Theory of planetary formation and migration

Summer School

Astroseismology and Exoplanets

Horta, Faial, Portugal, July 2016

Christoph Mordasini, University of Bern, Switzerland

Outline of lectures

- Lecture I

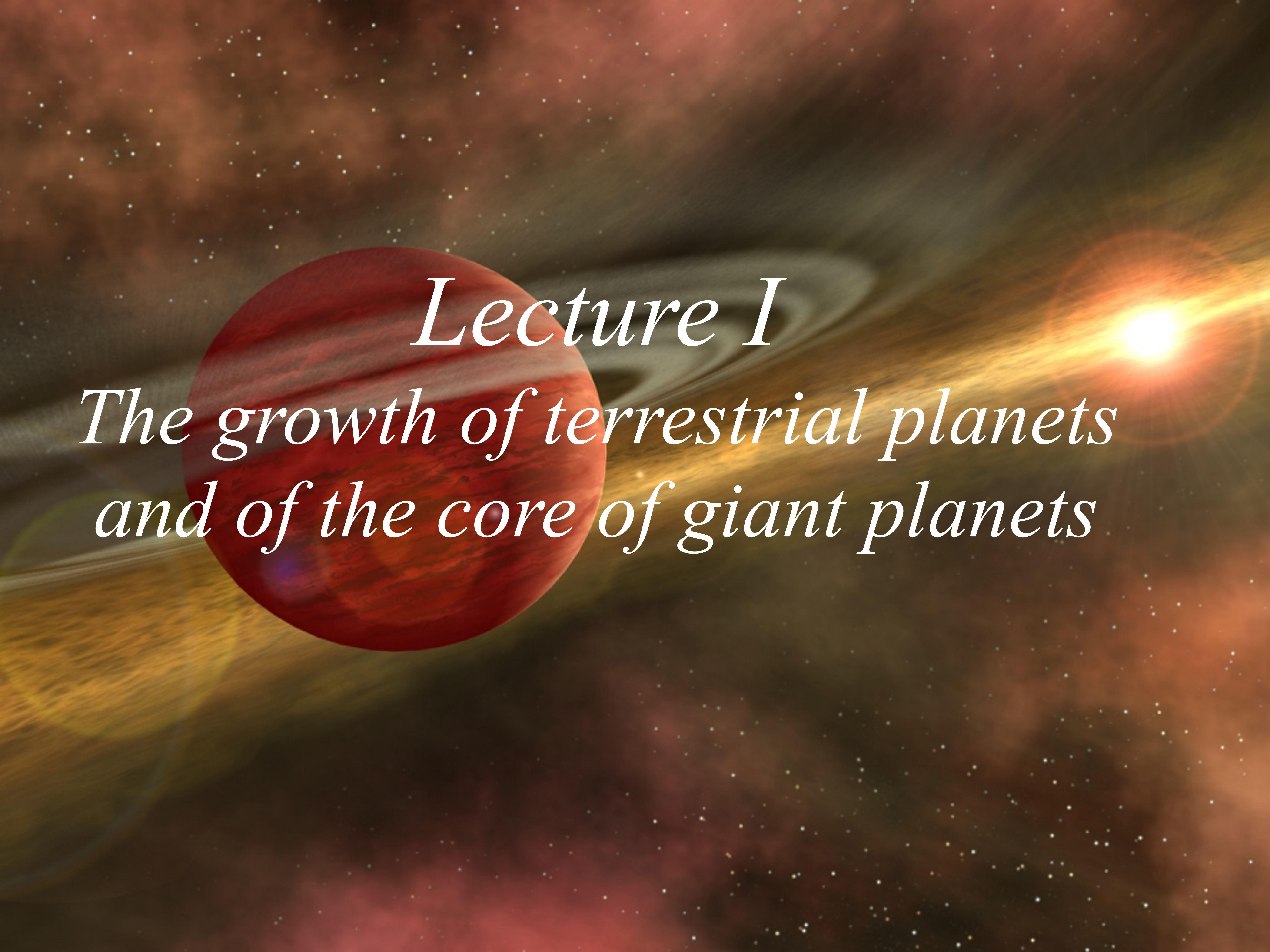
The growth of terrestrial planets and of the core of giant planets

- Lecture II

The formation of giant planets and migration

- Lecture III

Planetary population synthesis



Lecture I

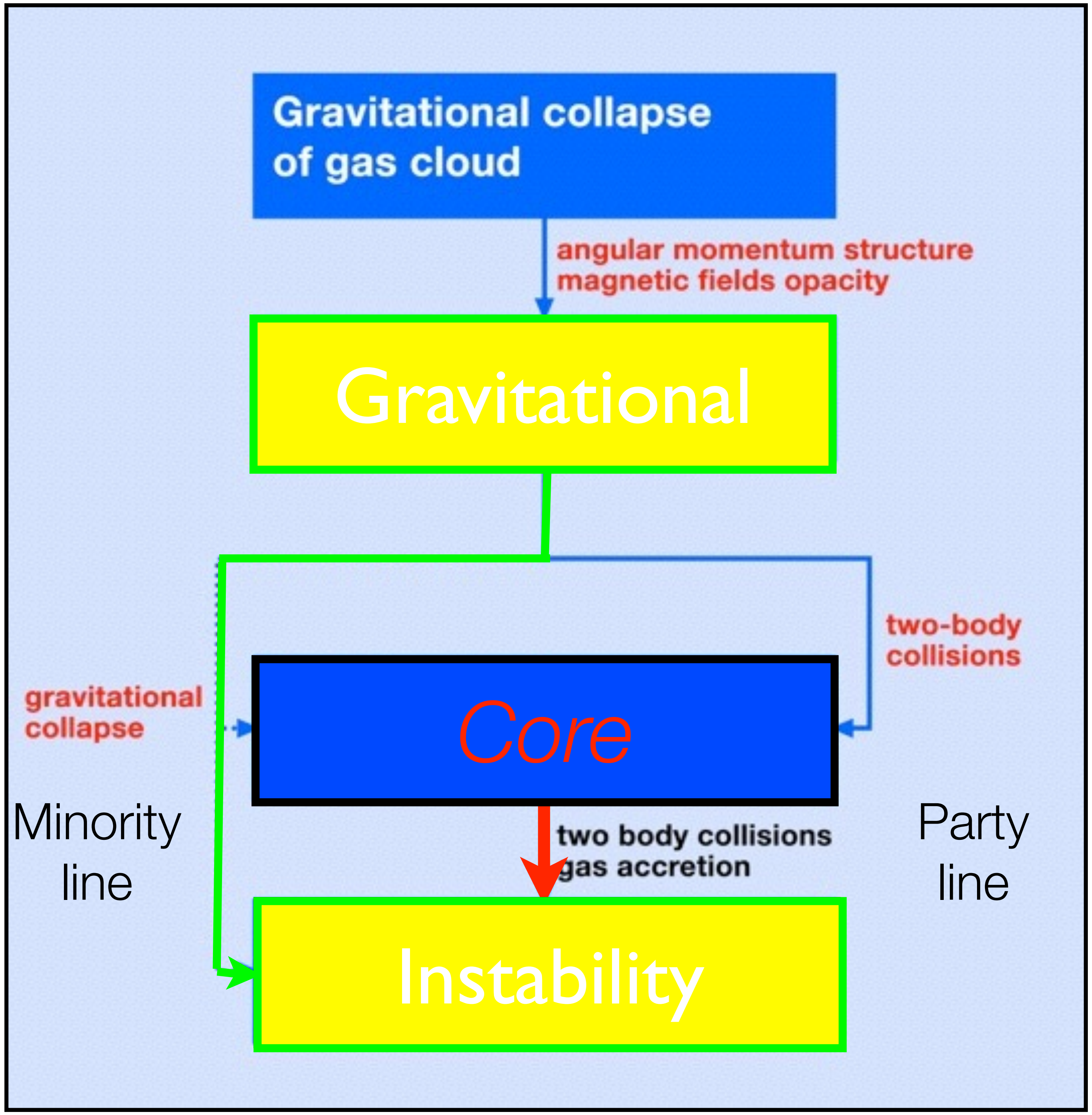
The growth of terrestrial planets and of the core of giant planets

