular state, dust grains, etc.

» **Molecules detected**

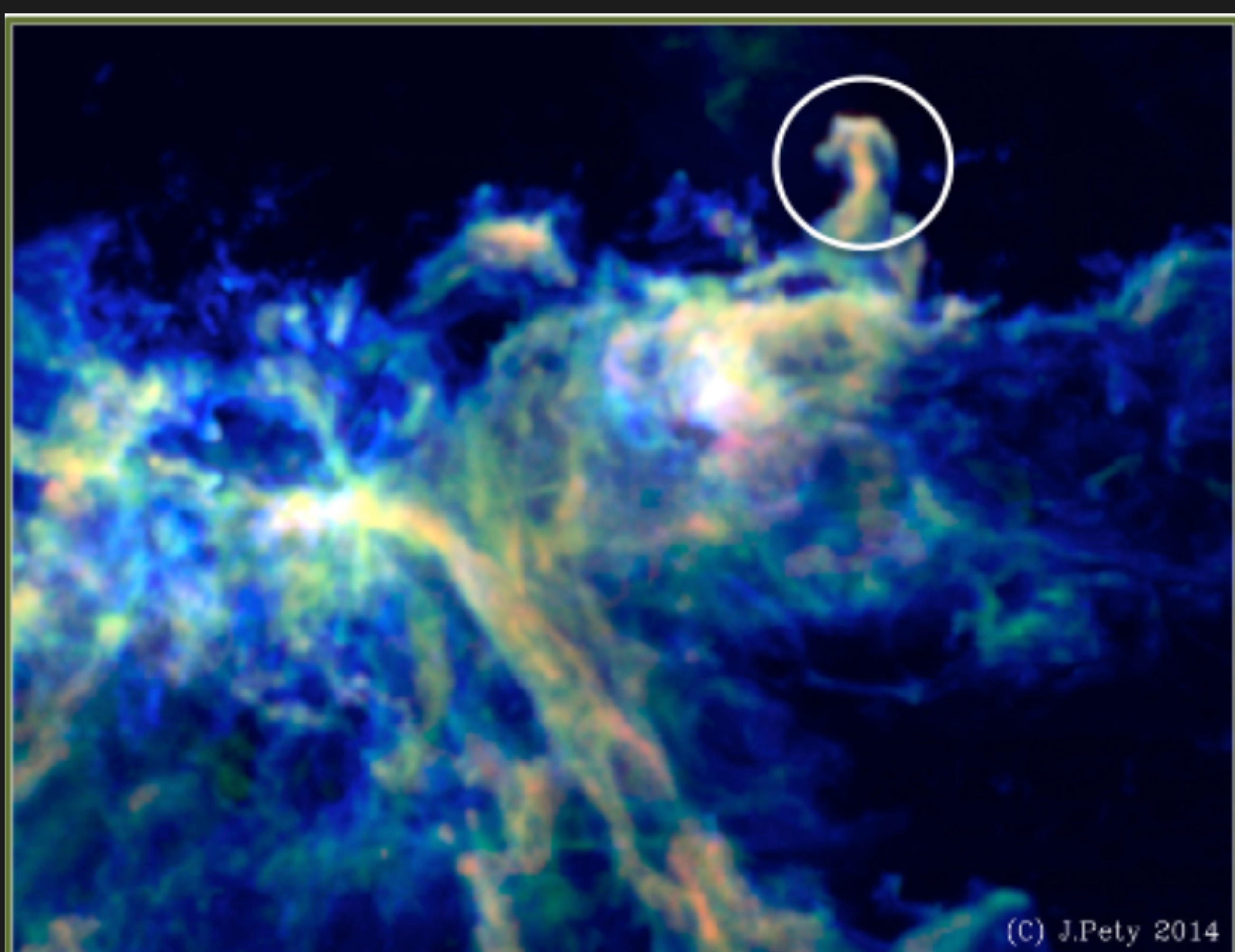


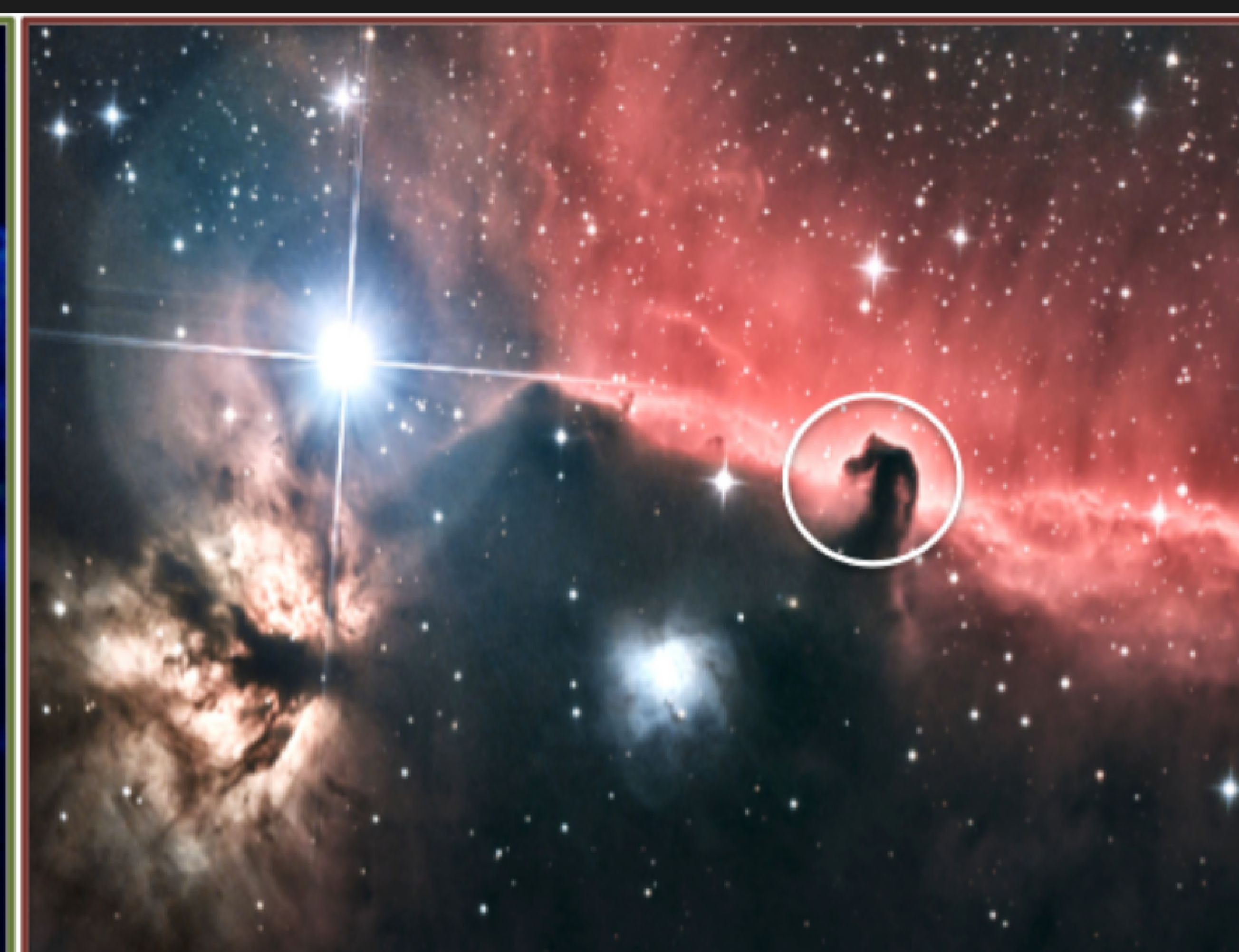
Why Radio

Another Important aspect is the complementarity of the radio wavelenghts with respect to the optical

- OPTICAL = hot matter at 10^3-10^4 K (star or HII ionized regions)
- MILLIMITER = cold matter at 10-100 K (dust/molecules)

Stars grow in cold matter, so radio frequencies are used to study star forming regions.



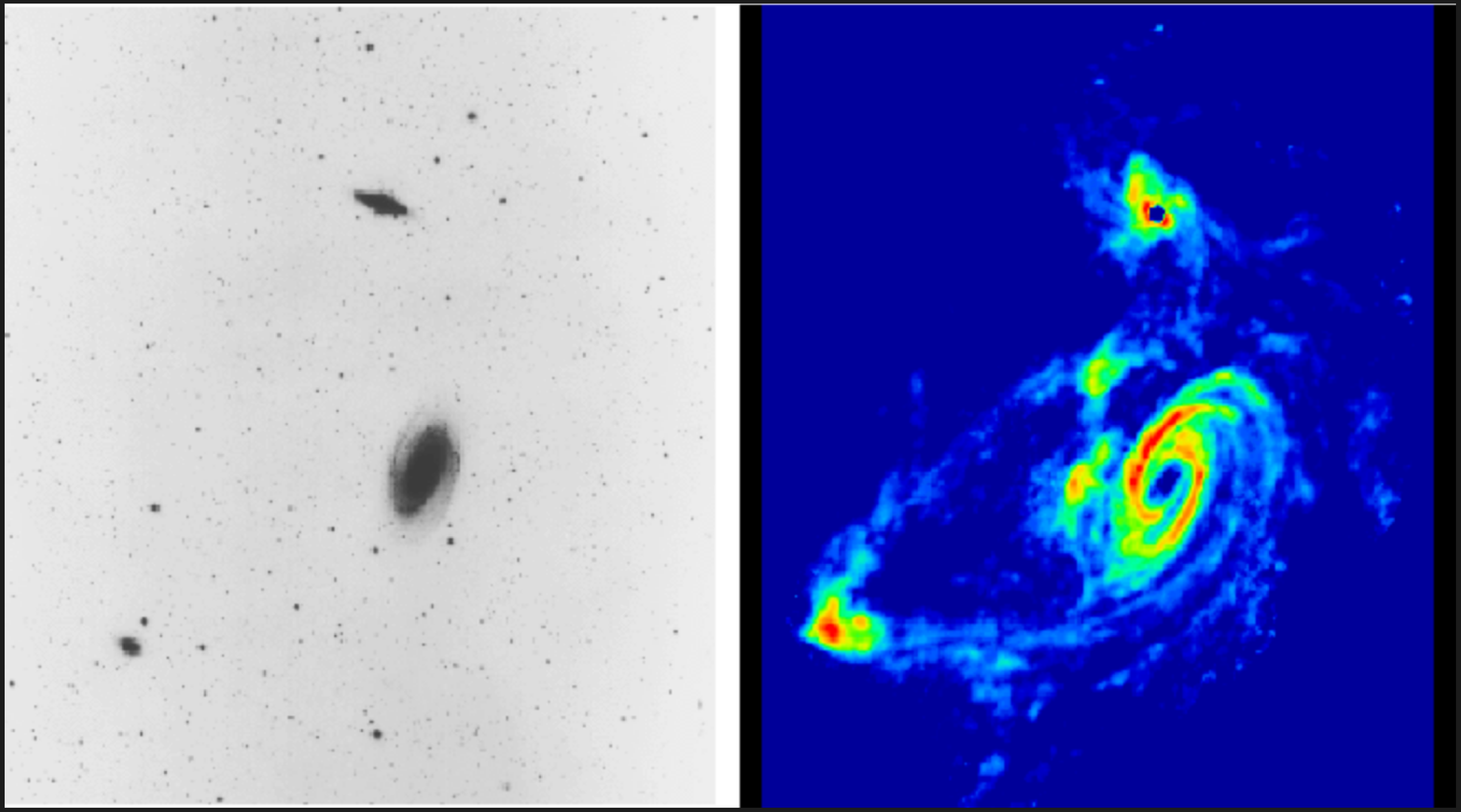


Why mm

Another Important aspect is the complementarity of the mm wavelenghts with respect to the optical

- OPTICAL = hot matter at 10^3-10^4 K (star or HII ionized regions)
- MILLIMITER = cold matter at 10-100 K (dust/molecules)

Stars are less affected than gas in galaxies interactions, so Radio is used to study for environmental effects (M81)



Why mm

Also important for non thermal emission (synchroton): violent Universe

Single Dish telescope

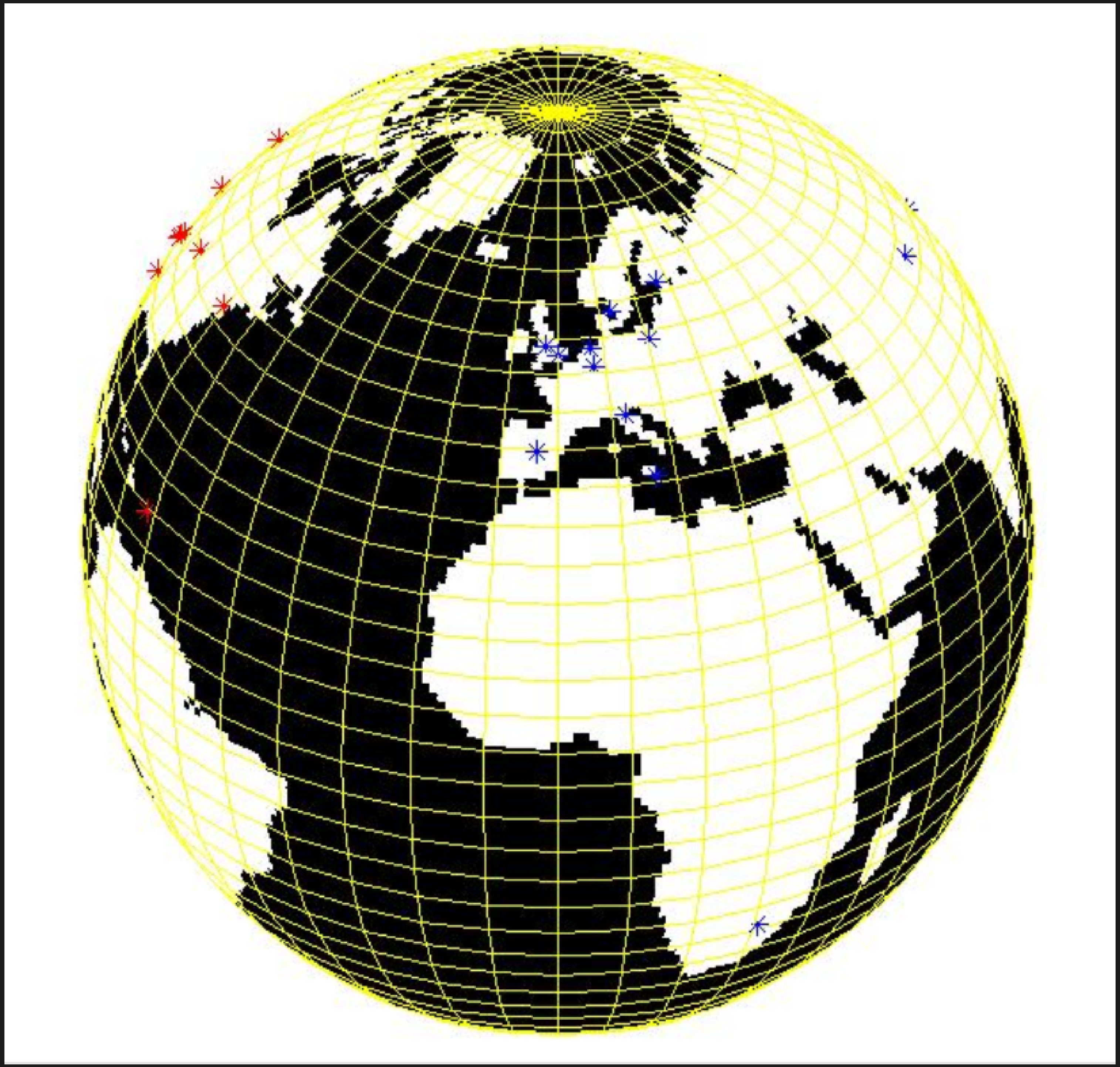
Two examples:
- Arecibo (306 m)
- GBT (100 m)

Interferometer

Two examples:
- ALMA
- VLA

Interferometer

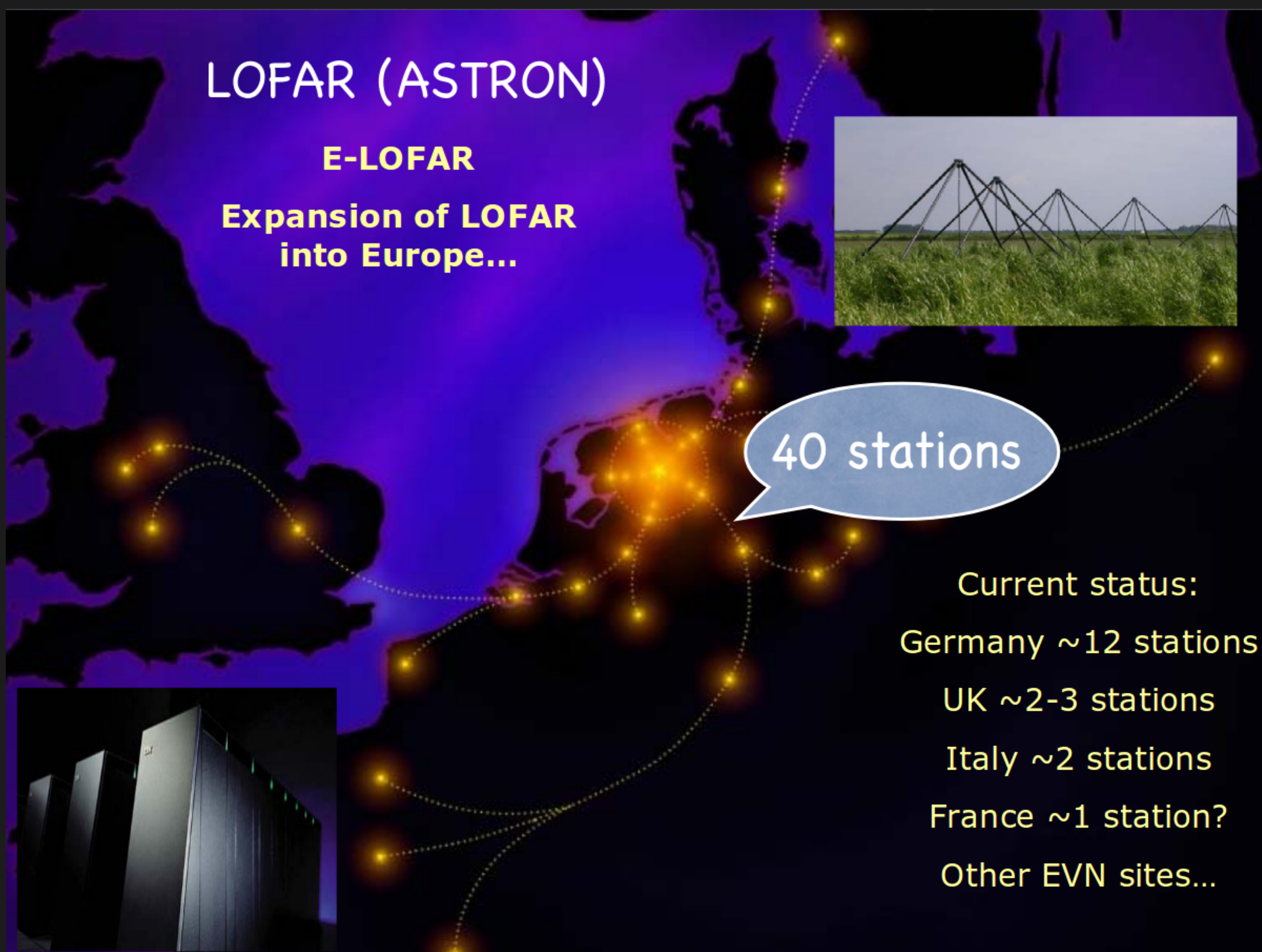
Special mention: VLBI
Make an interferometer the size of the Earth
– combine signals from antennas all around Earth
– Tape record data and combine later 'off-line' or send in real-time over internet