Lecture 1 overview

1. Protoplanetary disks
2. From dust to planetesimals
3. From planetesimals to protoplanets
    - 3.1 Focussing factor
    - 3.2 Growth rate
    - 3.3 Isolation mass
    - 3.4 Growth regimes
    - 3.5 Growth as a function of orbital distance
4. Terrestrial planet formation

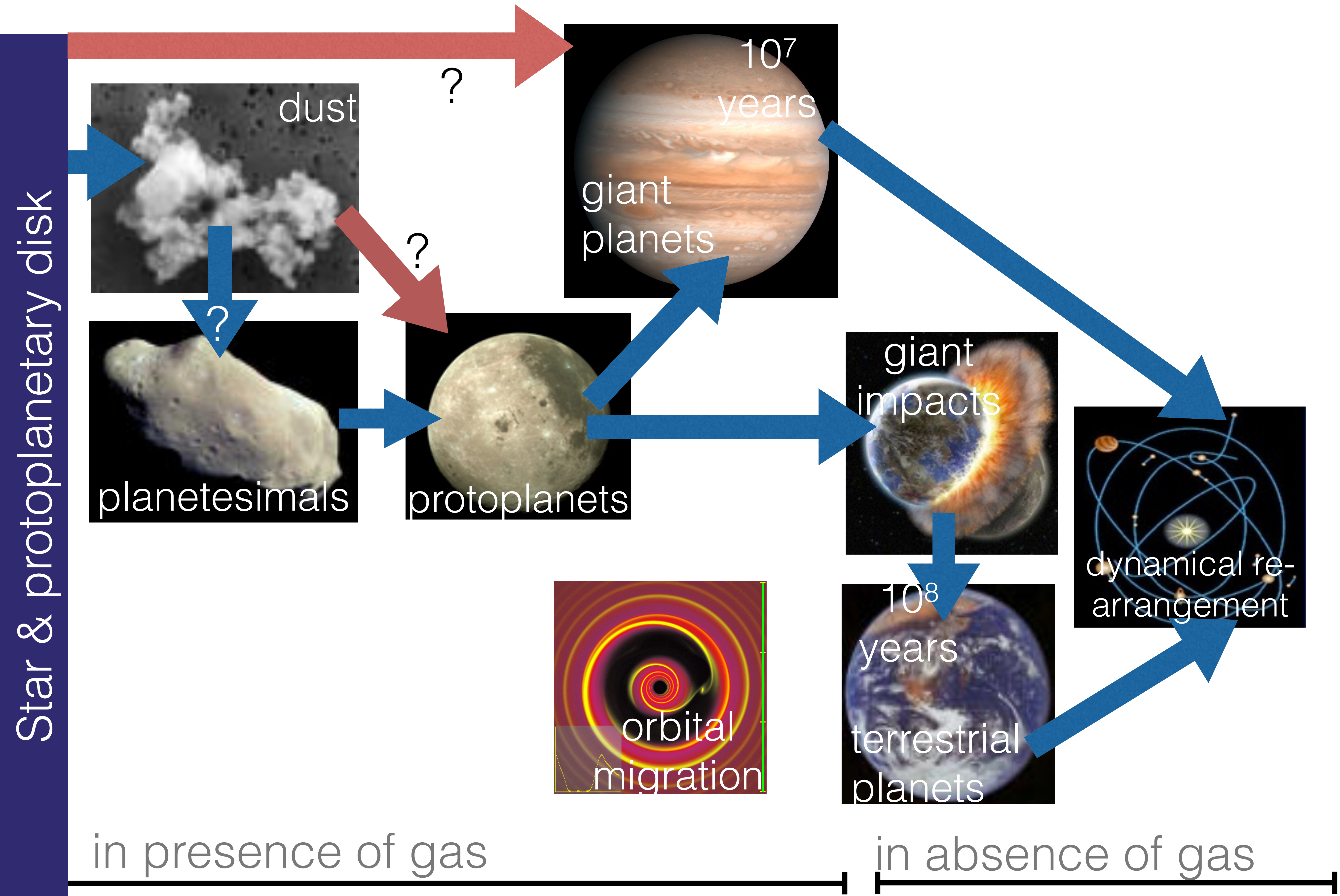
Introduction

Planet formation: The paradigm

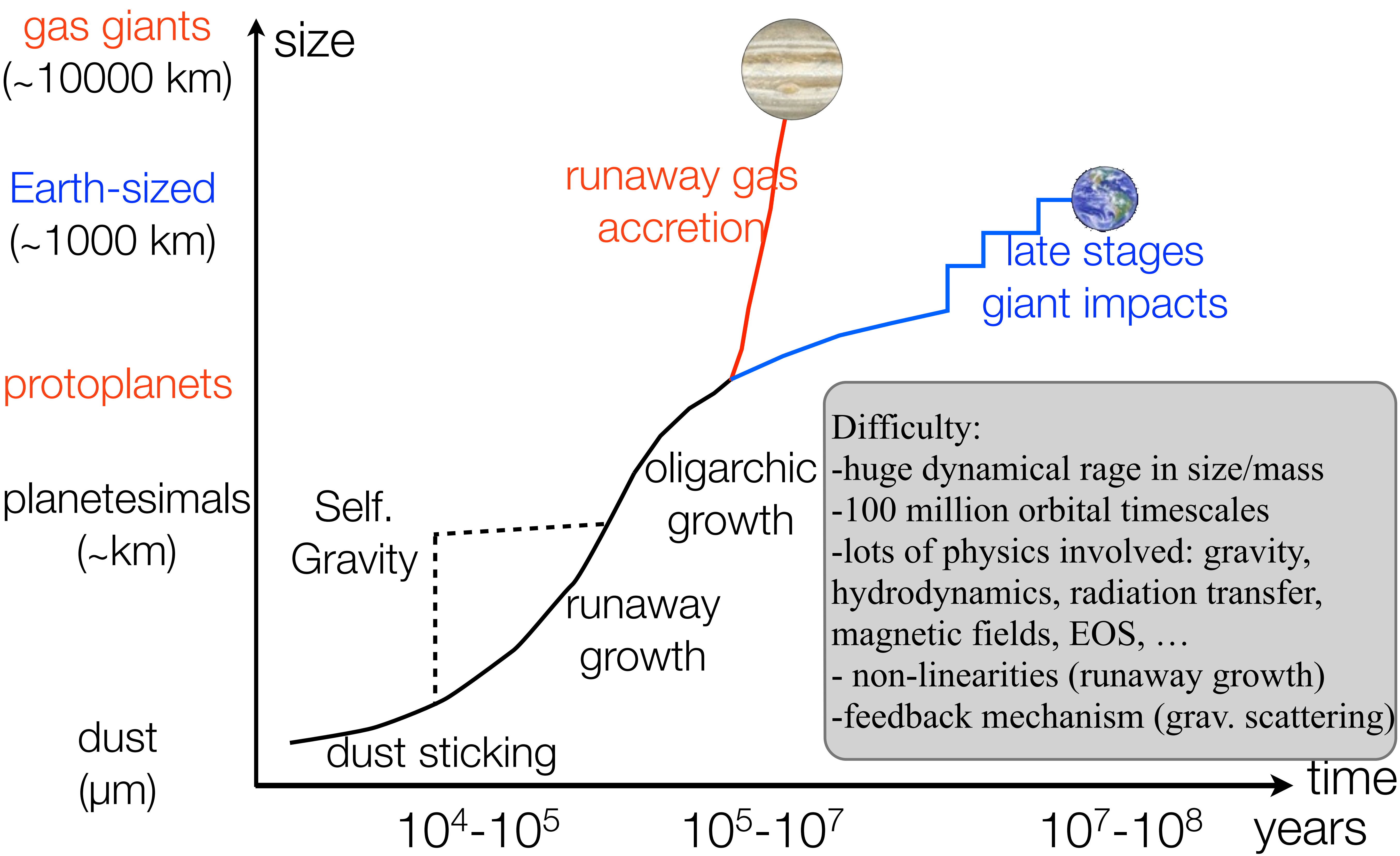


A satisfactory theory should explain the formation of planets in the solar system as well as around other stars.

Sequential picture of planet formation

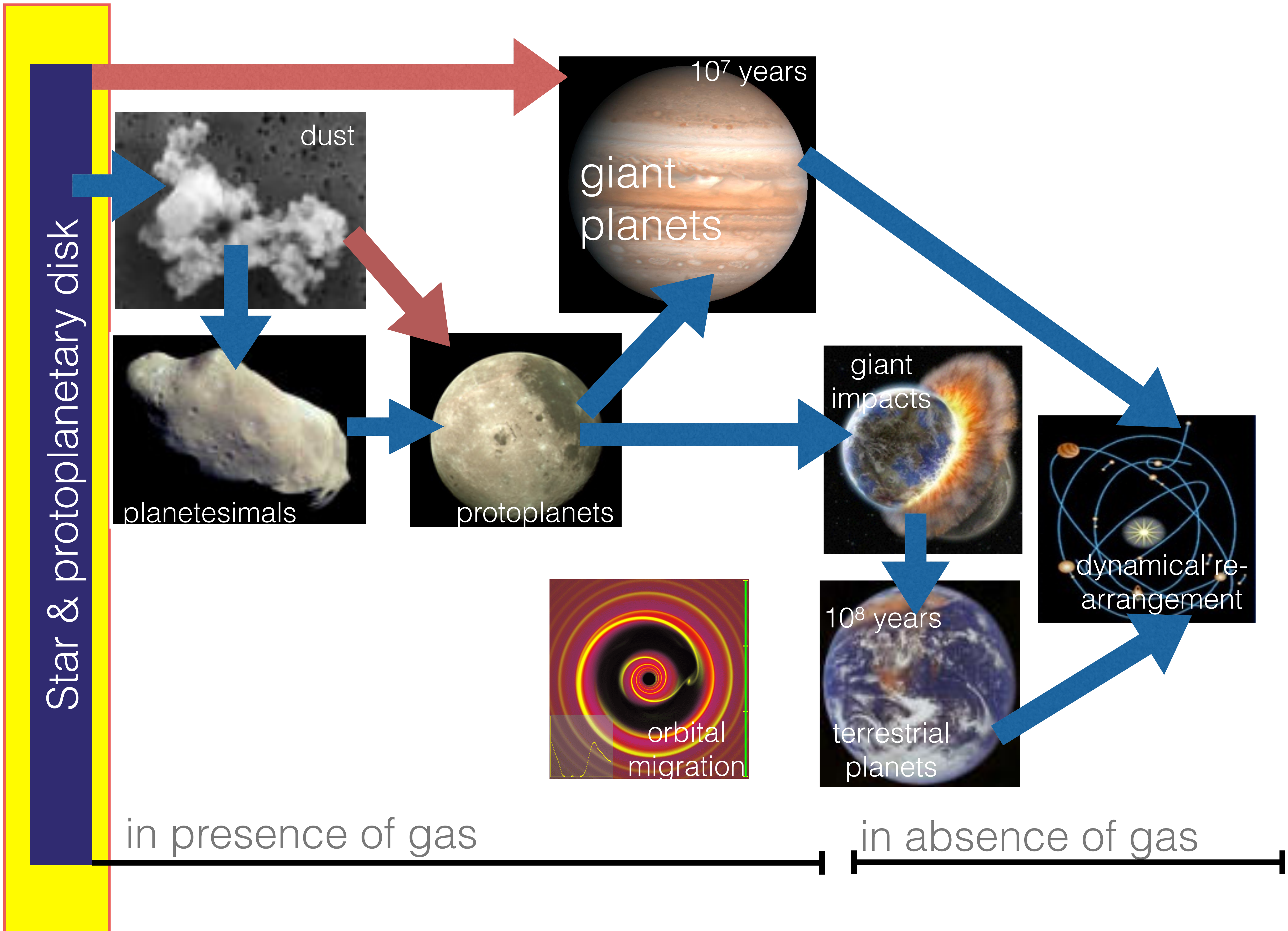


Challenges in planet formation

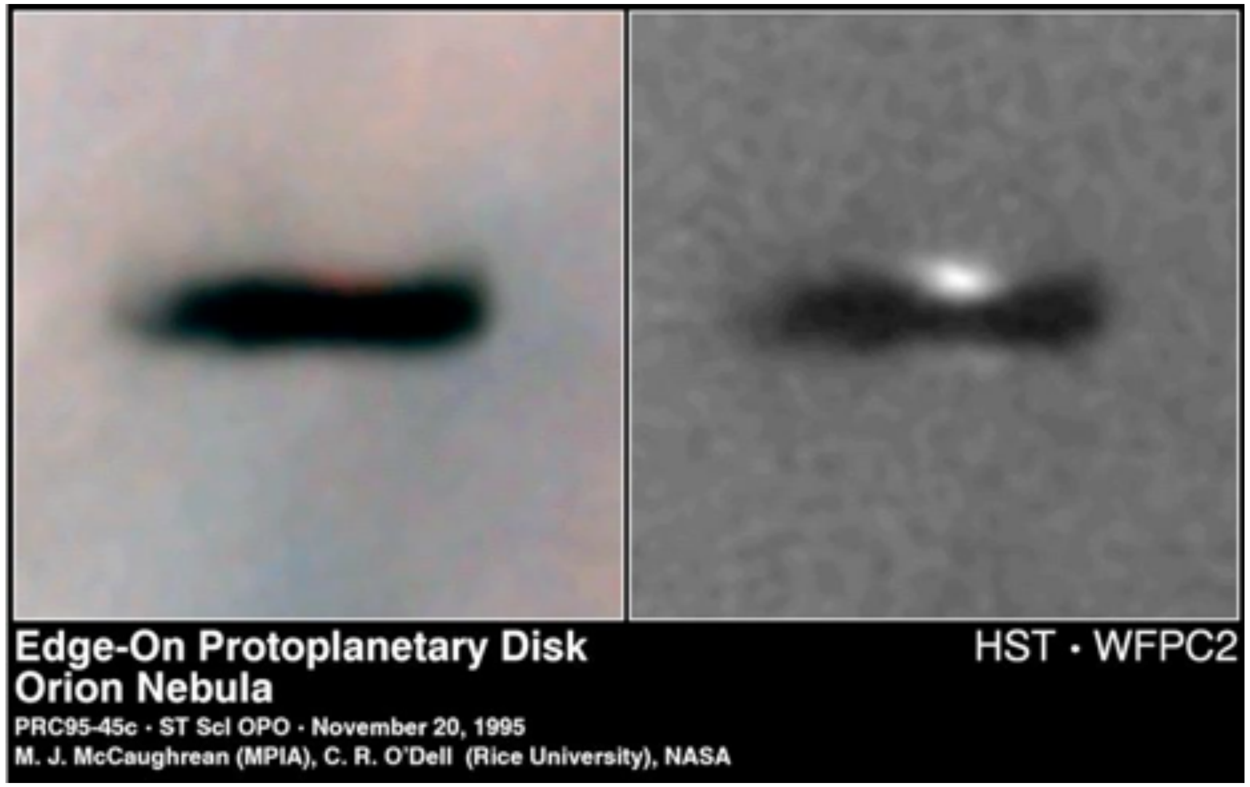


Difficulty:
-huge dynamical rage in size/mass
-100 million orbital timescales
-lots of physics involved: gravity, hydrodynamics, radiation transfer, magnetic fields, EOS, …
- non-linearities (runaway growth)
-feedback mechanism (grav. scattering)