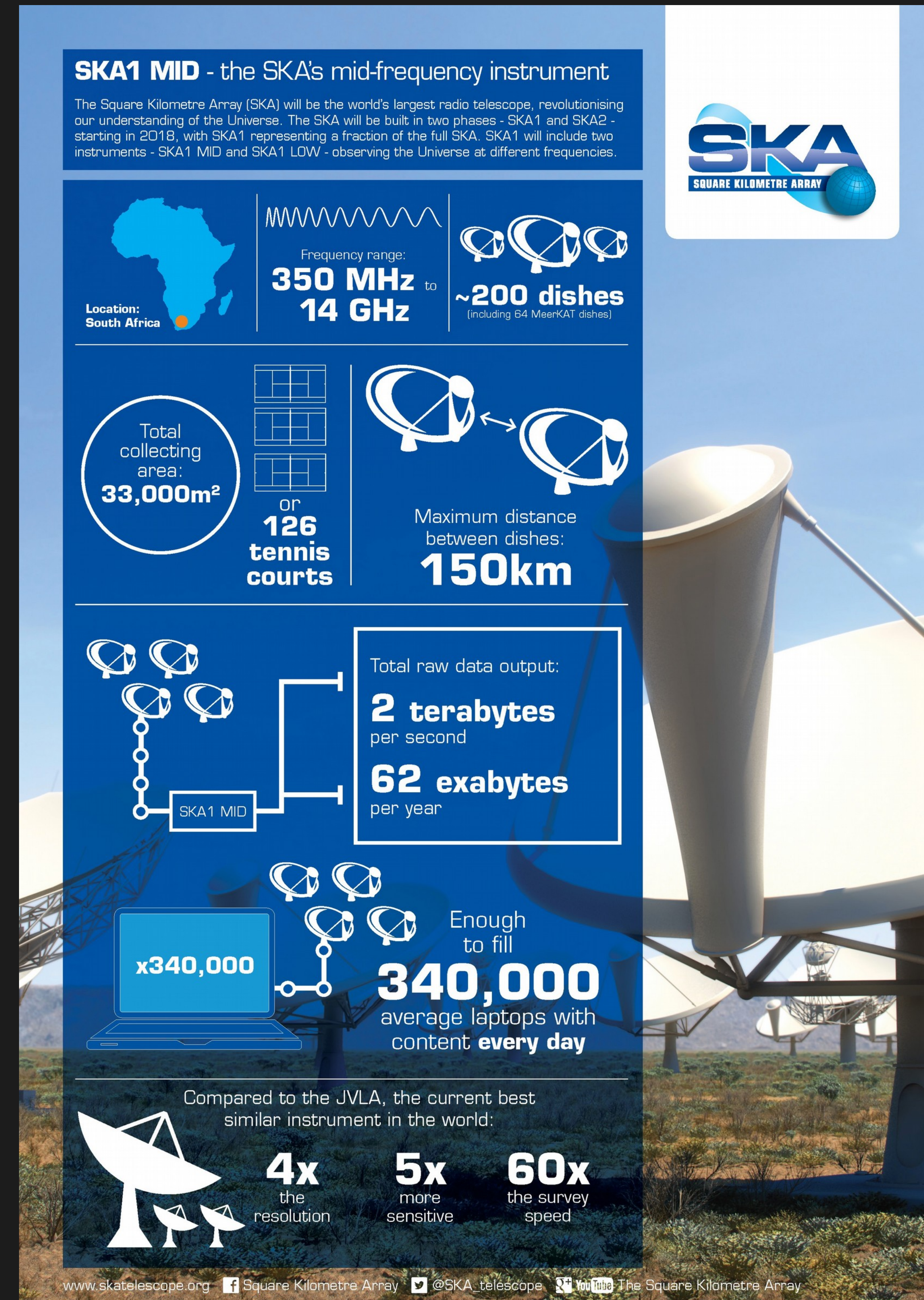


Future

Many telescopes are upgrading e-MERLIN, e-VLBI

New telescope growing MeerKat, LOFAR, ASKAP





Conclusions

IT ISA GOOD MOMENT TO DO RADIOASTRONOMY

Why we need interferometry?

The angular resolution of single dish radio telescope is given by:

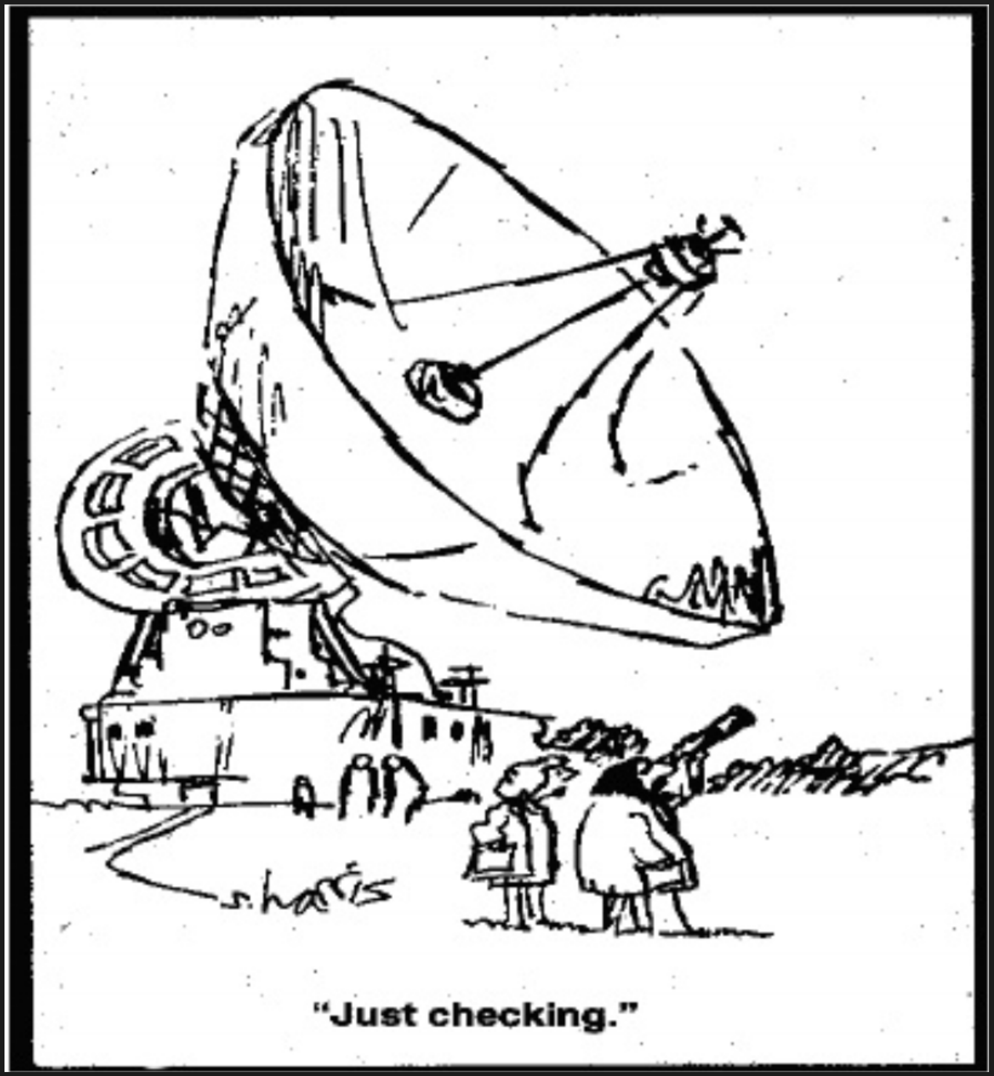
$\theta \sim \lambda/D$

where:

λ = wavelength of the signal received

D = telescope diameter

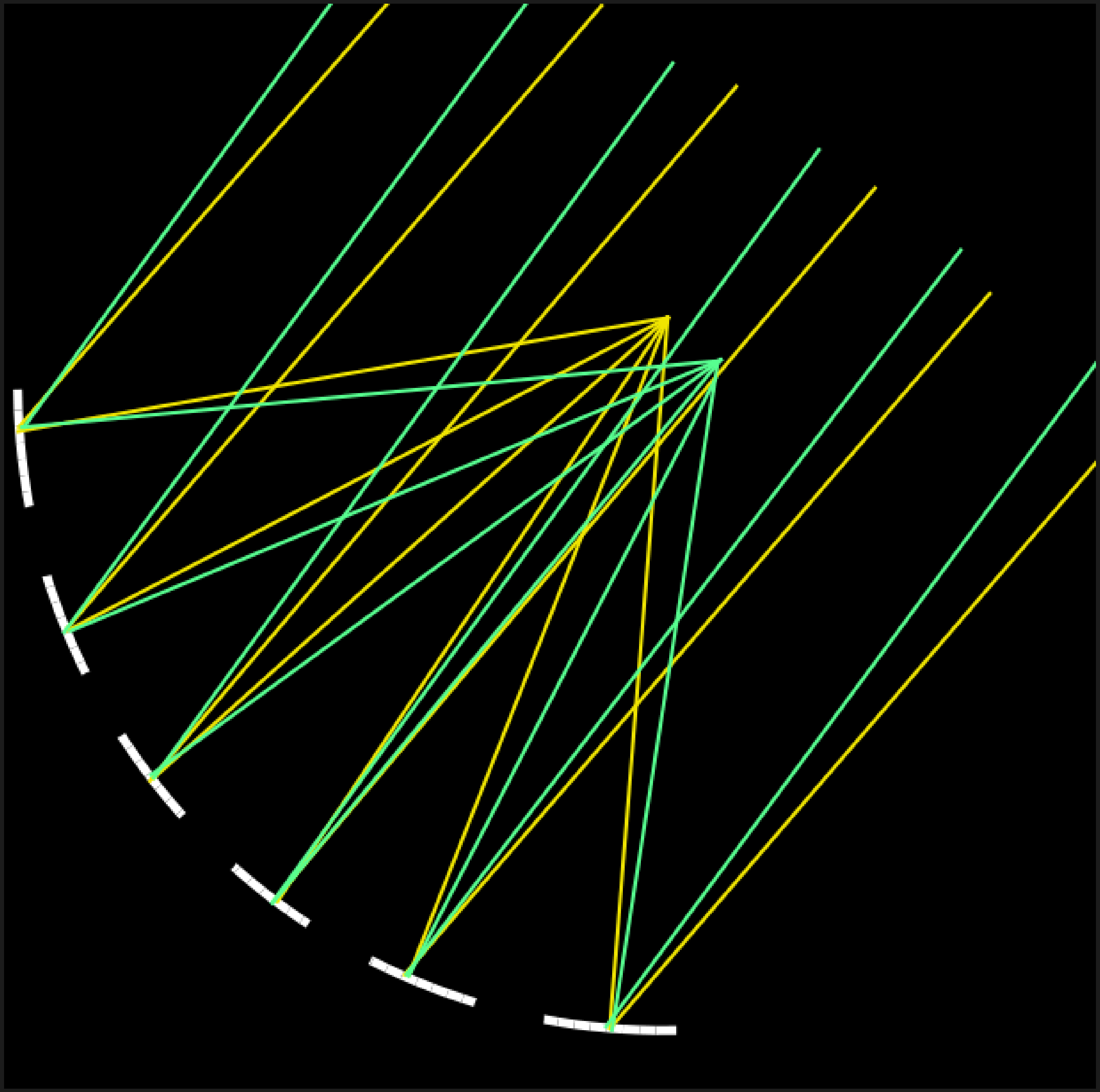
For example at 6 cm the Effelsberg telescope (D = 100 m) has a resolution of 2 arcmin....the human eye resolution is 1 arcmin



Interferometry

The idea is to use an array of separate telescopes working together as a wider single telescope.

SINGLE DISH



Interferometry

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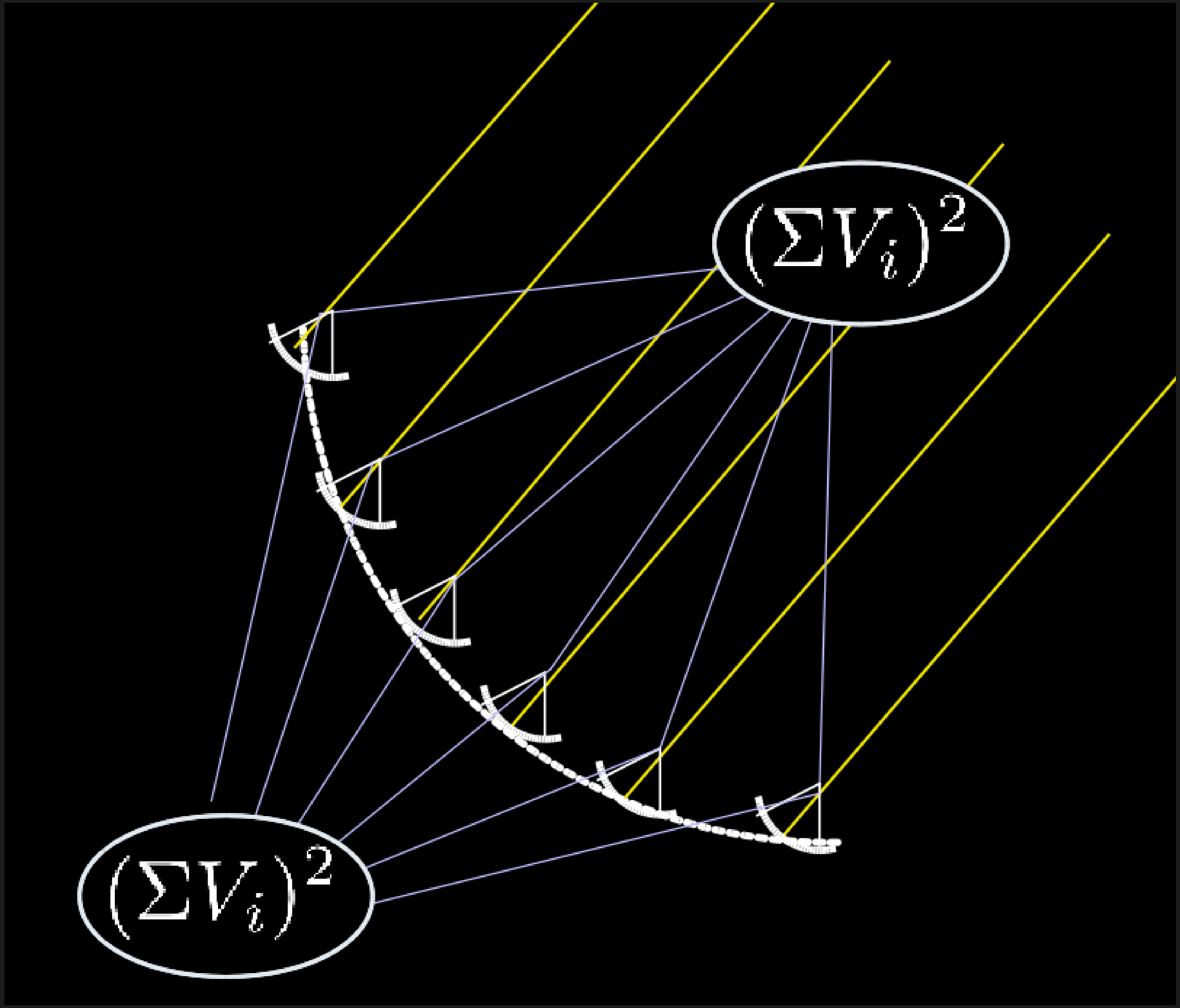
IDEAL INTERFEROMETER



Interferometry

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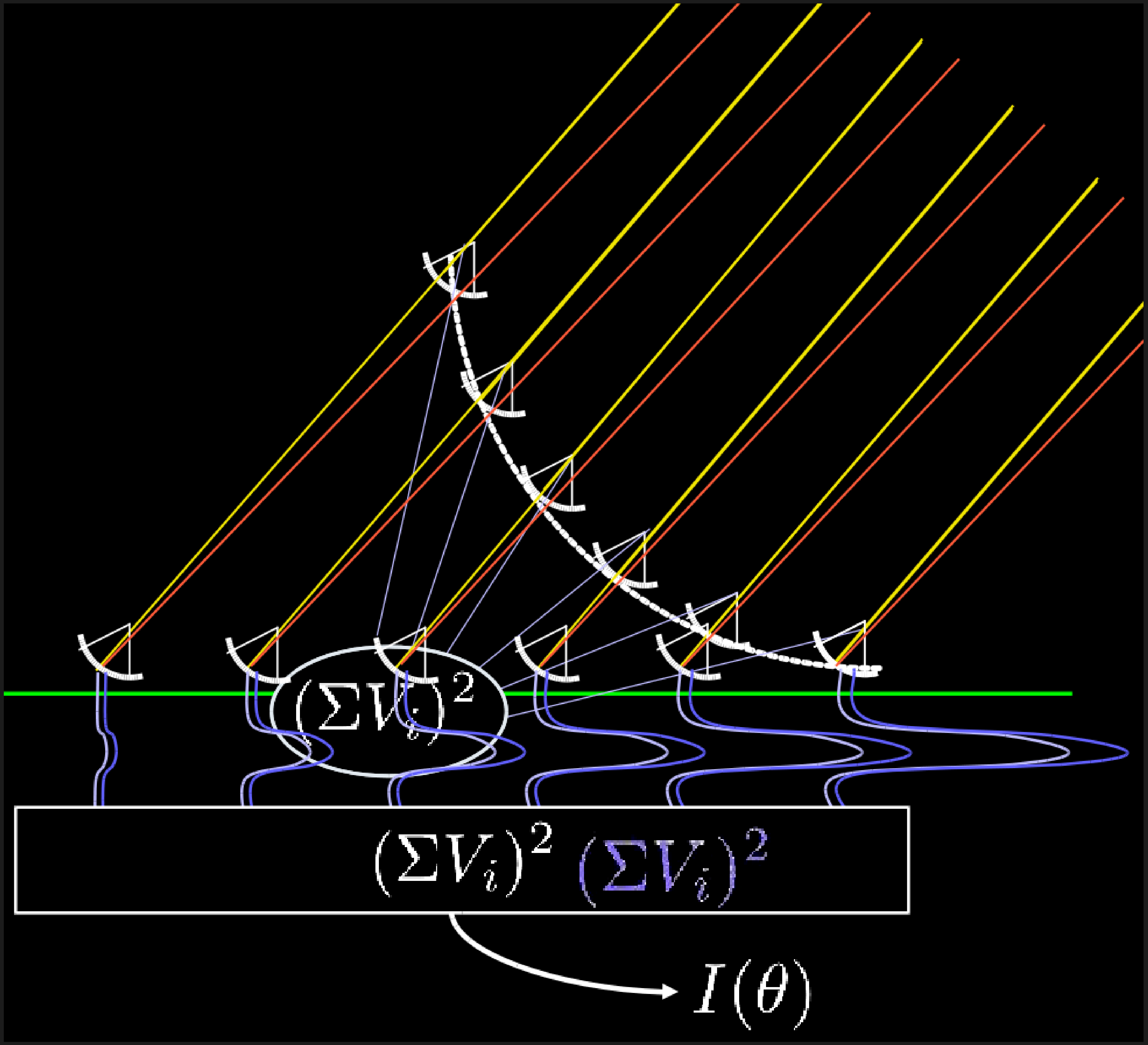
REAL INTERFEROMETER