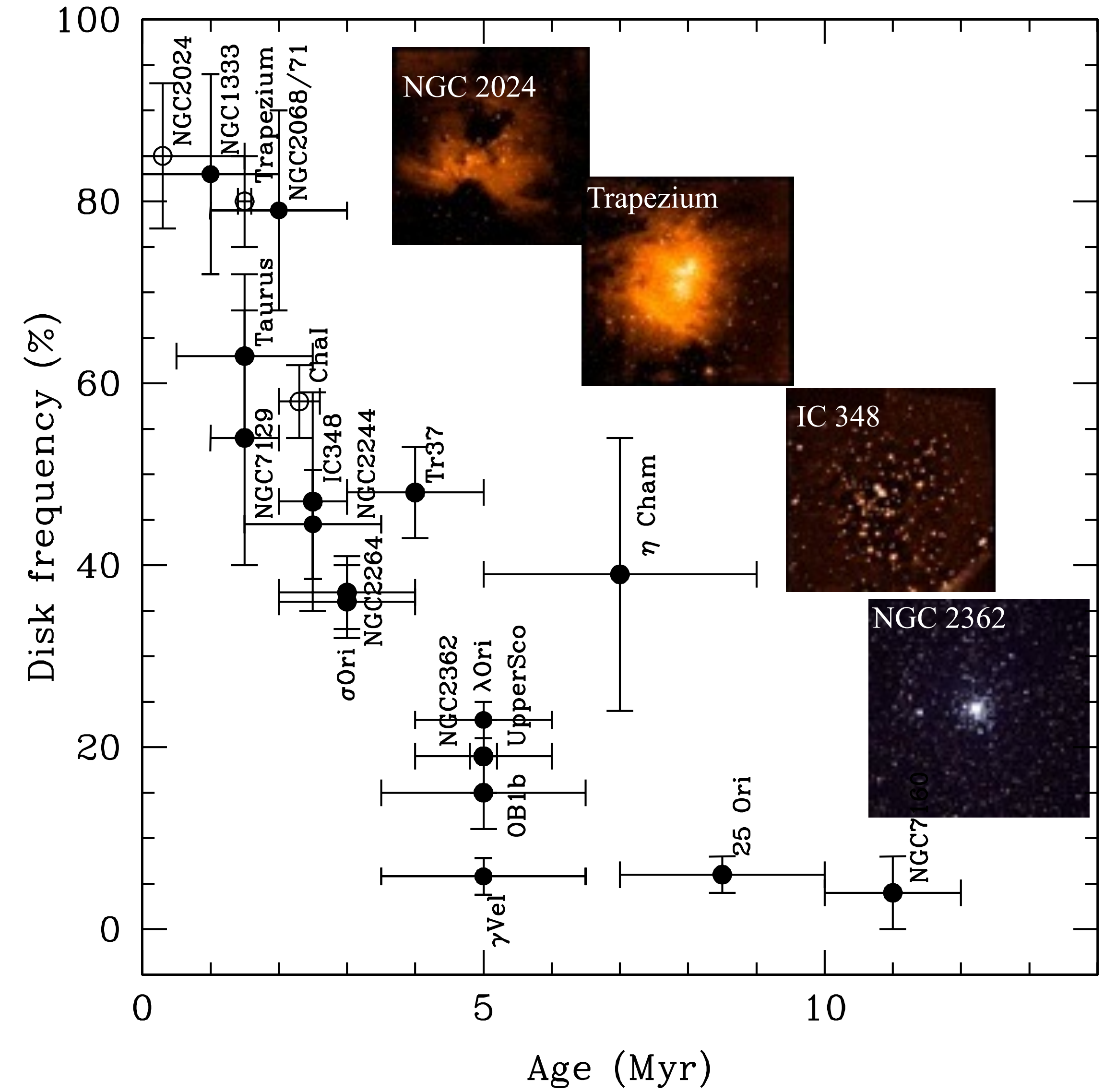
1. Protoplanetary disks

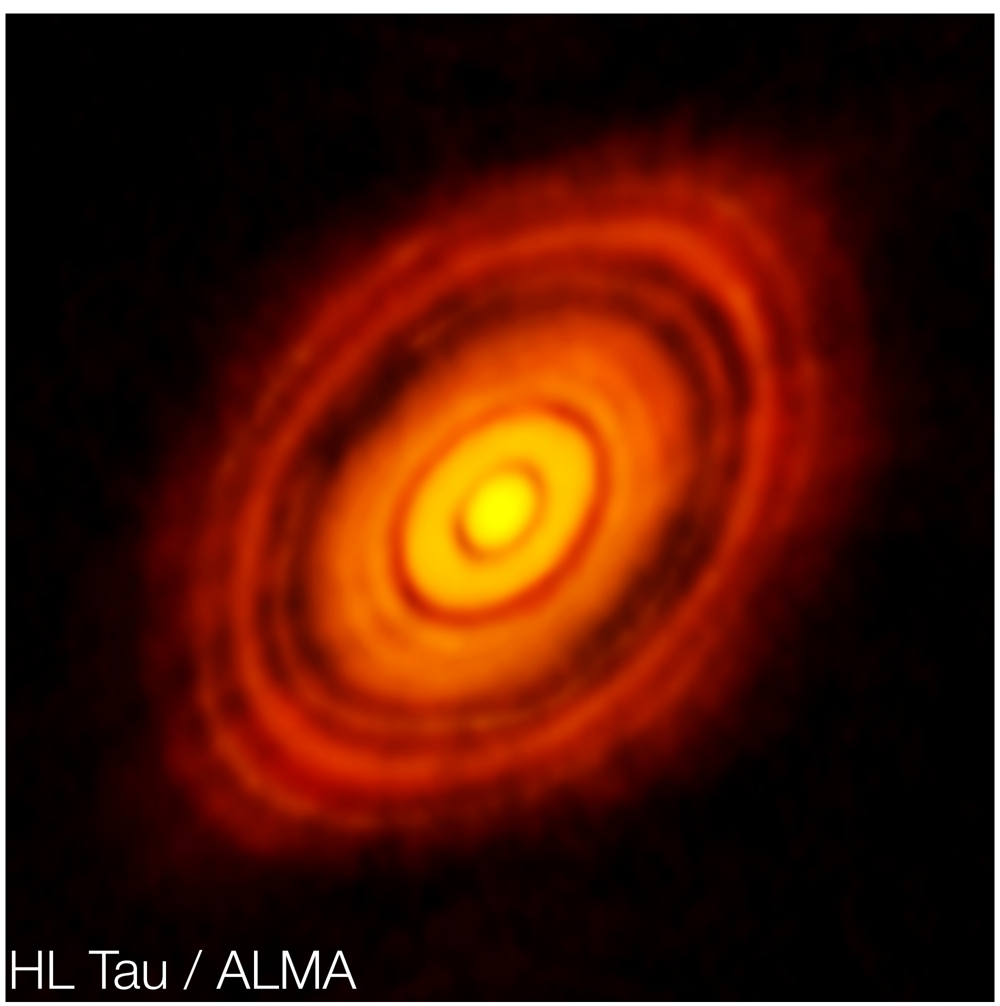


Protoplanetary disk



Hernandez et al. 2008





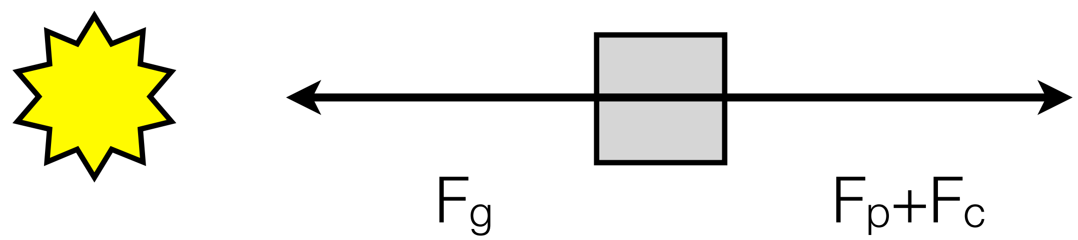
-astrophysical accretion disks (angular momentum conservation)

-size: several tens to hundreds of AUs

-thin: aspect ratio H/r 0.01 to 0.1 (H=vertical pressure scale height)

Rotation of solids and gas

In the radial direction: equilibrium of gravity, pressure and centrifugal force



$$\frac{v^2}{r} = \frac{GM_s}{r^2} + \frac{1}{\rho}\frac{\partial p}{\partial r}$$

solids

-the solids orbit in Keplerian rotation

$$\Omega = \sqrt{\frac{GM_s}{r^3}} \quad T_{orb} = \frac{2\pi}{\Omega}$$

-gas slightly pressure supported: rotates slightly slower than solids/planets

Initial solid surface density profile

First solids in the disk: Condensation into micrometer sized dust.
In reality inheritance in the outer disk…

Simplistic assumption: fraction of material that condenses constant except for increase at the iceline

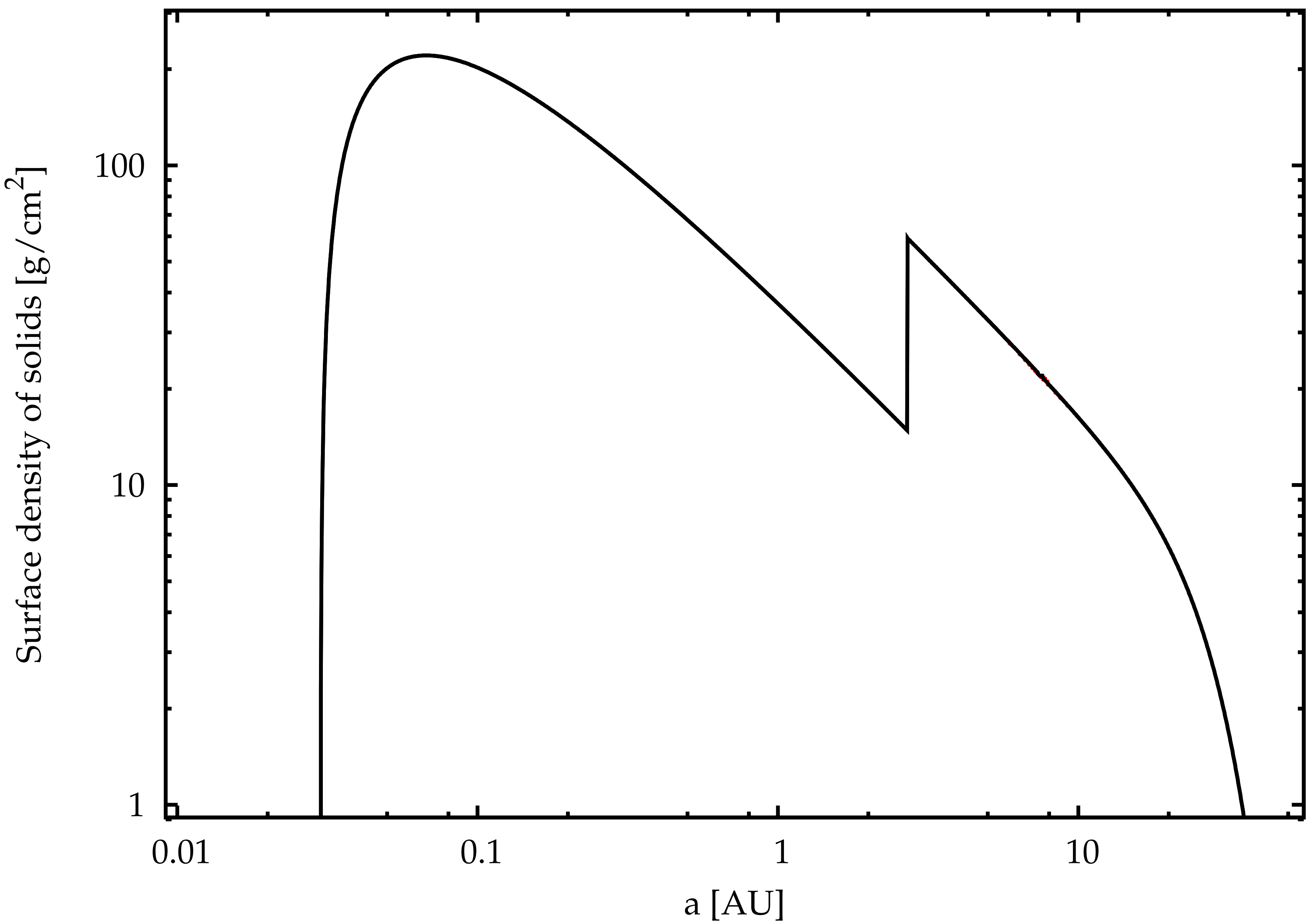
$$\Sigma_D(r, t=0) = f_{\mathrm{D/G}} f_{\mathrm{R/I}} \Sigma(r, t=0)$$

-Σ(r,t=0): gas surface density at t=0 (obviously ill defined)
-$f_{D/G}$ is the dust to gas ratio $f_{D/G}$ (assumed that it is the same in disk and star)
-Iceline: $f_{R/I}$ rock to ice ratio

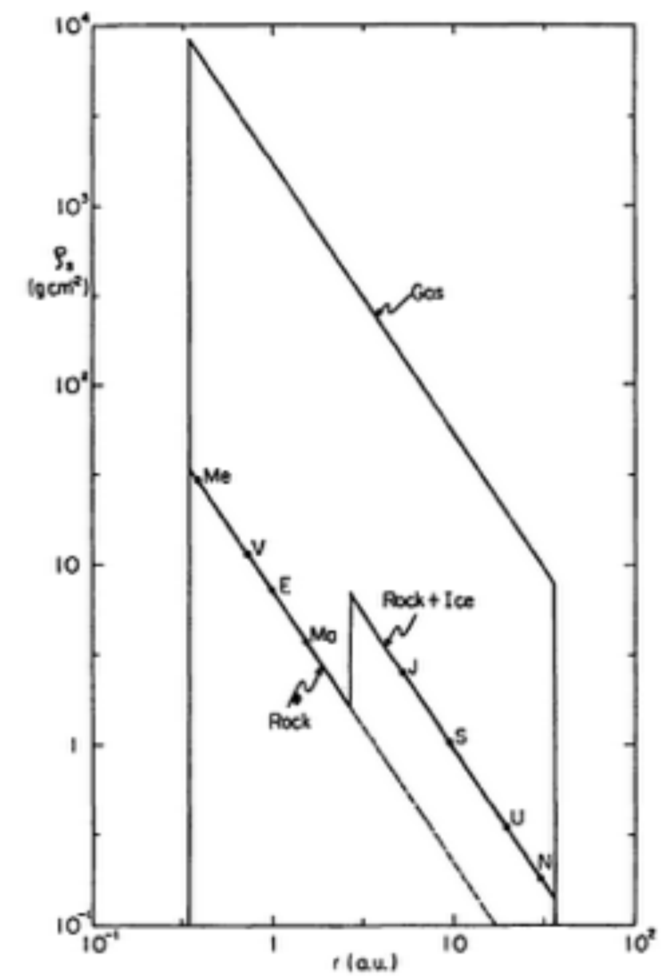
Relate it to stellar metallicity [Fe/H]: $$\frac{f_{\mathrm{D/G}}}{f_{\mathrm{D/G},\odot}} = 10^{[\mathrm{Fe/H}]}$$