Interferometry

The idea is to use an array of separate telescopes working together as a wider single telescope.
In this way the resolution increases up to:

$$\theta \sim \lambda / B_{MAX}$$

where:

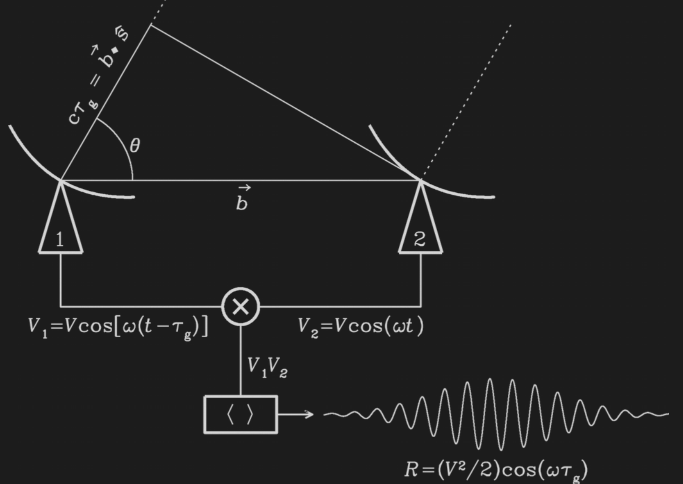
λ = wavelength of the signal received

B_{MAX} = maximum distance of two array telescopes

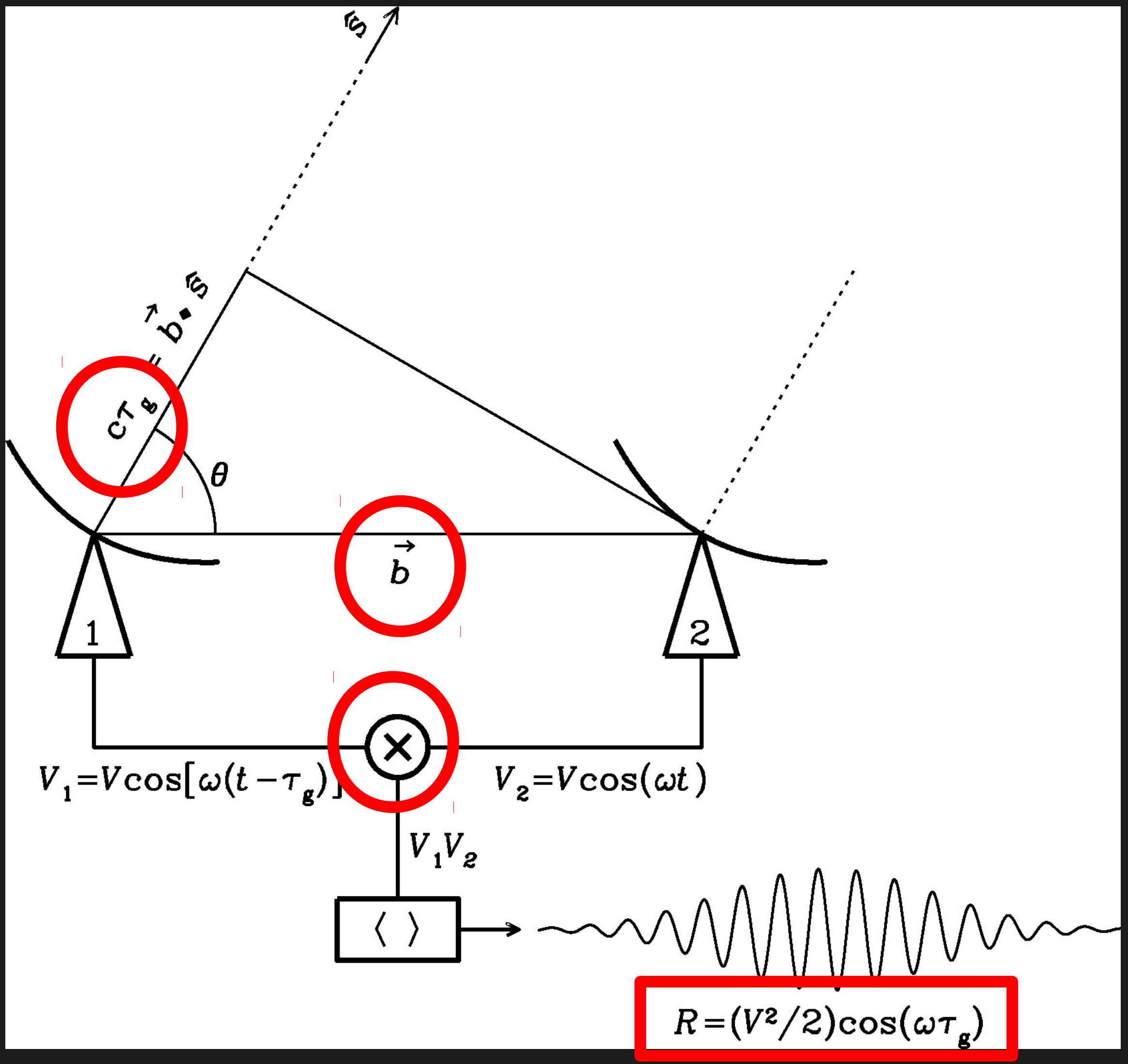
This process is possible by means of the interferometry technique.

A radio interferometer measures the coherence of the electric field between the 2 receiving elements.

Since the problem of N interferometers can be treated as a defined number of interferometer pairs, the basic of interferometry is to study the simplest case, a two-element narrowband interferometer.



Interferometry



2 identical dishes separated by a distance b, called ***BASELINE****.*

Plane waves from the space reach the two antennae at different time.
The output of the voltage measured by the two antenna is the same, with a lags of τ_g, called ***GEOMETRIC DELAY****.*
e.g. a geometrical delay of about 1 millisecond is expected for B ~ 300 km

The ***CORRELATOR*** *multiplies the two voltages and averaging in time gives:*

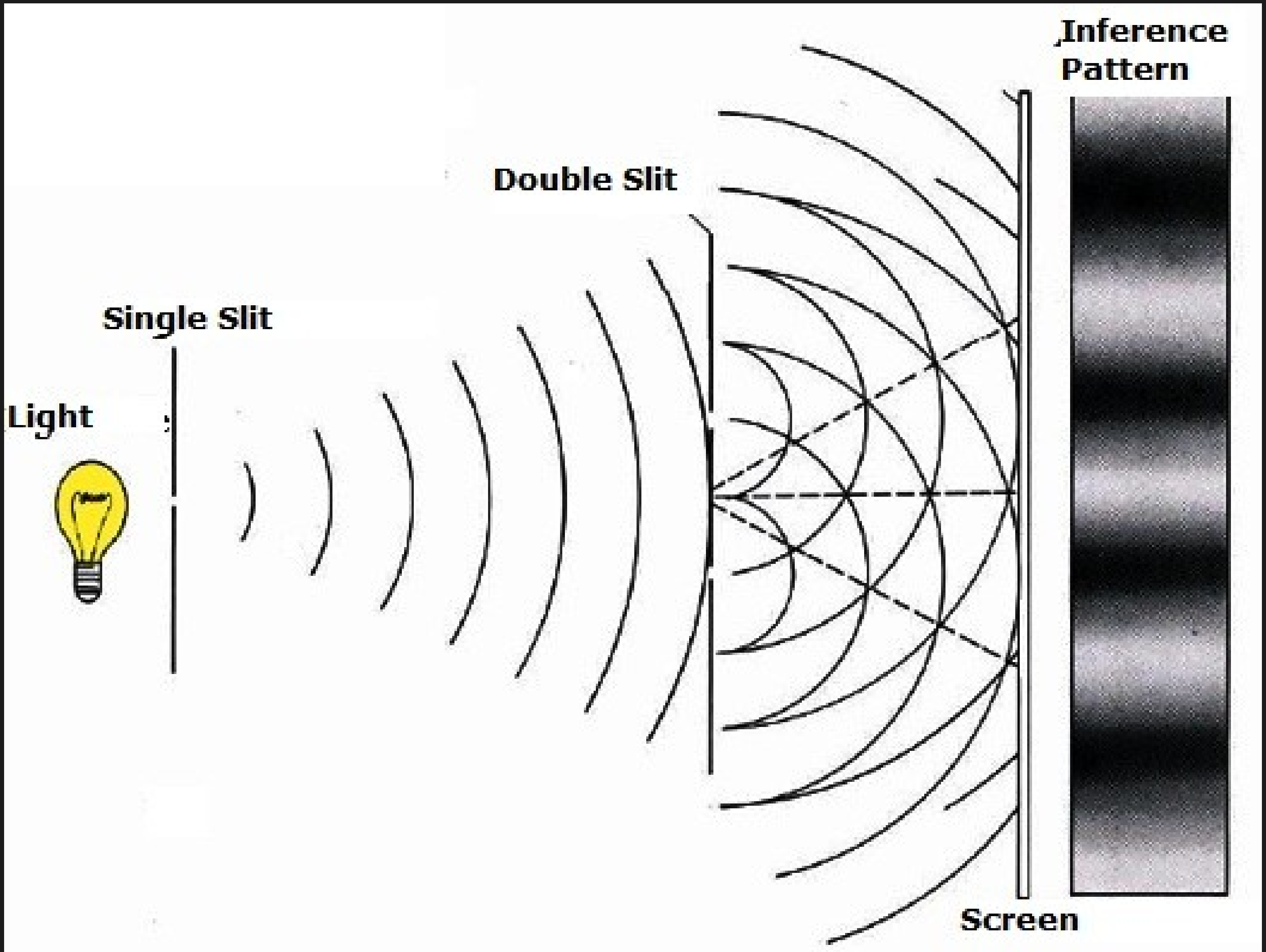
$$R = (V^2/2)\cos(\omega\tau_g)$$

i.e. the correlator output depends on the voltages and the geometric delay. This function R varies sinusoidally with the change of source direction in the interferometers frame.

Interferometry

These sinusoids are called ***FRINGES*** *where V2/2 is the amplitude (proportional to the point source flux density S), and ωt is the phase, depending on the source position.*

This is similar to what happens in Young's experiment with double slits:



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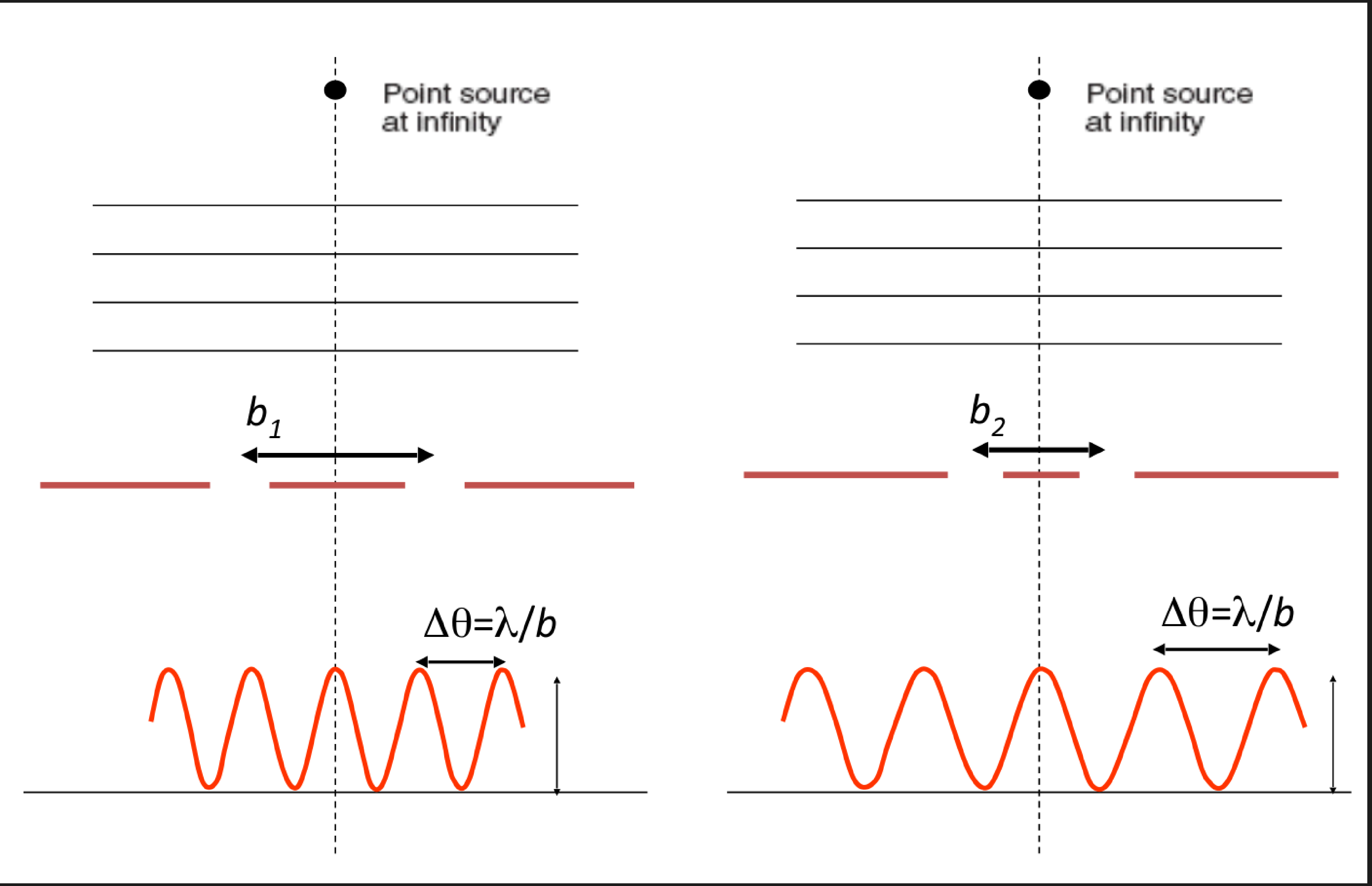

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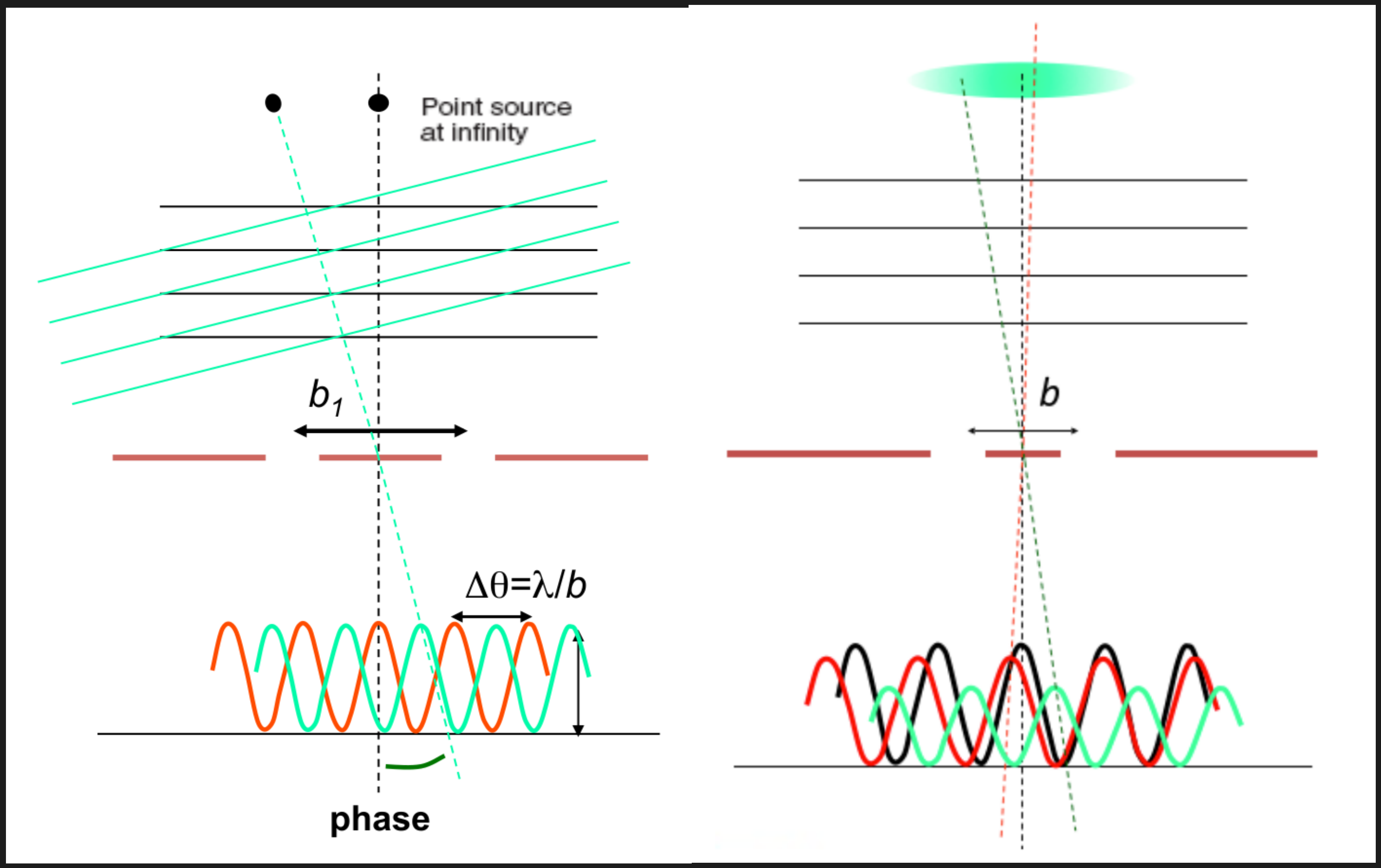
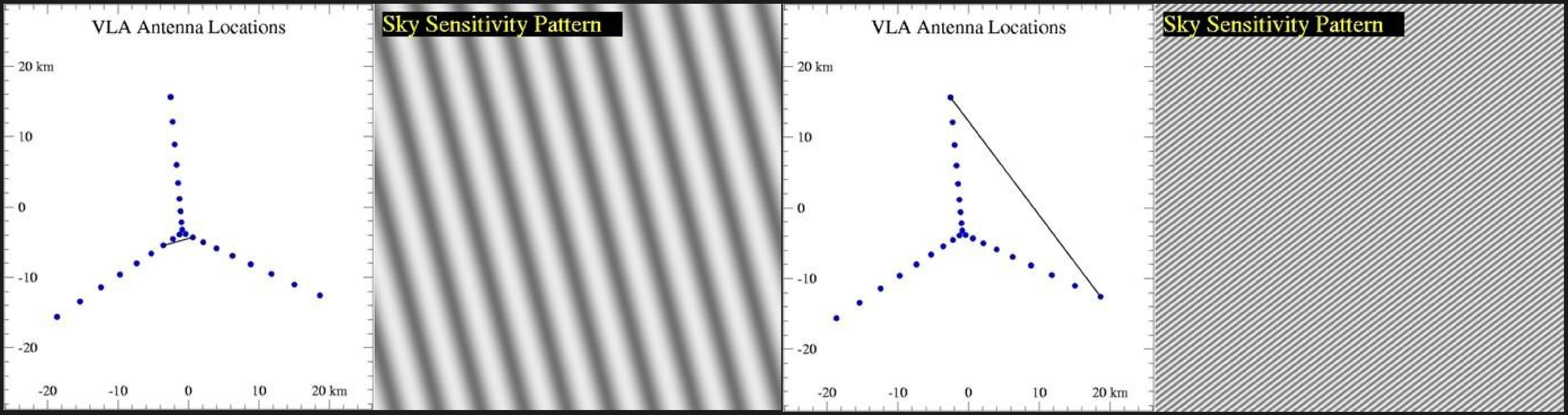