Link of disk and stellar properties influencing planet formation process

Initial solid surface density profile



5 x Minimum mass solar nebula

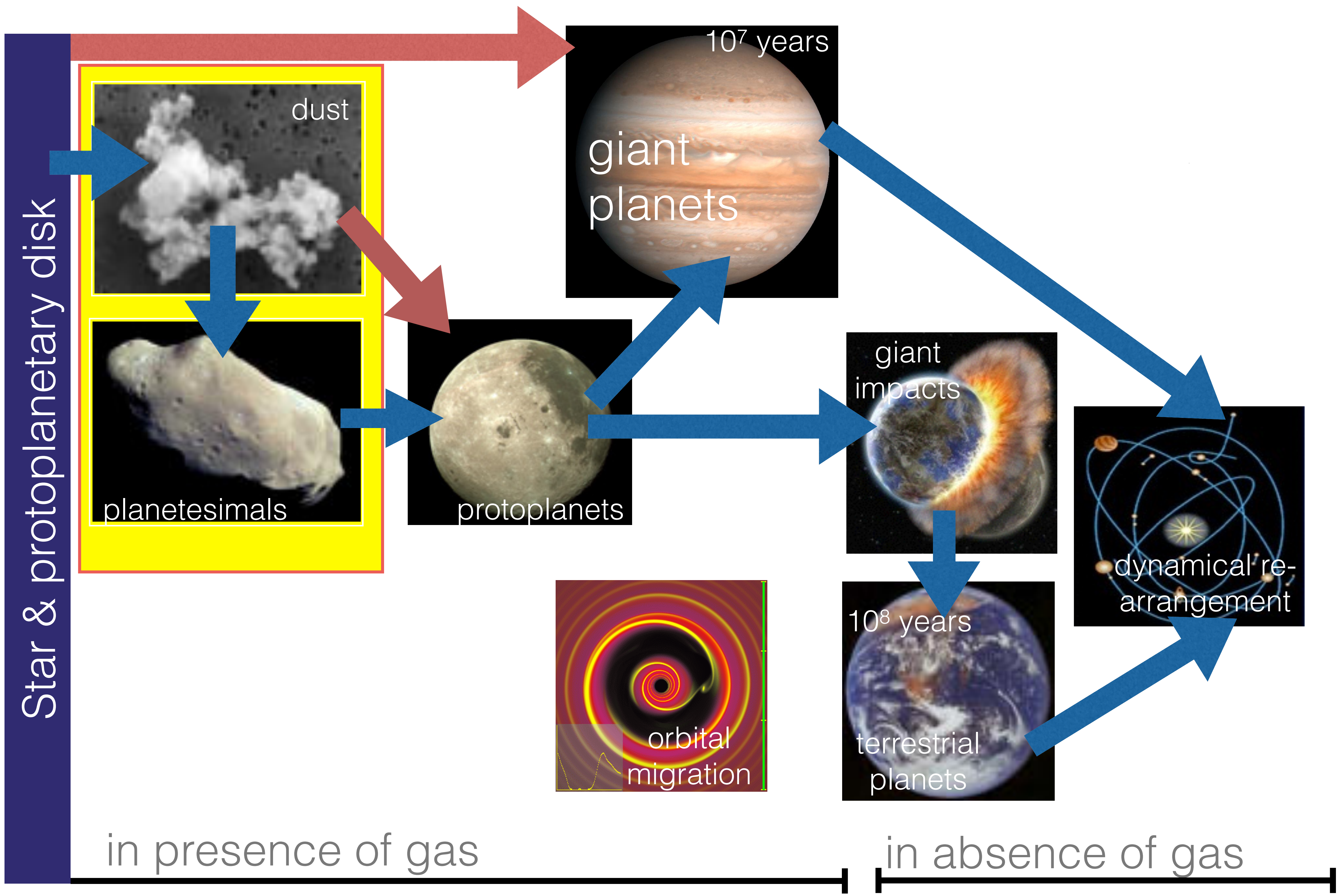


Inside (hot, T>~180 K): rocks only (silicates and metal)

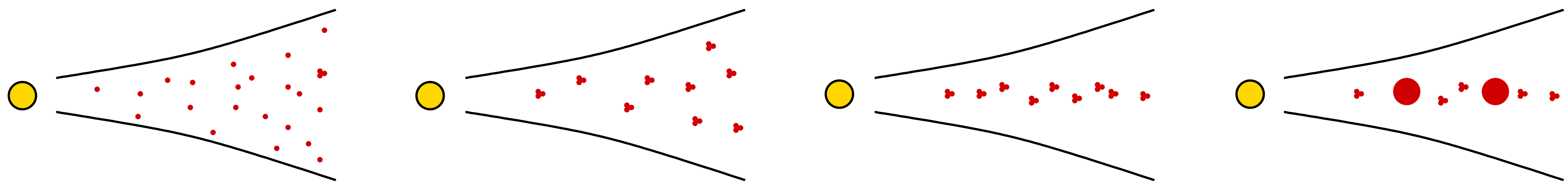
Jump at "iceline": Disk temperature small enough for ice to condense.

Outside(cold): ice and rocks

2. From dust to planetesimals



Early phases



The basic picture of the early stage of planet formation (growth from dust to km sized planetesimals) is the following:

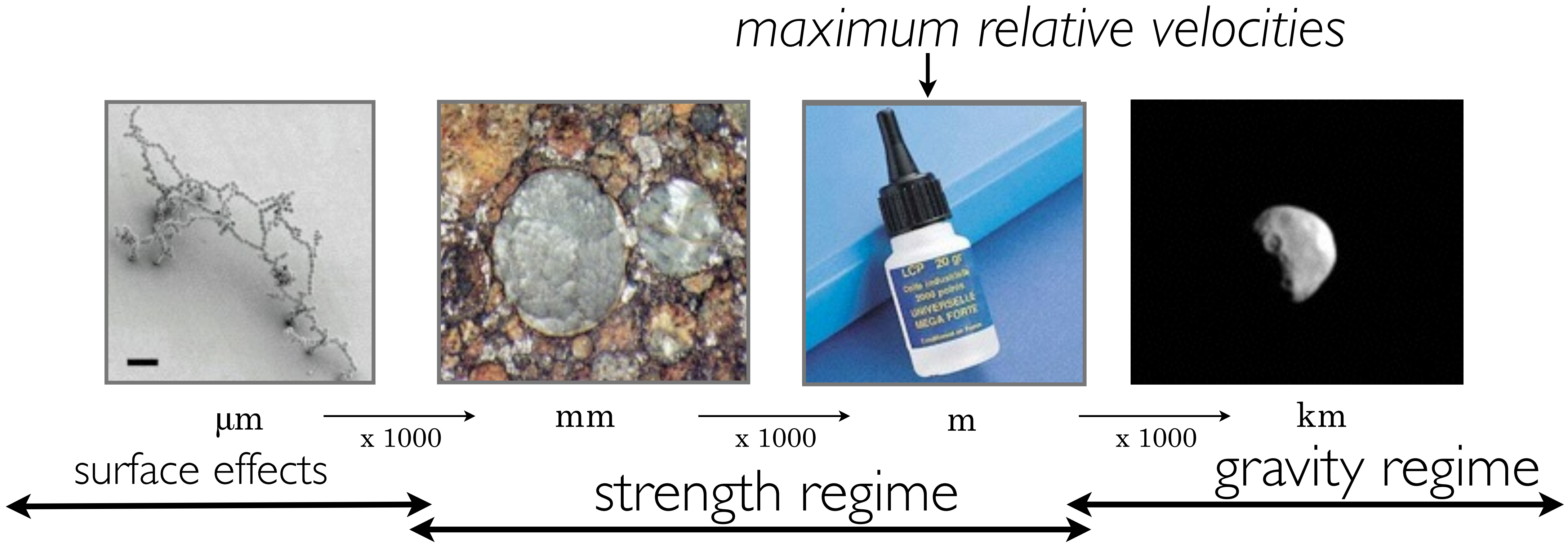
- The dust grains settle into a thin mid-plane layer in the disk (no vertical pressure gradient for solids).

- Dust grains condense, coagulate and gradually decouple from the gas. Gas drag is very important.

- Planetesimals (~km sized) form by continued coagulation (two body collisions) or a self-gravitational instability of the dust (or a combination of the two) in the dense mid-plane layer.

Dust to planetesimals

- solids and gas do not orbit the star at the same speed
  → gas drag causes dust to drift towards the star
  → gas drag & turbulence determines the collision velocities

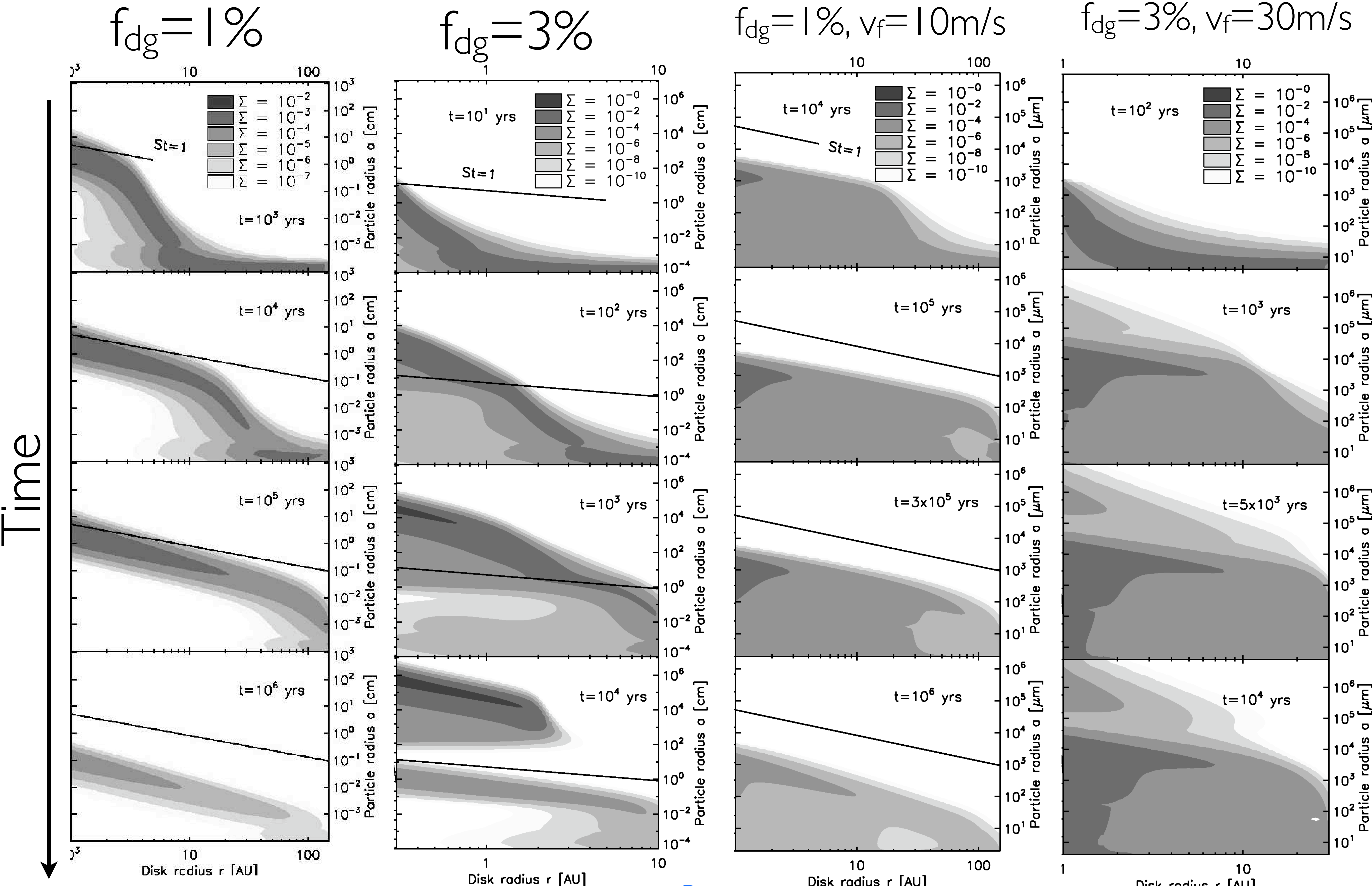


So called "meter-barrier" for classical coagulation. Double trouble:

-Drift barrier (drift timescale only 100 yr for 1 m body at 1 AU!)

-Fragmentation barrier (typical relative velocities for 1 m bodies lead to destructive collisions)