Interferometry

These sinusoids are called ***FRINGES*** *where V2/2 is the amplitude (proportional to the point source flux density S), and ωt is the phase, depending on the source position.*

If the antennae were isotropic the response function would be a sinusoids spanning the sky, and sensitive to only the Fourier component of the brightness with angular period $\lambda/(b \sin\theta)$.
A baseline samples only specific spatial frequencies components of the signal emitted: $m\lambda/(b \sin\theta)$ (m is an integer number)



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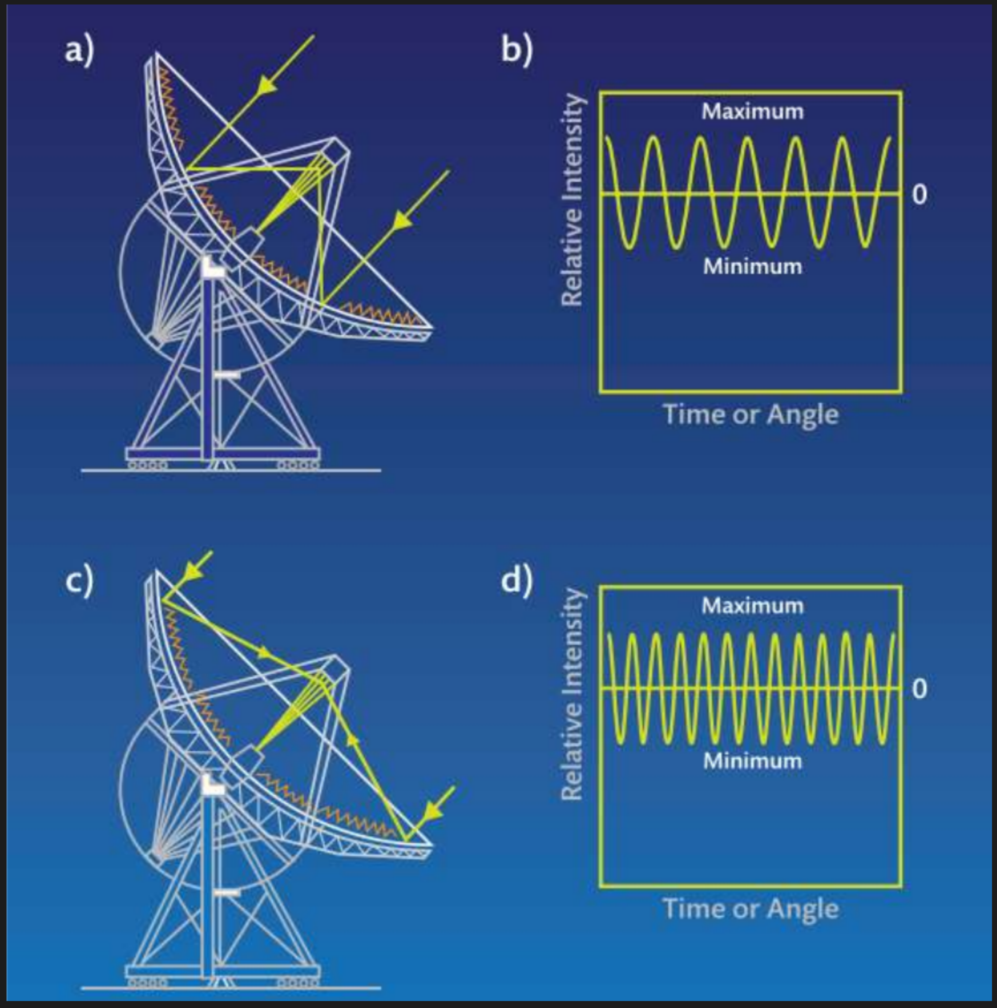


Foreword before the Visibility

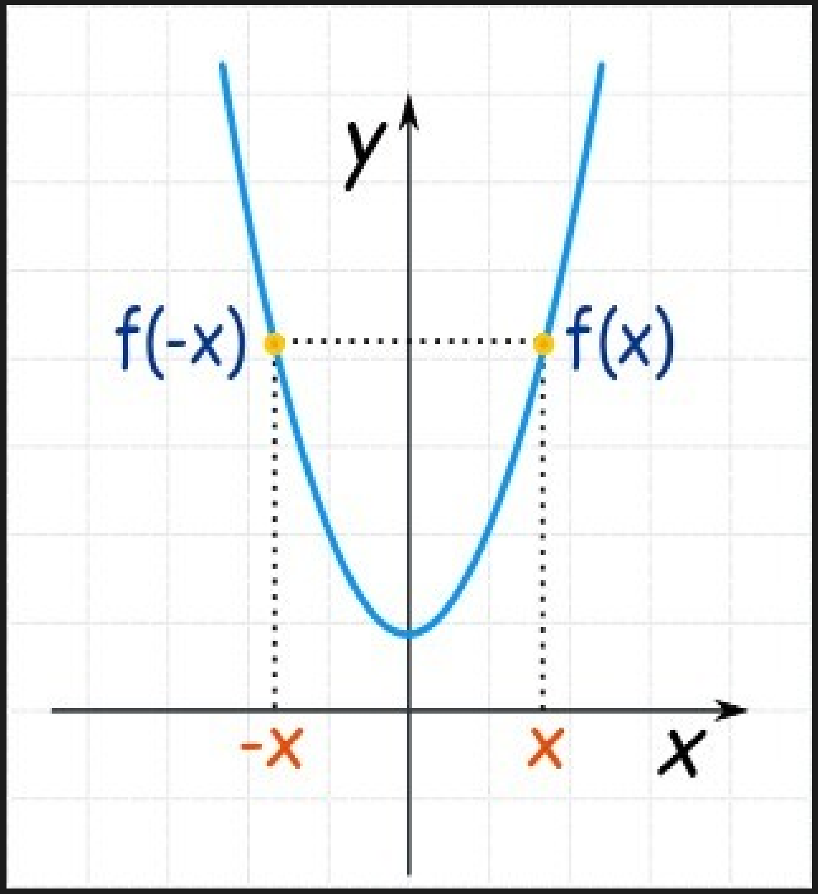
Any real function can be decomposed into an even and odd part:

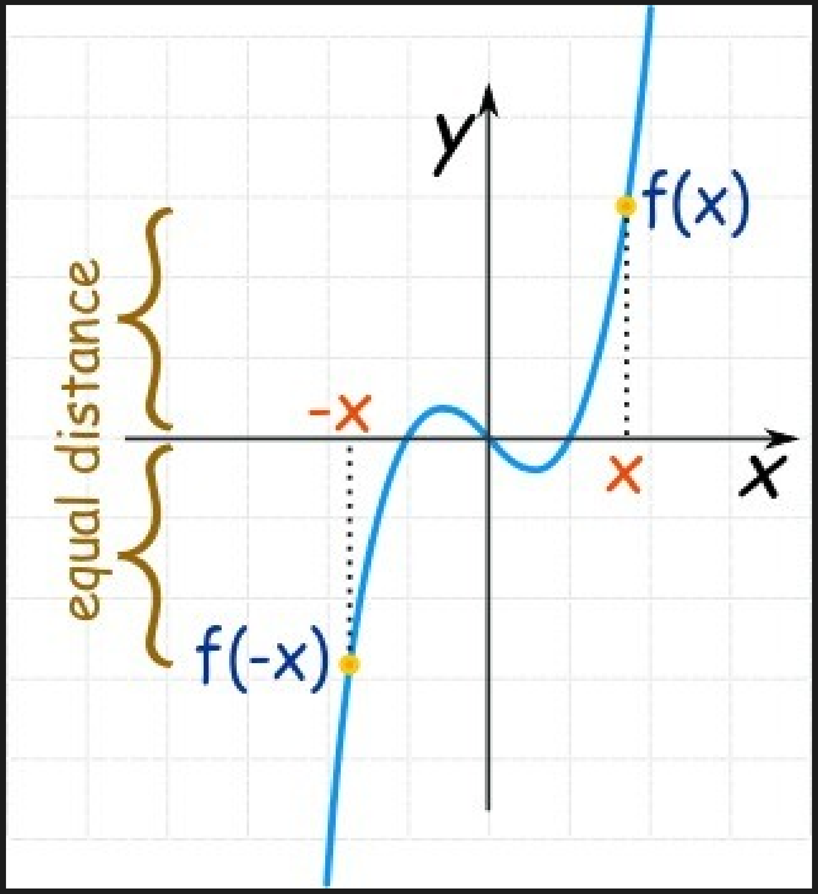
$$I(x,y) = I_E(x,y) + I_O(x,y)$$

Such that:

$I_E(-x,-y) = I_E(x,y) \rightarrow$ *symmetric function*

$I_O(-x,-y) = -I_O(x,y) \rightarrow$ *antysimmetric function*





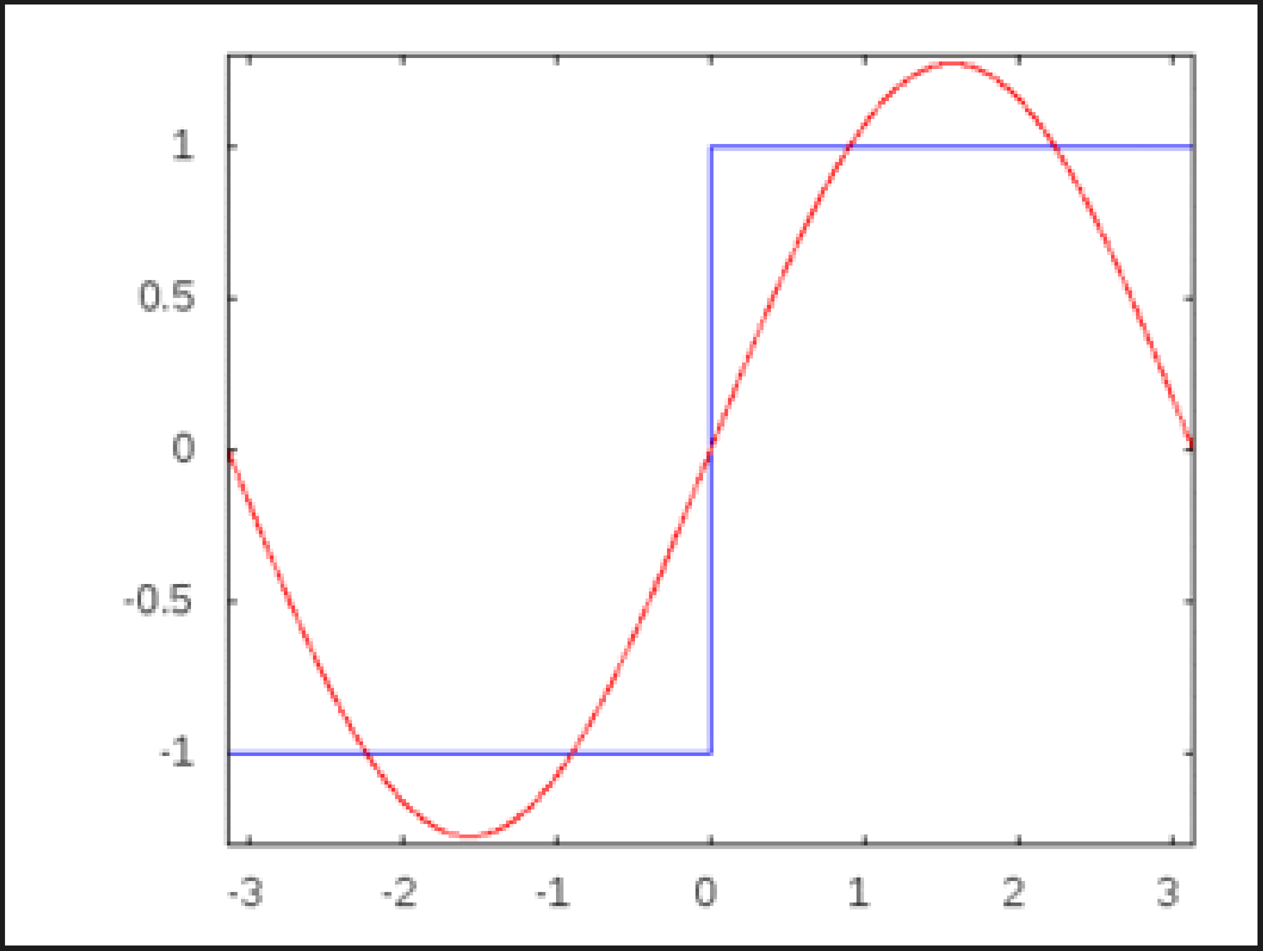
Foreword before the Visibility

Fourier theorem states that any periodic function (of perdio P) or periodic signal can be decomposed into a sum of simple oscillating functions, namely sines and cosines, expressed with:

$$f(x) = \sum [A \cos(2\pi n x/P) + B \sin(2\pi n x/P)]$$

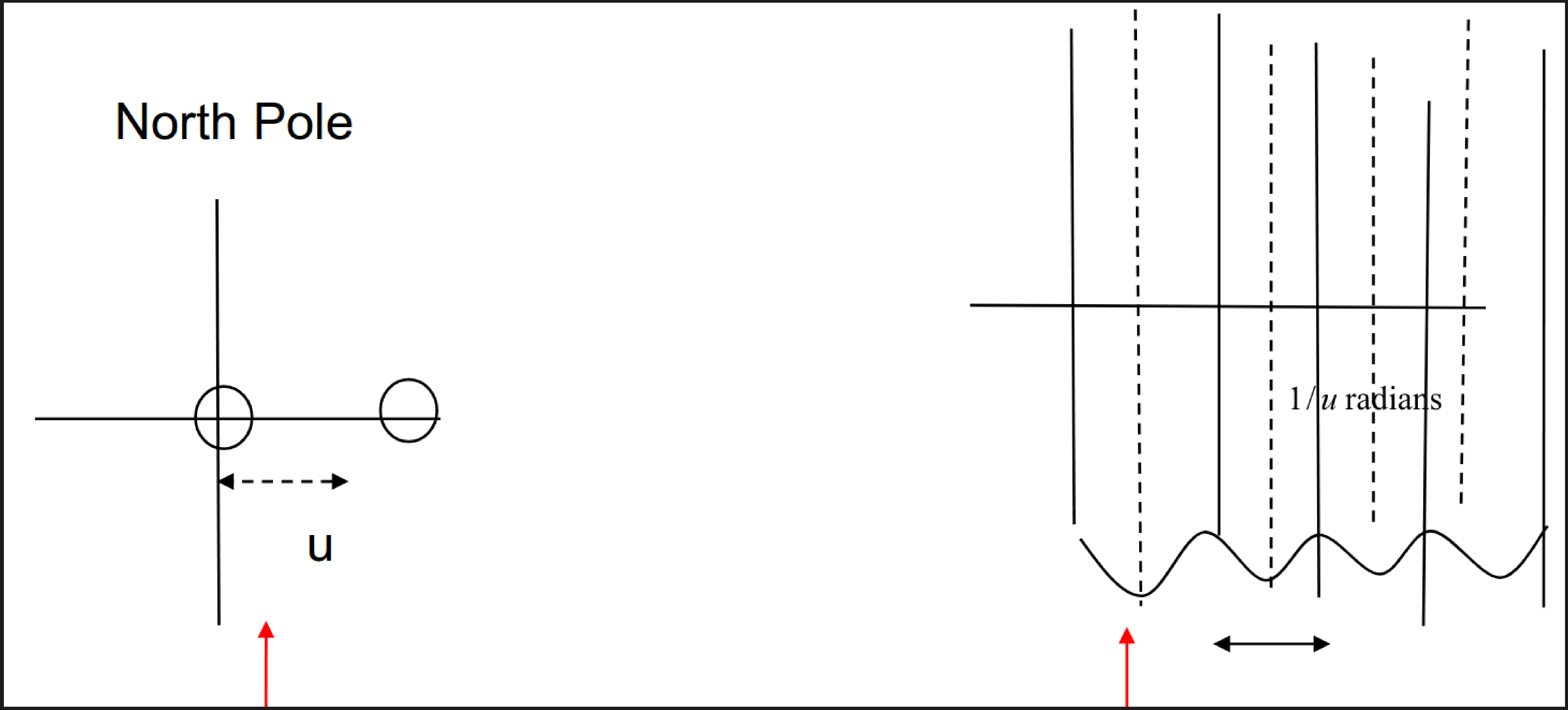
or in complex notation

$$f(x) = \sum C\, e^{i(2\pi n x/P)}$$



Visibility

We can treat extended sources as the sum of independent point sources.
However, with ‘cosine’ correlator we are sensitive only to the even part of the intensity emitted.



Antennae Position

Response Function (Fringes)

Visibility

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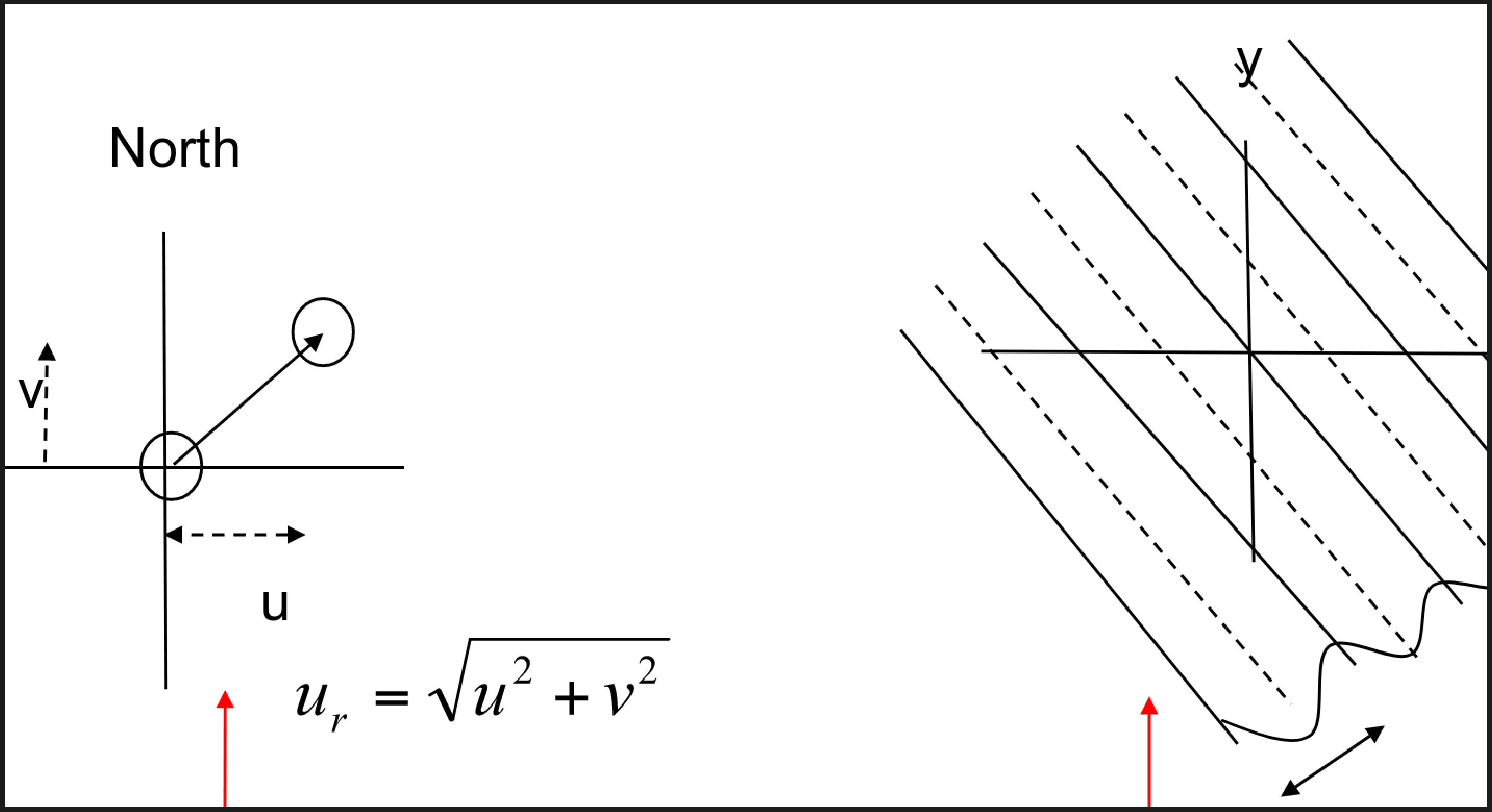


$$u_r = \sqrt{u^2 + v^2}$$

Antennae Position

Response Function (Fringes)

Visibility

We can treat extended sources as the sum of independent point sources.
However, with 'cosine' correlator we are sensitive only to the even part of the intensity emitted.
To detect the odd part we need a 'sine' correlator, usually implemented using a second correlator with a 90 deg phase delay inserted in the output on an antenna.
The combination of 'cosine' and 'sine' correlators is called 'complex' correlator, because using the correlation:

$$e^{i\phi} = \cos(\phi) + i\sin(\phi)$$

we can write the total answer of the complex correlators as:

$$V = R_{cosine} - iR_{sine}$$

V *is the* ***COMPLEX VISIBILITY****, can be written as:*

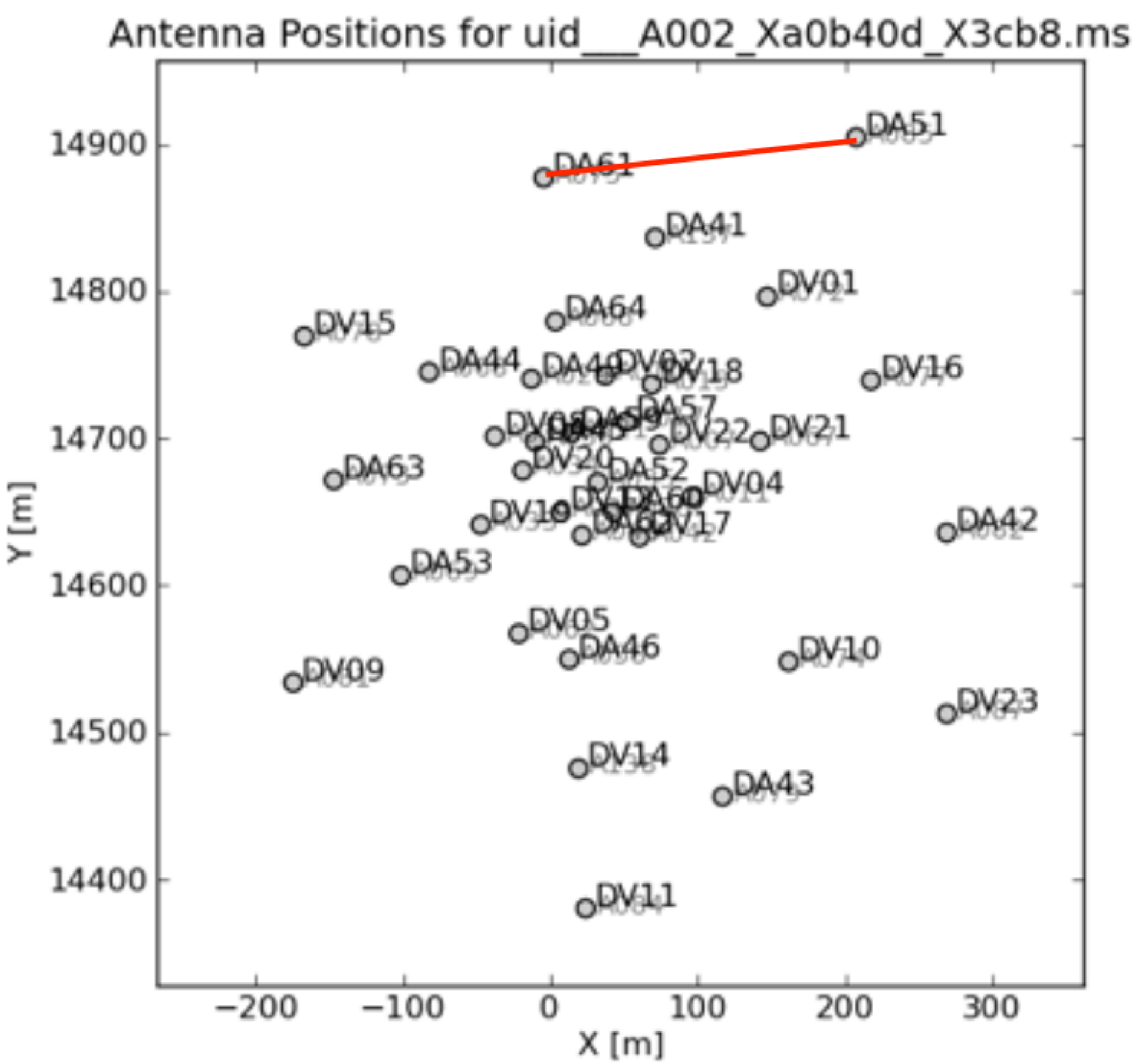
$$V = Ae^{-i\phi}$$

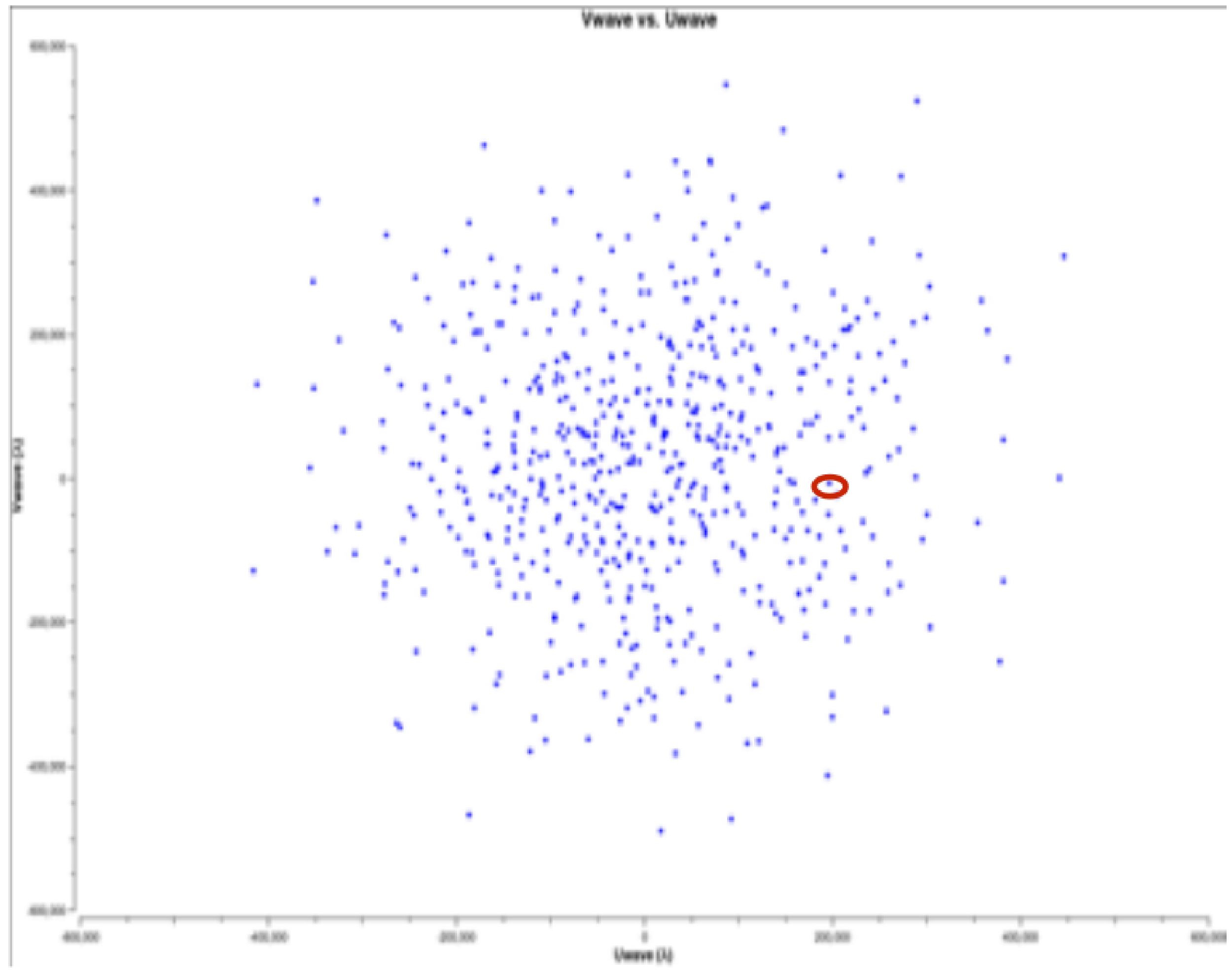
A = AMPLITUDE *and* ***ϕ = PHASE***

- The phase is the offset of the filter that maximizes the total transmitted intensity
- The amplitude is the value of this maximal transmitted intensity

Visibility

*The **COMPLEX VISIBILITY** in the uv plane. Each antenna pairs has its own point in the uv plane, which represents the separation between each pair as seen from the source*



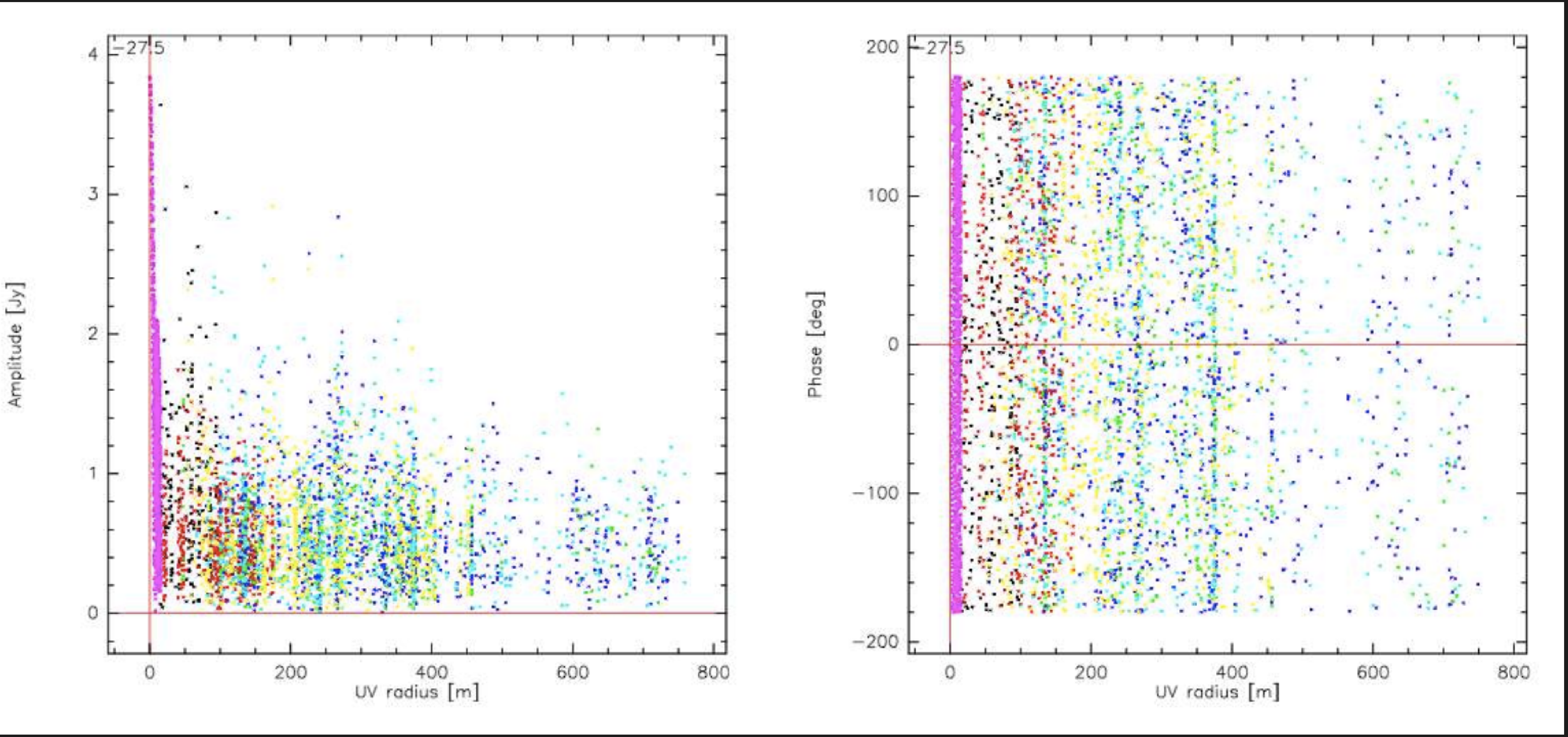


In this plane each visibility is the integral of the sky brightness T(x,y) in the Fourier space:

$$V(u,v) = \iint T(x,y)\, e^{2\pi i (ux+vy)}\, dx\, dy$$

Visibility

In conclusion the interferometer output is a set ot visibility measurements, done for each baseline



This approach introduces new definition of an interferometer:

A radio interferometer is an instrument that samples the visibility function, which is the Fourier transform of the sky brightness distribution

Visibility and the sky brightness

The possibility to convert visibilites into sky brightness measurements is given by the van Cittert-Zernike Theorem:

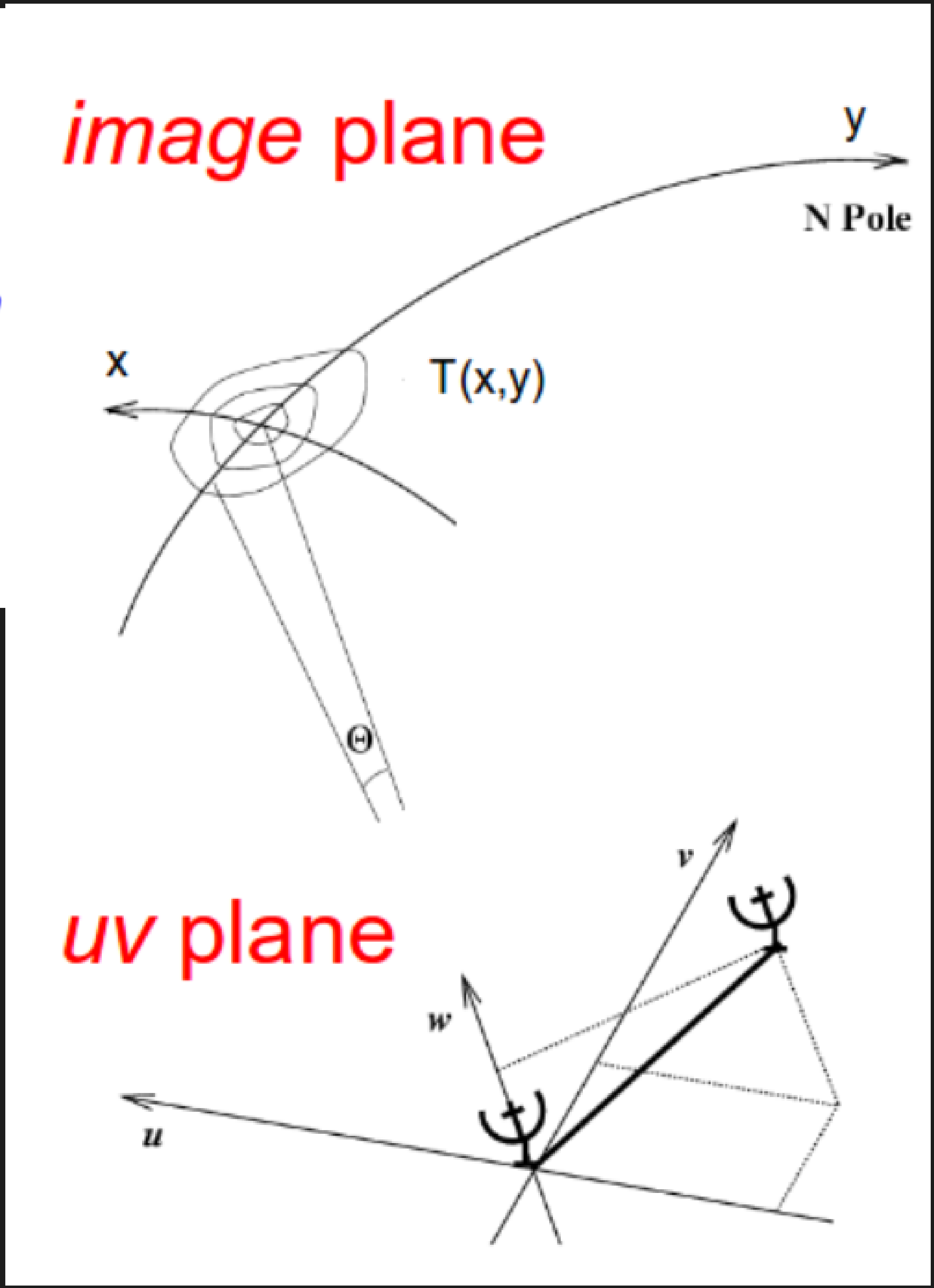
Fourier space/domain

$$V(u,v) = \int\int T(x,y) e^{2\pi i(ux+vy)} dx dy$$

$$T(x,y) = \int\int V(u,v) e^{-2\pi i(ux+vy)} du dv$$

Image space/domain

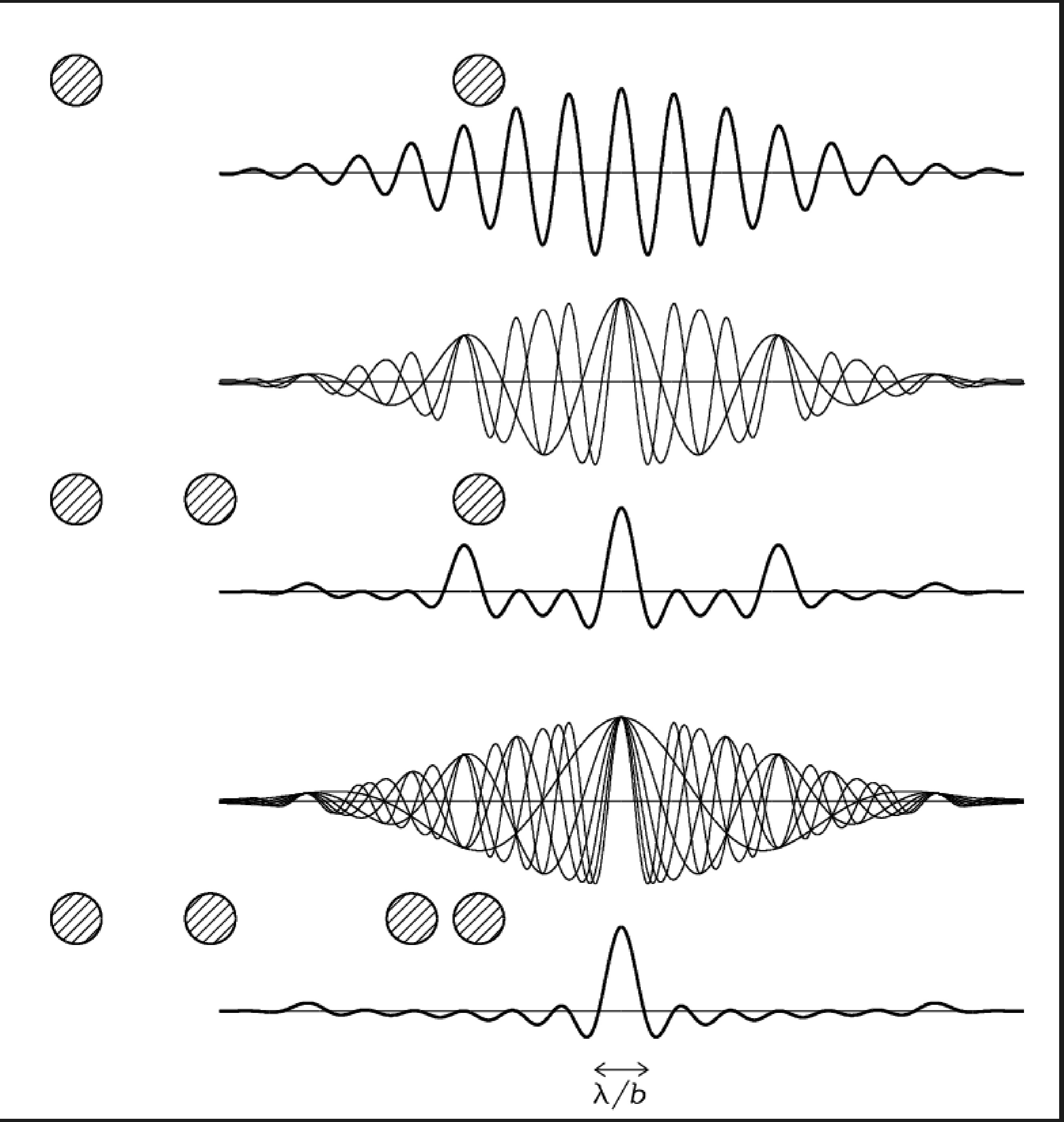
$$V(u,v) \rightleftharpoons T(x,y)$$



The visibility V(u,v) is the 2D Fourier transform of the sky brightness T(x,y). T(x,y) is the inverse Fourier transform of V(u,v).
But in reality we have finite sampling of the uv plane → T(x,y) must be reconstructed. The inverse Fourier transform of the actual finite set of V gives the ***DIRTY IMAGE***

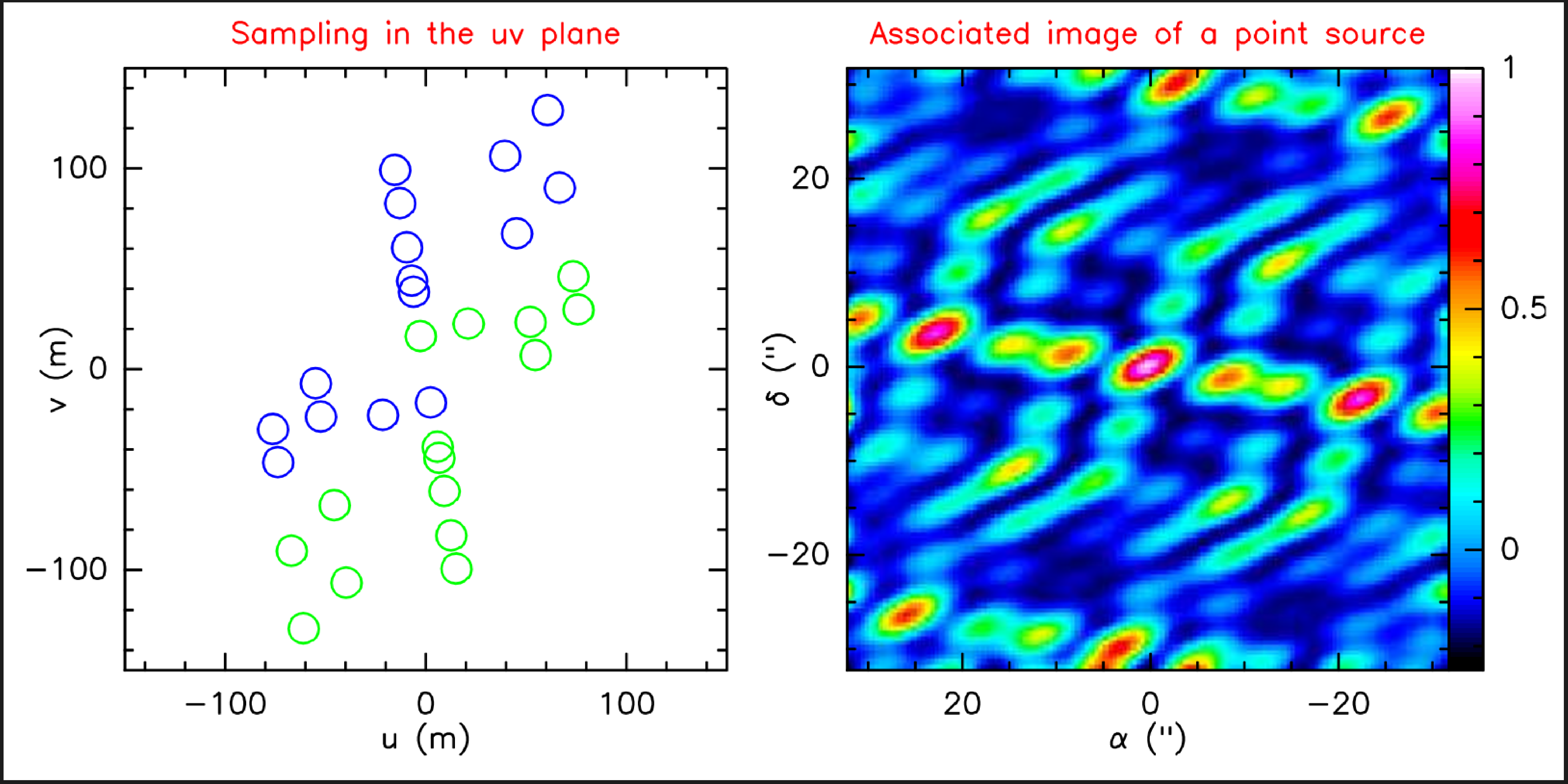
Aperture synthesis

To increase the point-source response we needs more Fourier components. Taking advantages of the earth rotation we have more interferometers, and we approach to a Gaussian, that correspond to the ***SYNTHESIZED BEAMS***



Aperture synthesis

As the hearts rotate the uv sampling increases, and the image of the point-source observed becomes a gaussian



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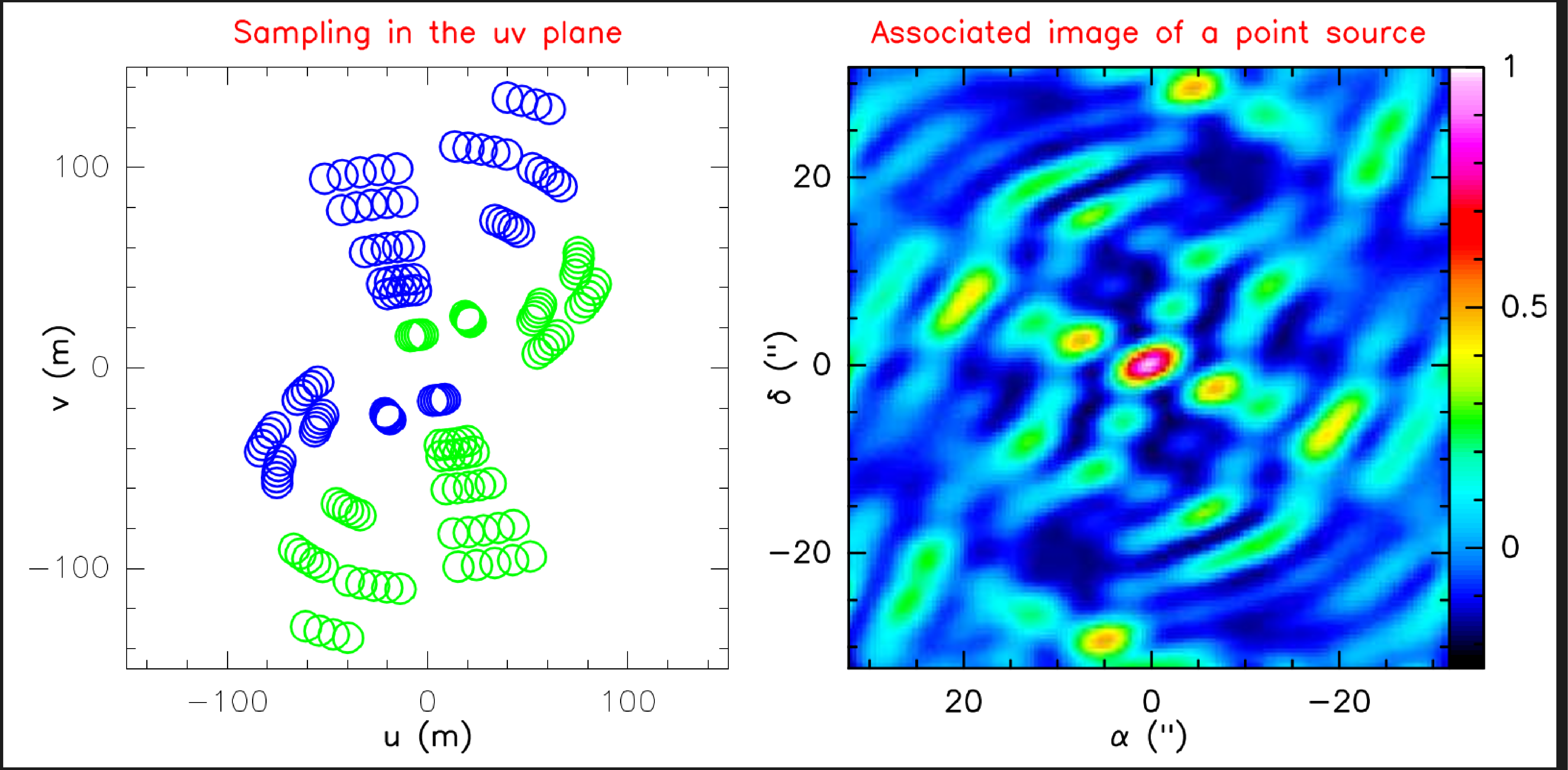

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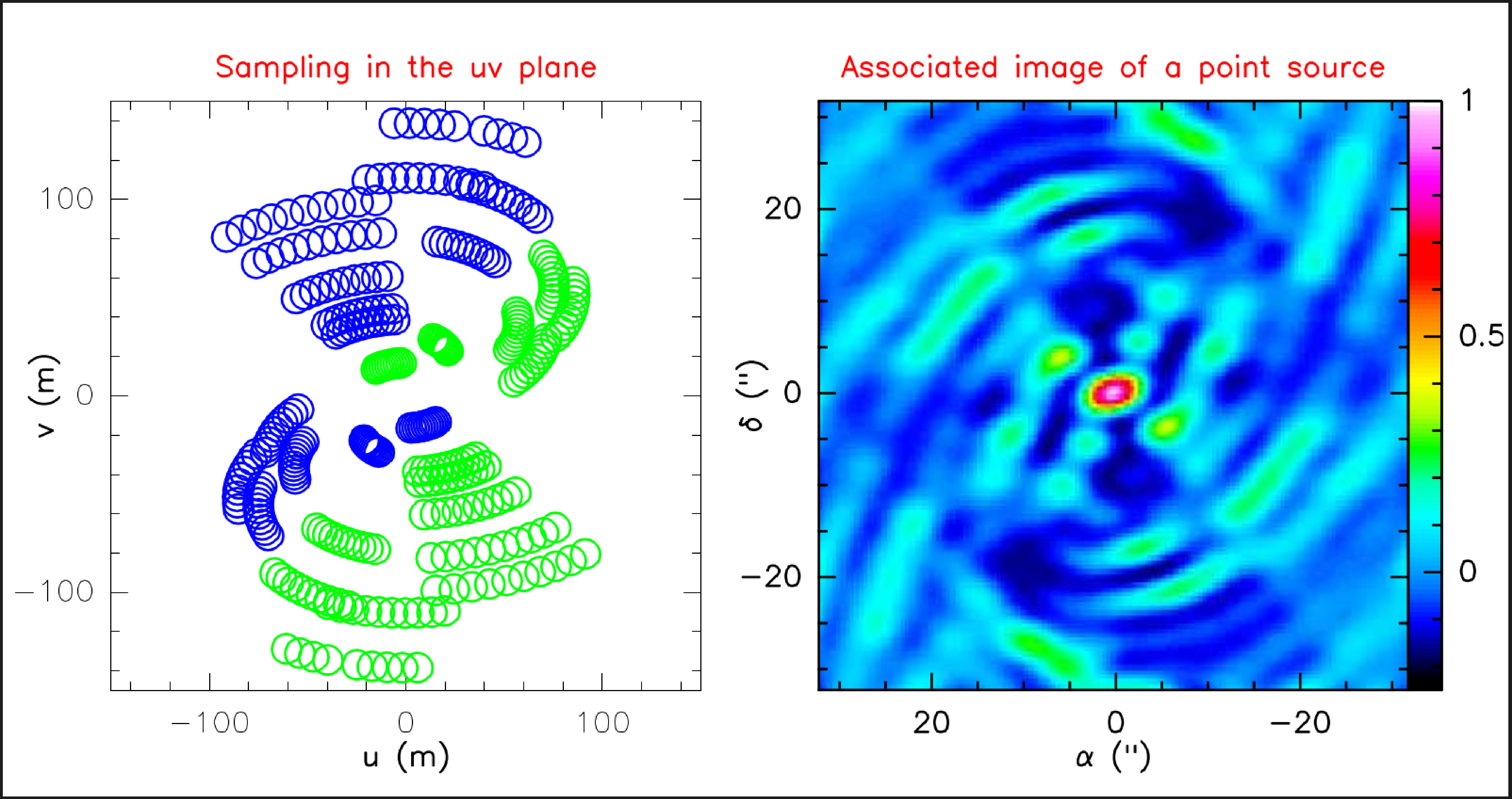

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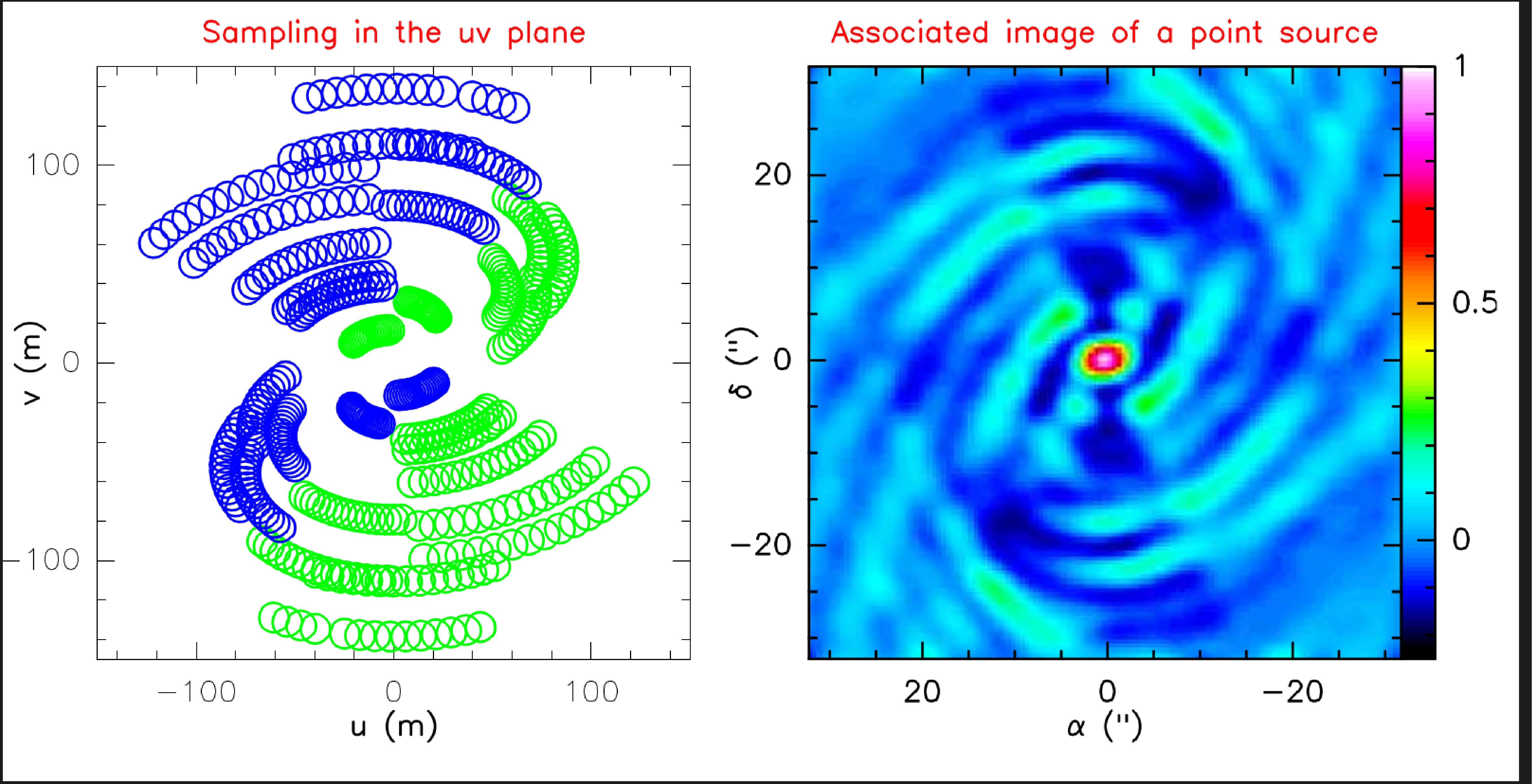

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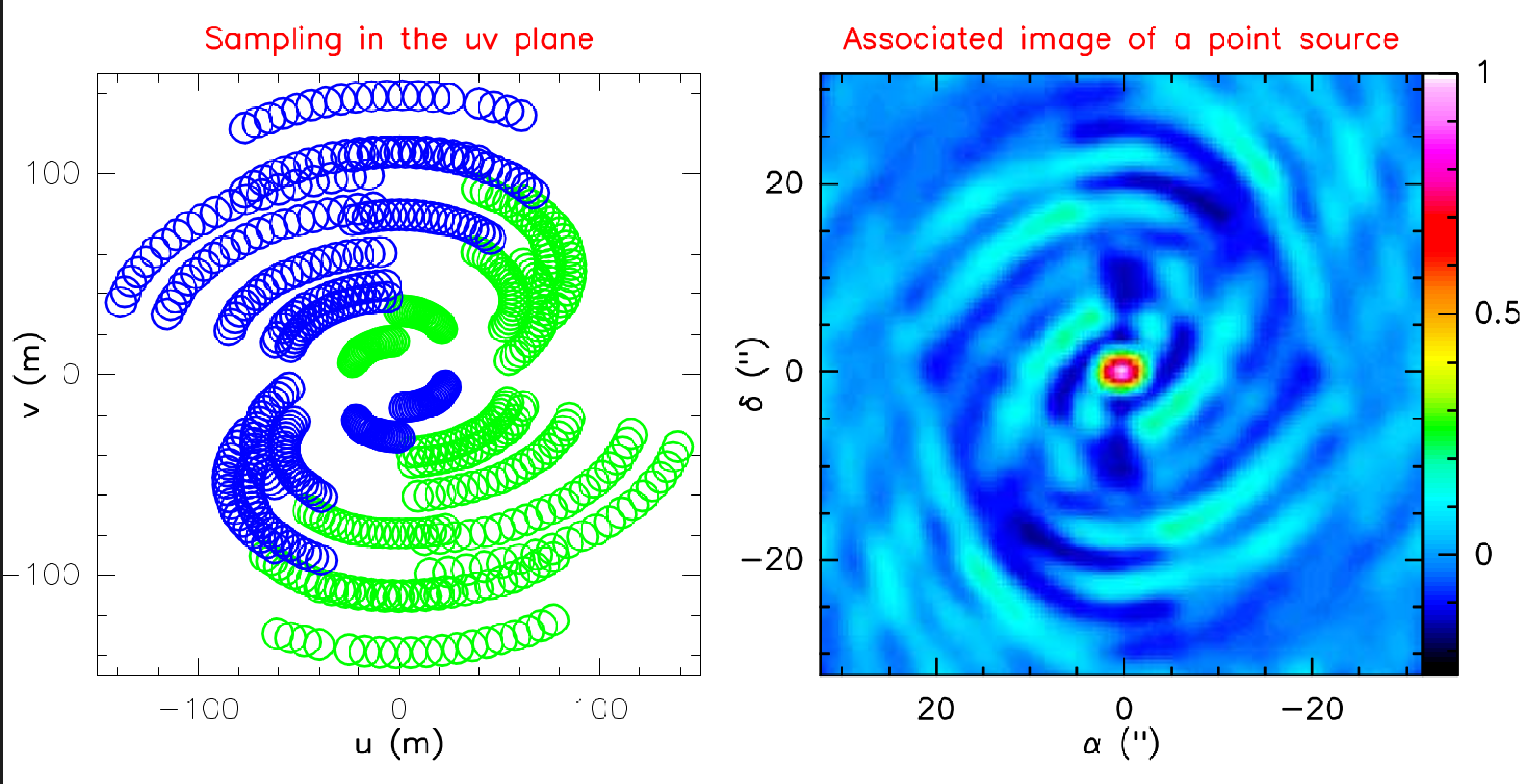

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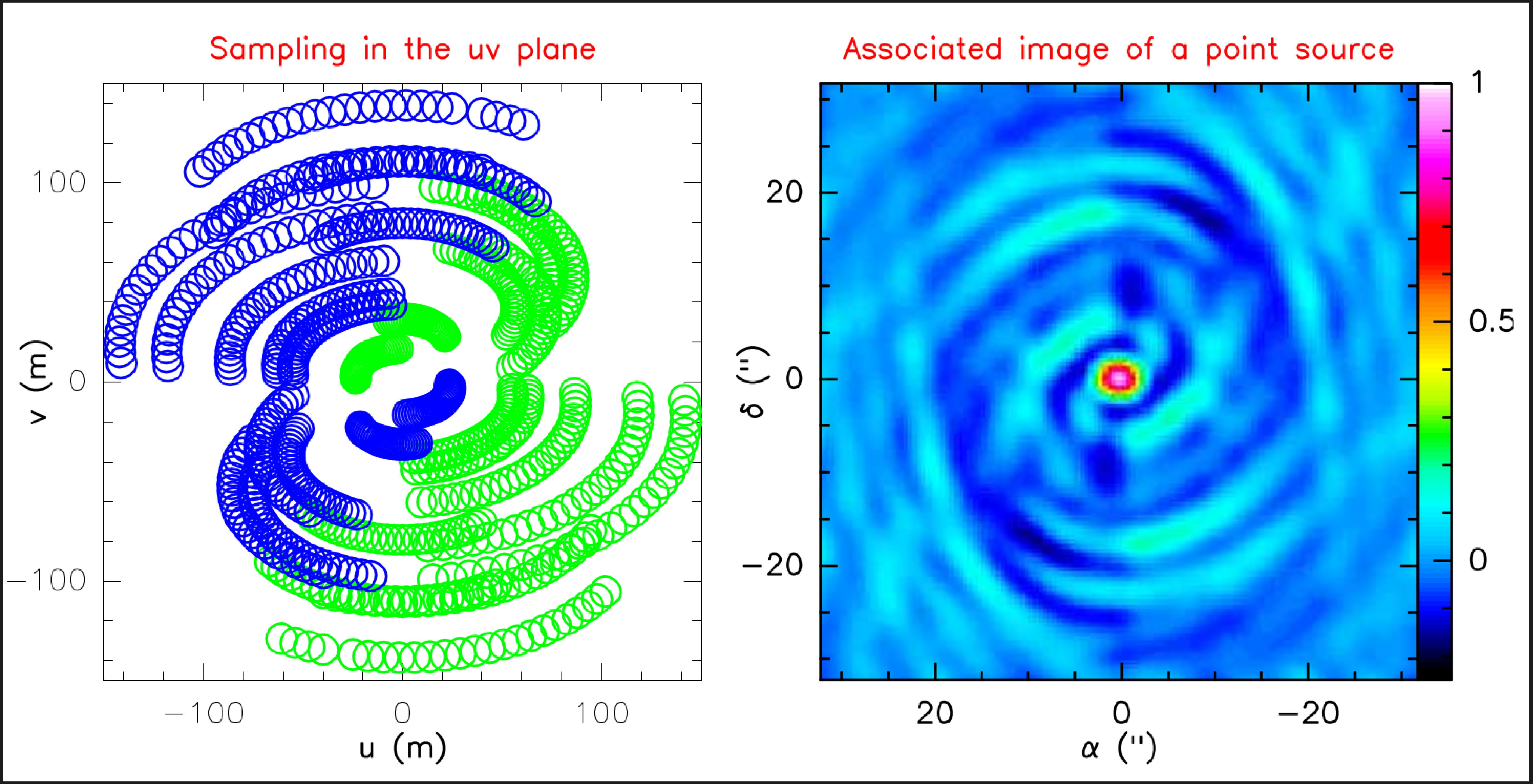

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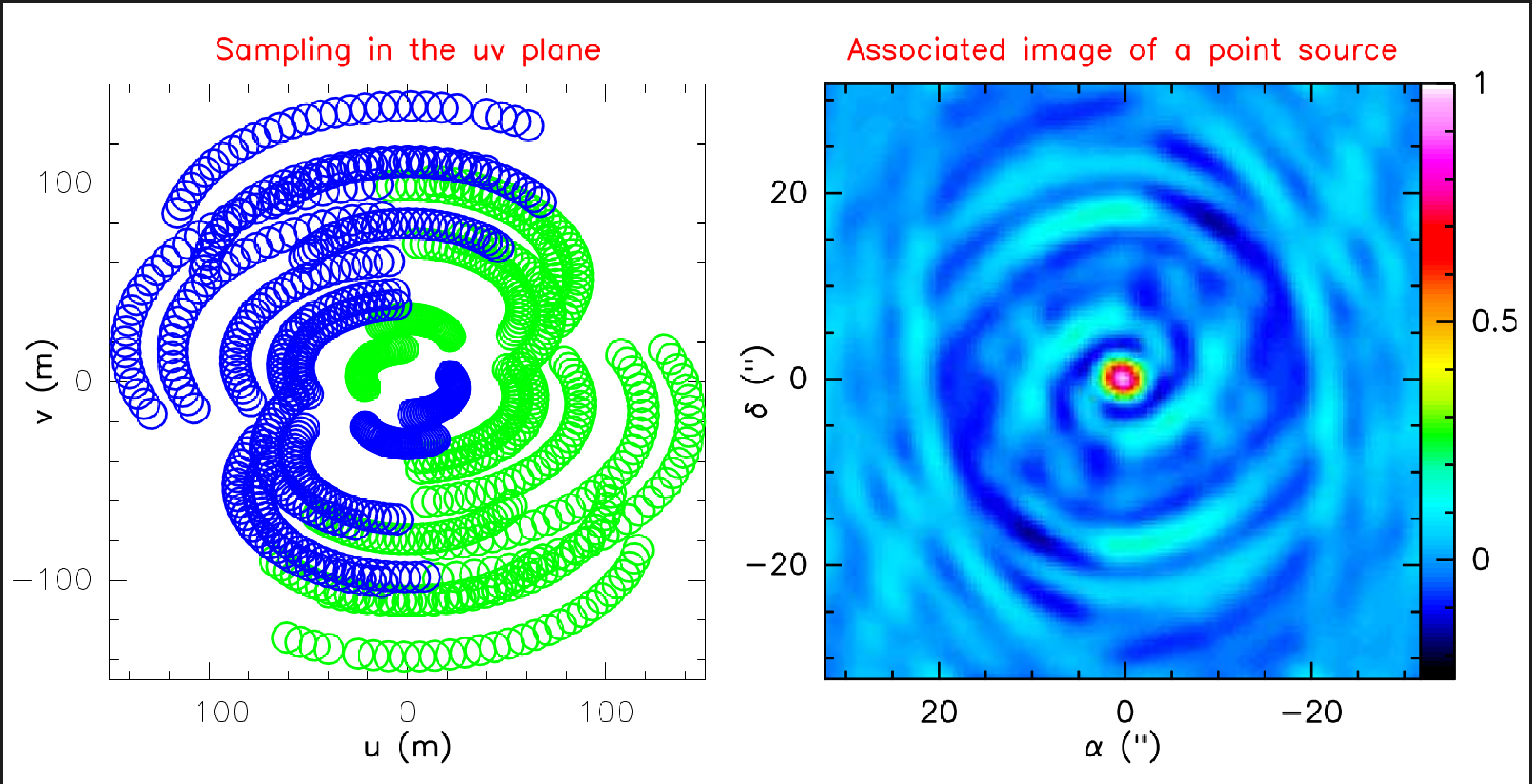

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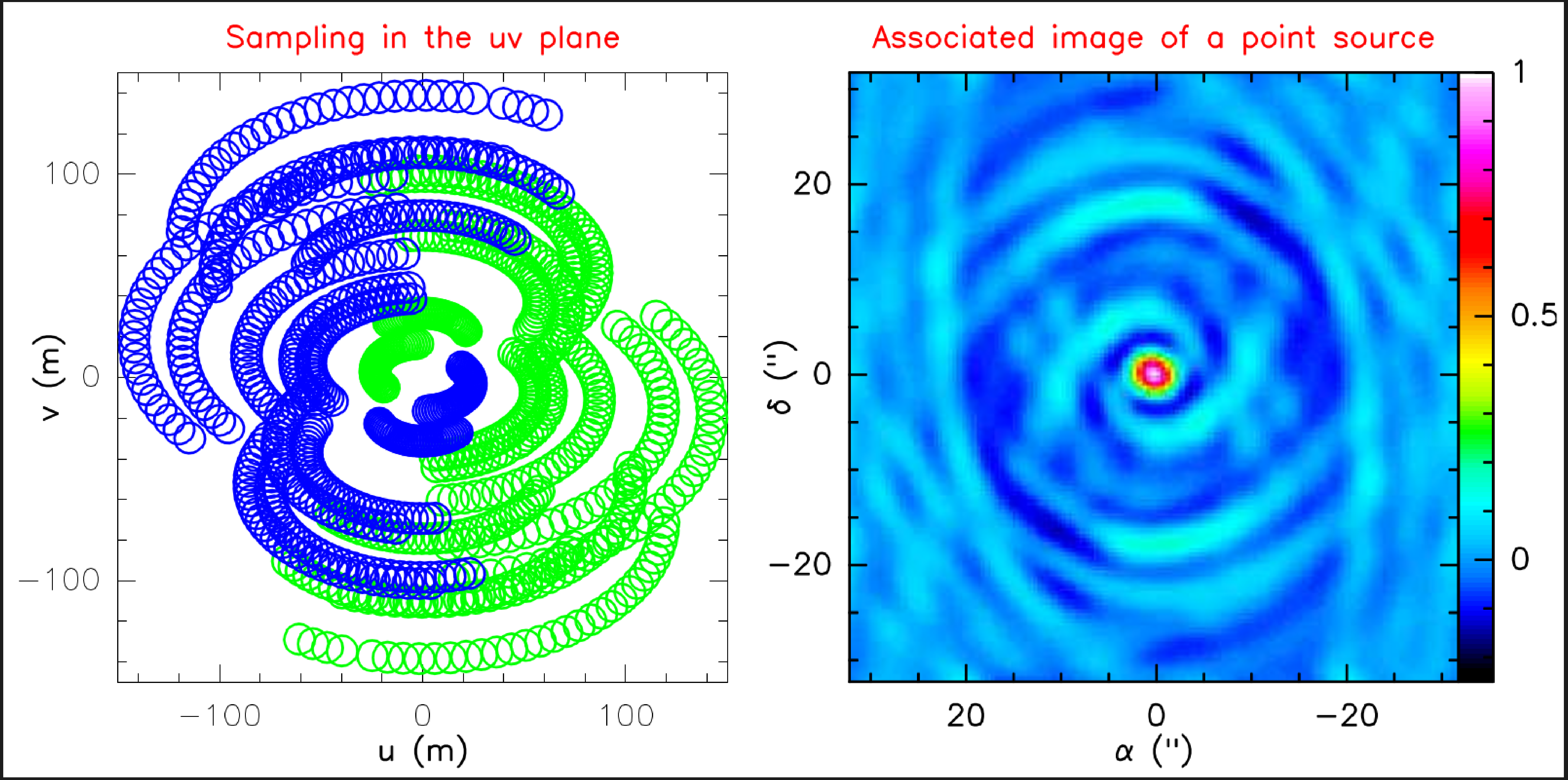


Further details: Primary beam (FOV)

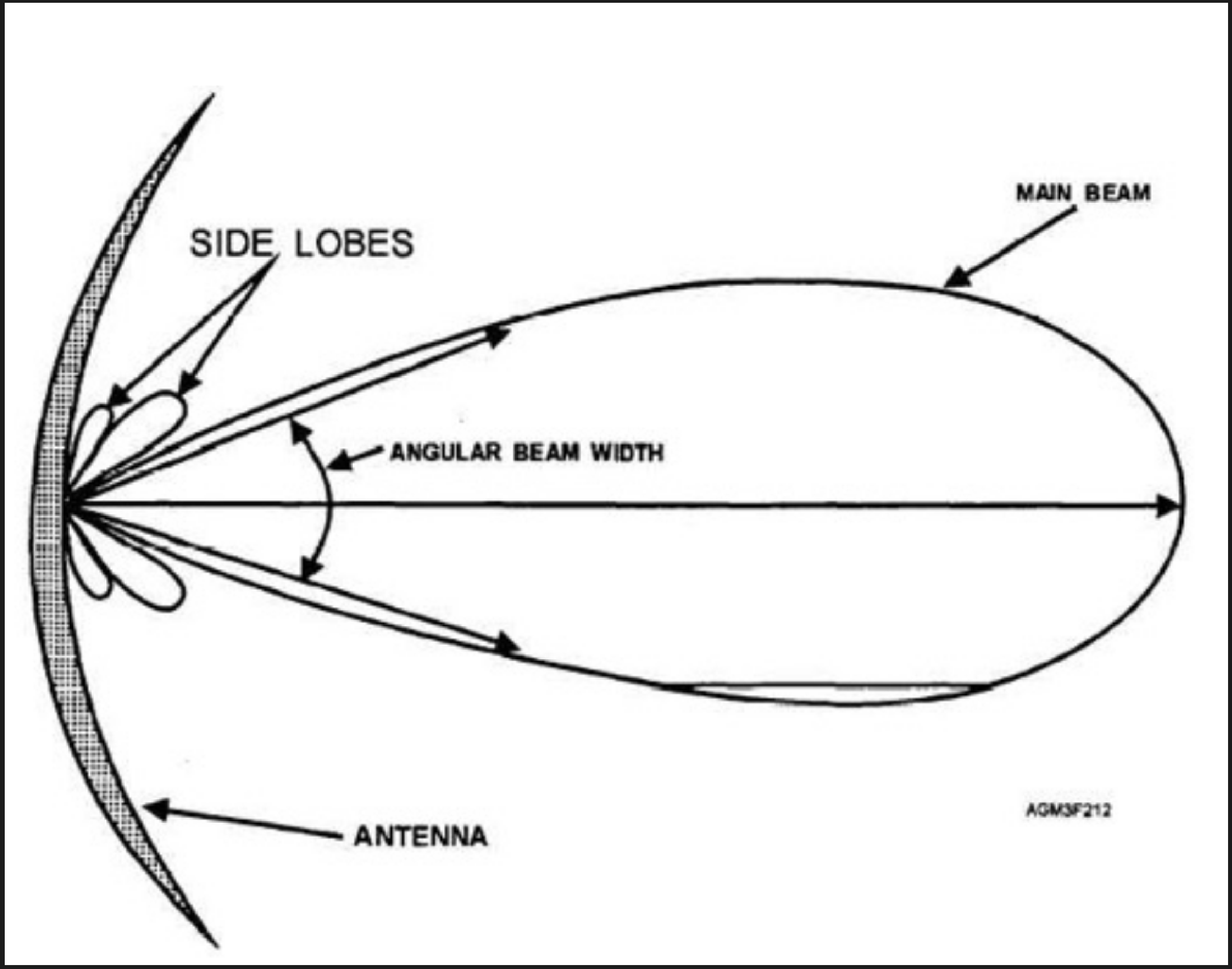
The field of view of an interferometer, also called ***PRIMARY BEAM****, depends on the single dish diameter:*

$$\boldsymbol{FOV = 1.22\ \lambda/D_{SD}}$$

Smaller dish → larger FOV then.

If the object is larger than the FOV → mosaic

N.B. = sensitivity decreases with the distance from the image centre



Further details: Sensitivity

Measurement of visibilities is limited by noise emitted by atmosphere, antenna, ground receivers.
The rms noise in a data cube is:

$$S = \frac{k \cdot T_{sys}}{A \cdot N^2 \sqrt{N_P \cdot \Delta\nu \cdot \Delta\tau}}$$

with:
T_{sys} = *system temperature*
A = *area of each antenna*
N = *number of antennas*
N_P = *number of polarizations*
$\Delta\nu$ = *bandwidth*
$\Delta\tau$ = *observing time*

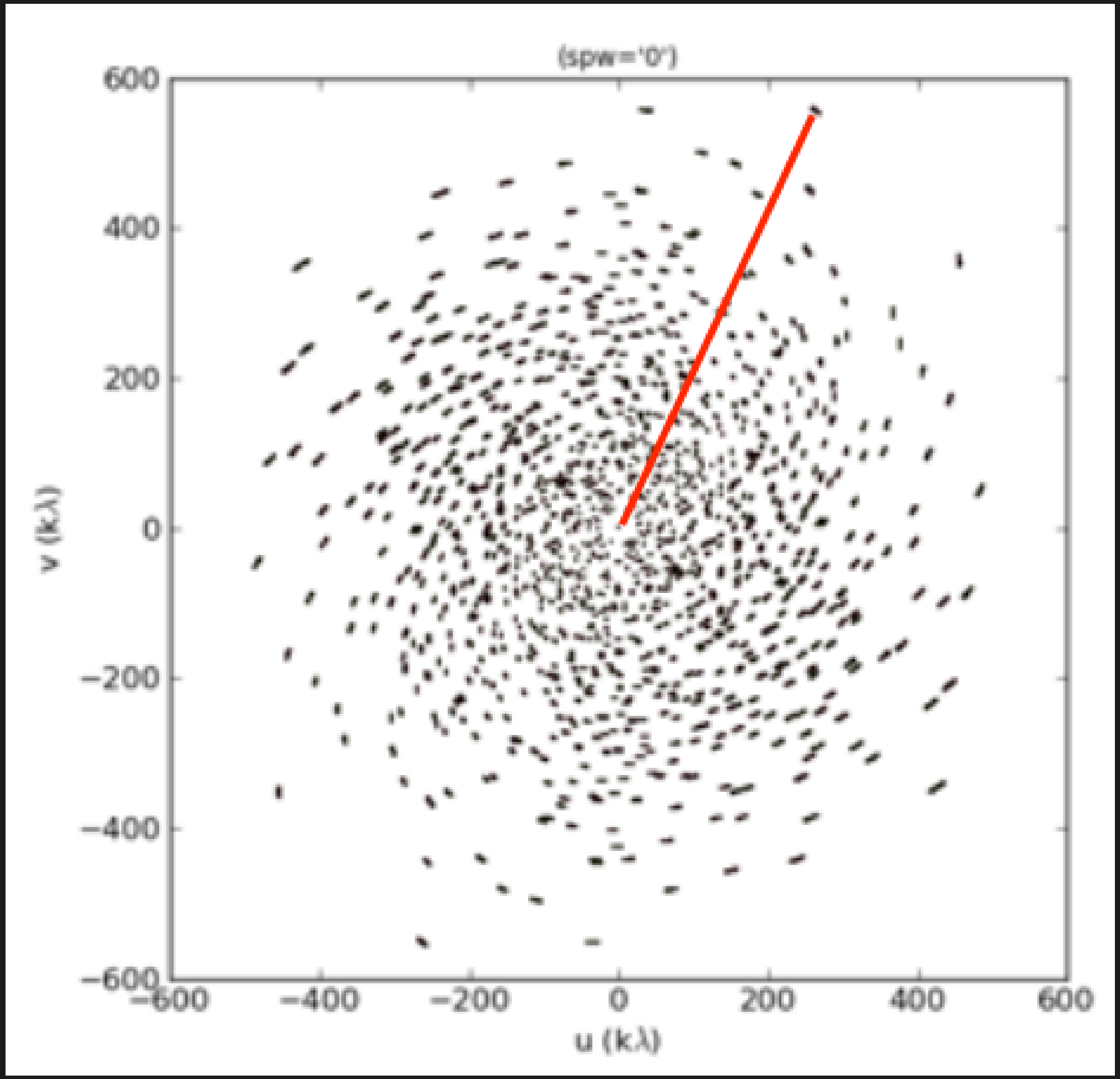
The sensitivity is a strong function of the atmospheric conditions. The troposphere has an effect on the optical depth, the atmospheric emission, and on the demands for calibration

Further details: Synthesized beam

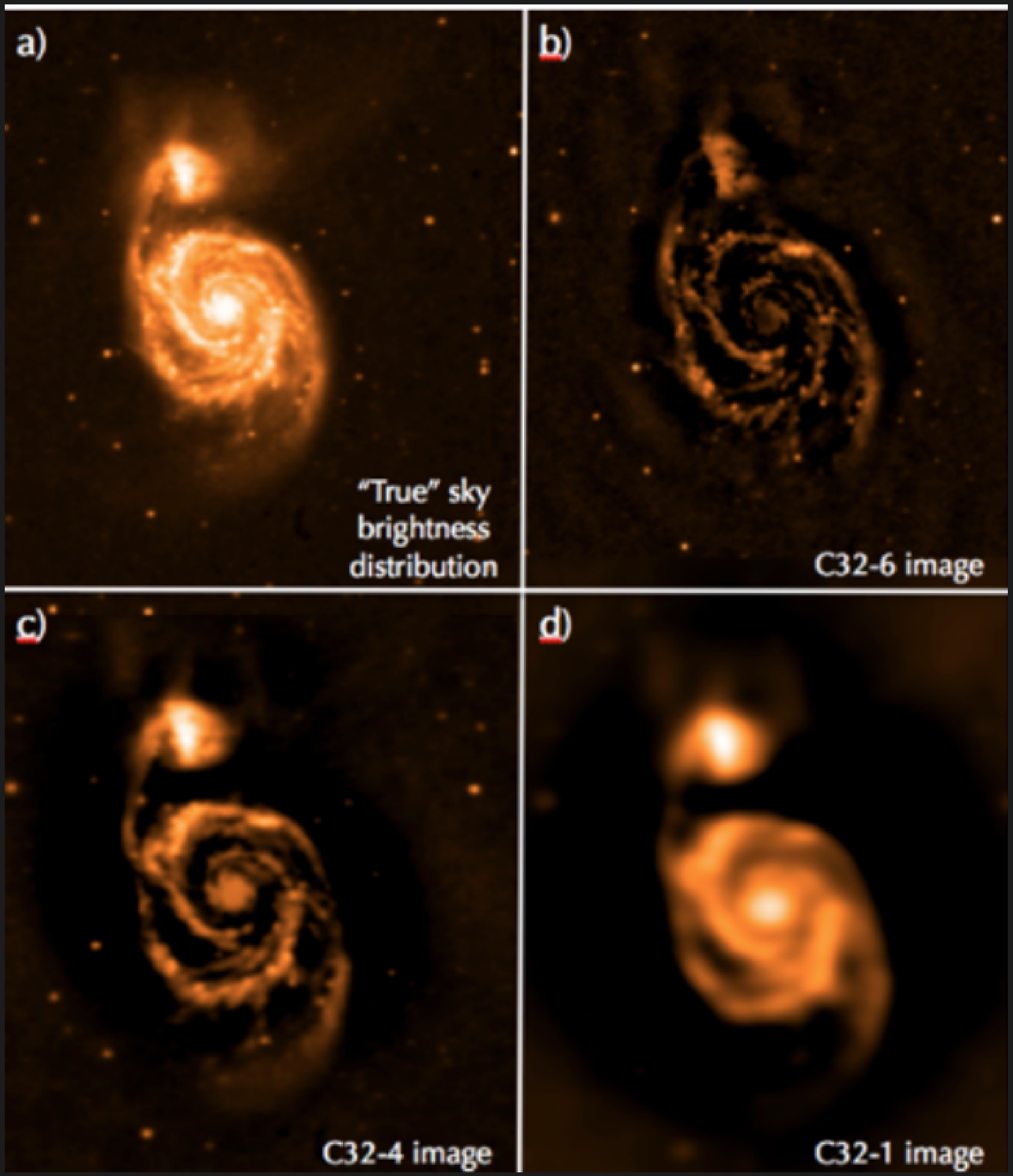
The angular resolution of the interferometer is the FWHM of the ***SYNTHESIZED BEAM****, and depends on the maximum distance between antennas:*

$$\theta = \lambda / B_{MAX}$$

Extended configuration → smaller angular resolution.



Further details: Synthesized beam



At the same time the maximum recoverable scale depends on the minimum baseline, and since by definition it cannot be less than the antenna diameter (there would be antennas overlap) the diffuse component cannot be recovered. For this reasons ALMA as an ACA and a TP with the goal of recovering this component (NOEMA for example uses the 30 m telescope)

Some hint about calibration

Once you have your visibilities the next step is to calibrate them.
There are 4 main steps to perform:

1 – Atmoshperic Phase Correction: to correct for the fluctuations of precipitable water vapor (PWV)

2 – Phase calibration, to determine the variations of phase and amplitude with time

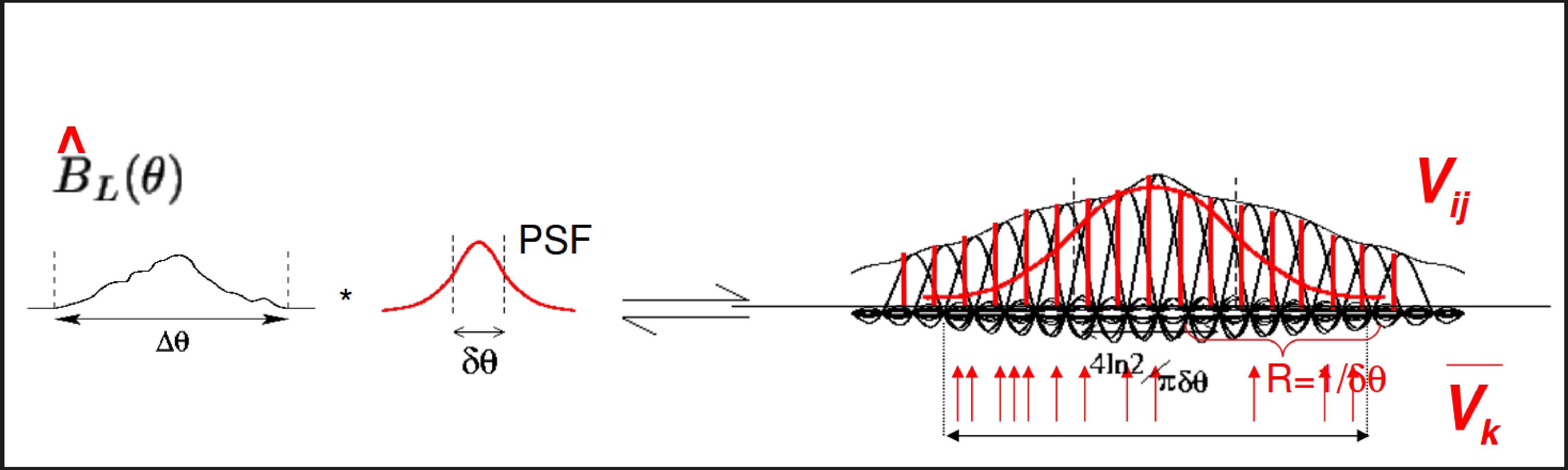
3 – Bandpass calibration, to correct for different receiver responses during the observations

4 – Flux calibration, to convert in the right units the visibilities observed

Some hint about Imaging

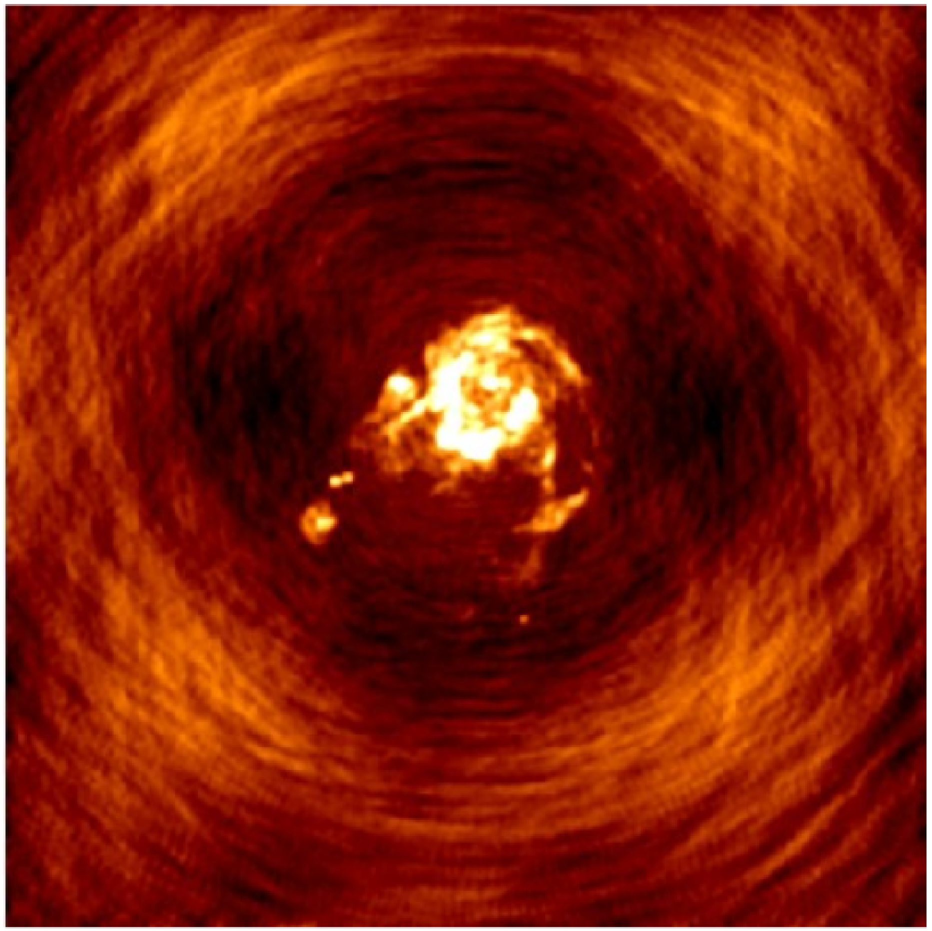
Once you have your calibrated visibility the next step is to do imaging.
This process is called ‘deconvolution’:

You first do a FFT of the interpolated visibilities to get a Dirty Image, then proceed with the ***CLEANING****:*
- assume the source brightness distribution is a sum of point sources
- Fit and subtract the synthesized beam iteratively

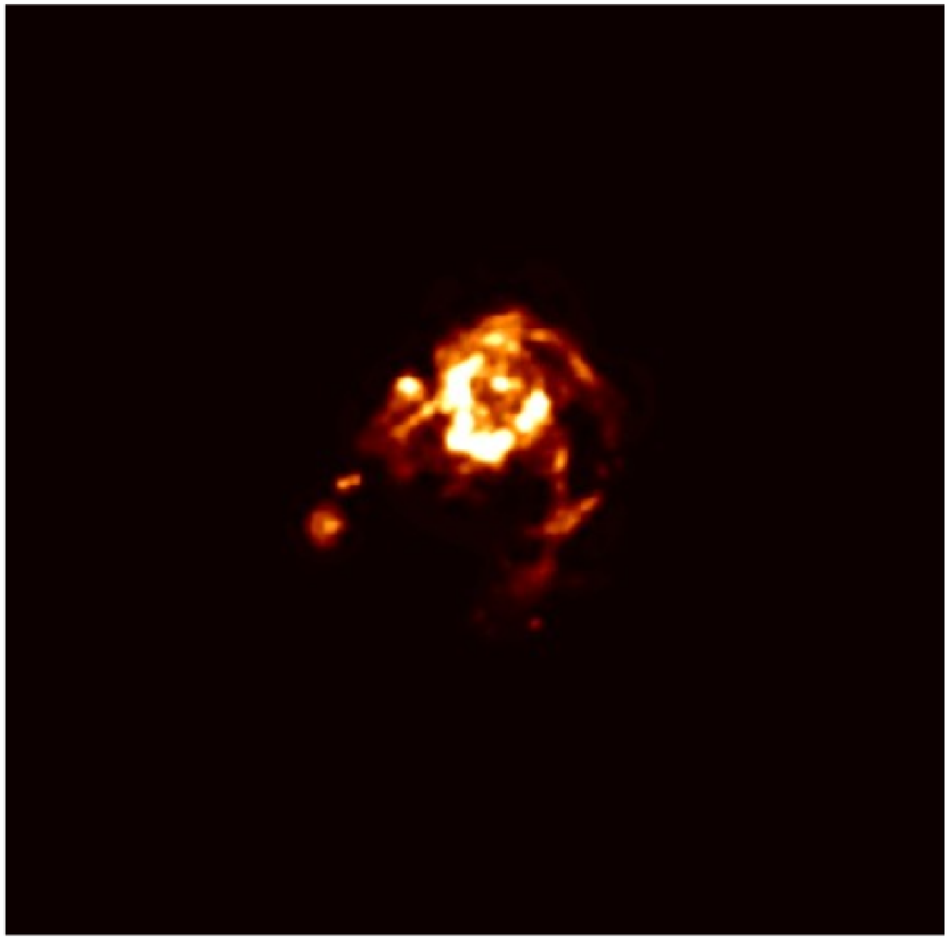


Some hint about Imaging

You first do a FFT of the interpolated visibilities to get a Dirty Image, then proceed with the ***CLEANING****:*
- assume the source brightness distribution is a sum of point sources
- Fit and subtract the synthesized beam iteratively



Dirty map



Clean map

Take Home Messages

1 - Interferometers use an array of separate telescopes working together as a wider single telescope

2 - A radio interferometer measures the coherence of the electric field between the 2 receiving elements

3 - A radio interferometer is an instrument that samples the visibility function, which is the Fourier transform of the sky brightness distribution

4 - The possibility to convert visibilites into sky brightness measurements is given by the van Cittert-Zernike Theorem

5 - The maximum recoverable scale depends on the minimum baseline, and since by definition it cannot be less than the antenna diameter (there would be antennas overlap) the diffuse component cannot be recovered