Classical coagulation

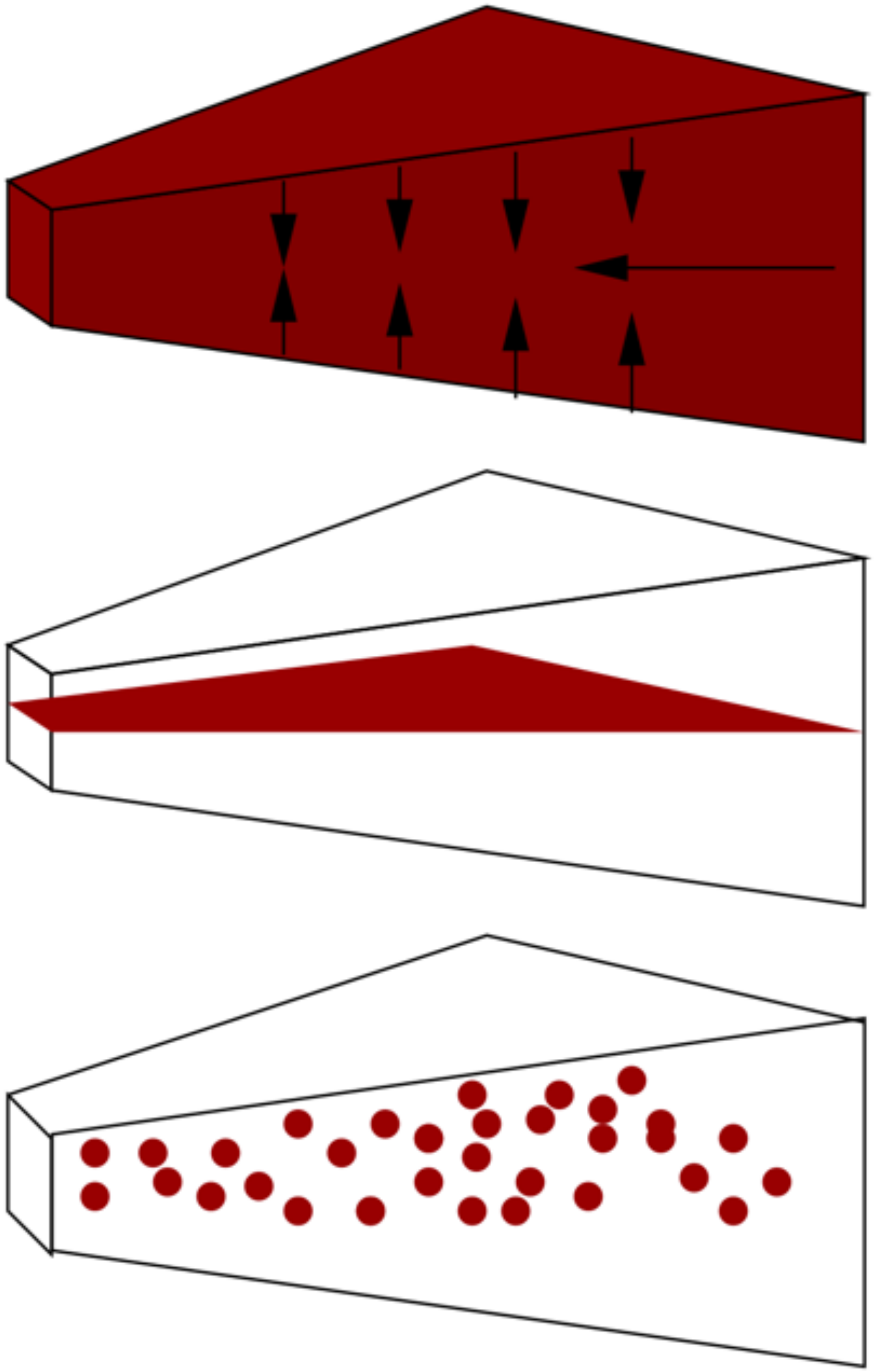


No fragmentation

Brauer et al. 2008

With fragmentation

Alternative: Goldreich-Ward mechanism

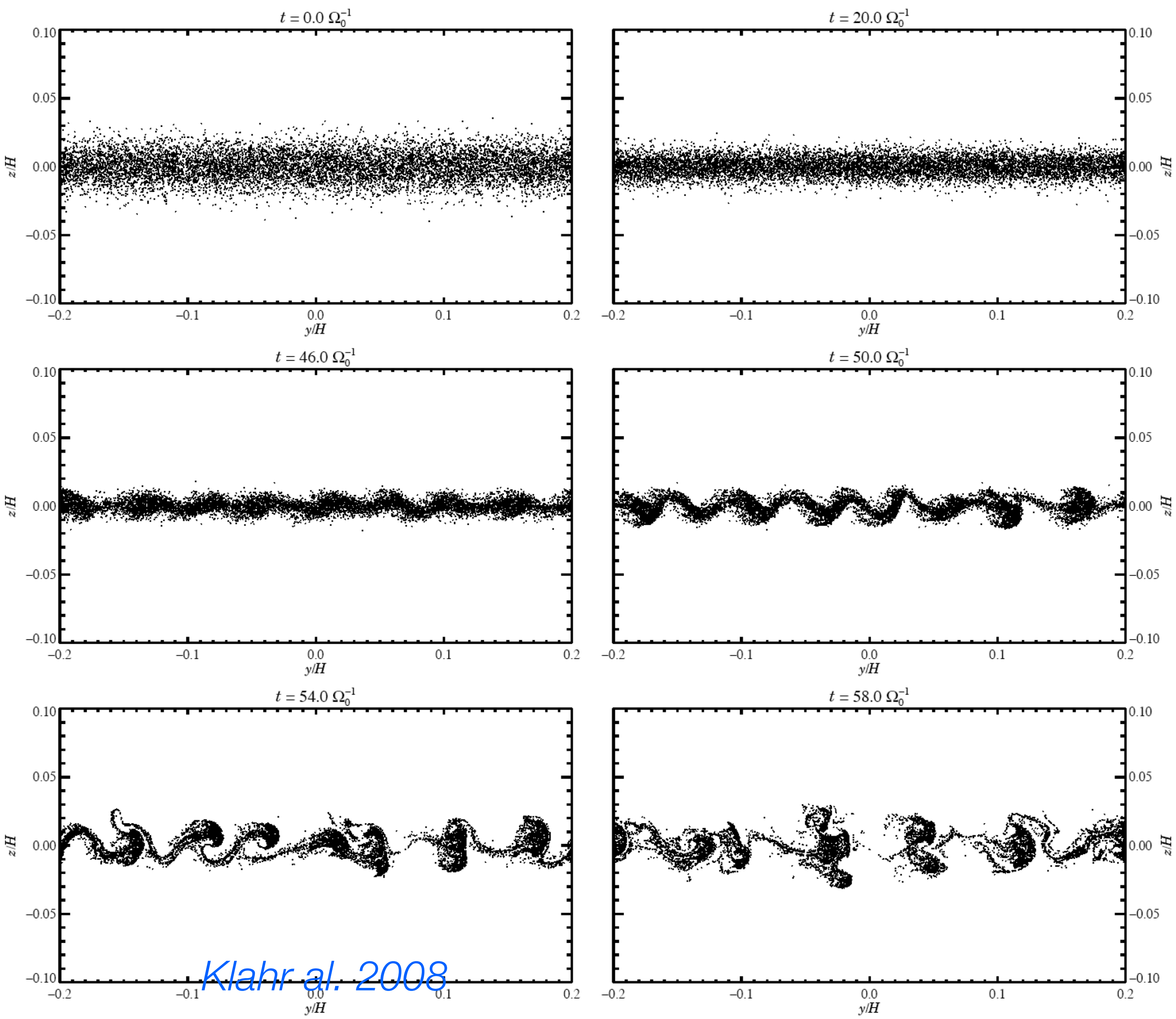


Dust settles into the midplane into a thin sheet: for sufficiently high dust concentration: unstable to a self-gravity. (Goldreich & Ward 1973)

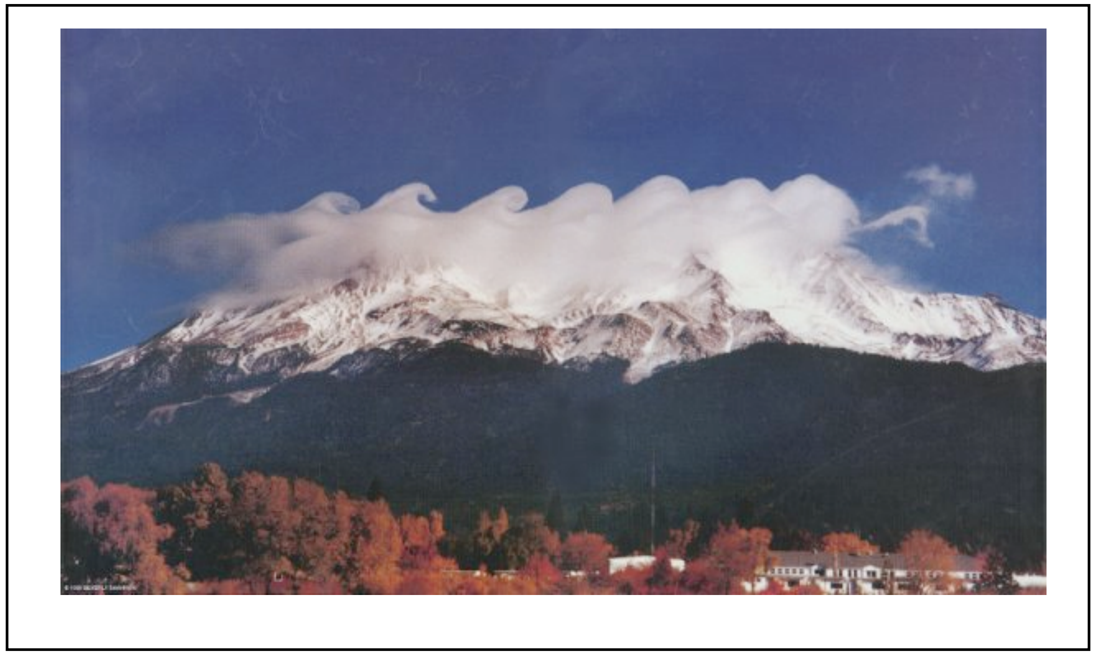
The turbulent speed of grains must however be low to reach the necessary concentration.

Alternative: Goldreich-Ward mechanism

Vertical shear between keplerian dust disk and subkeplerian gas above causes KH instabilities: stir up dust: no collapse possible.



Klahr al. 2008

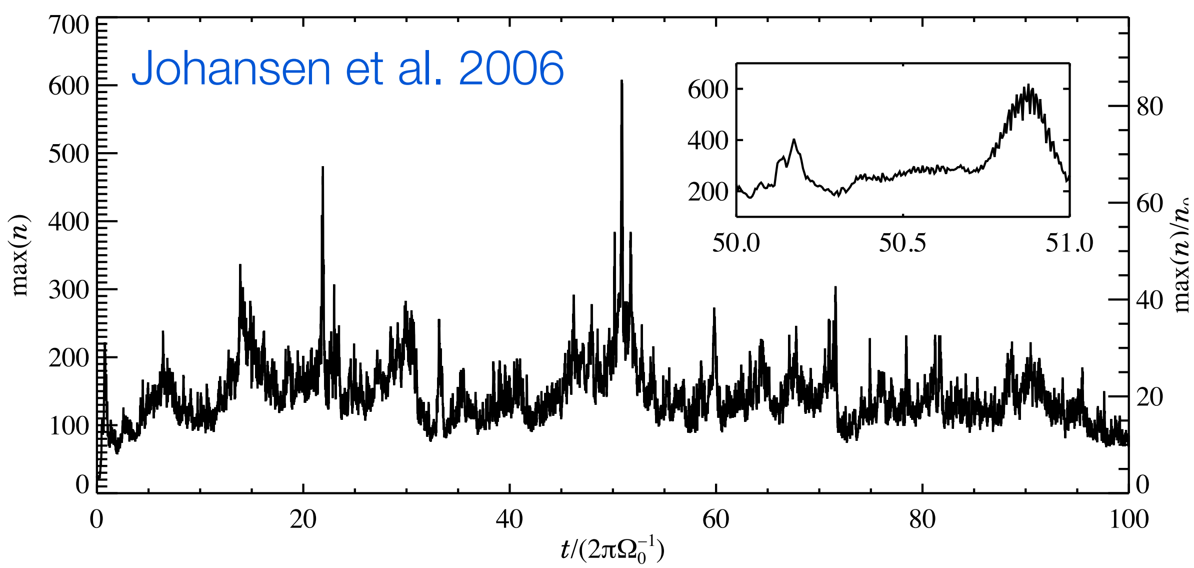


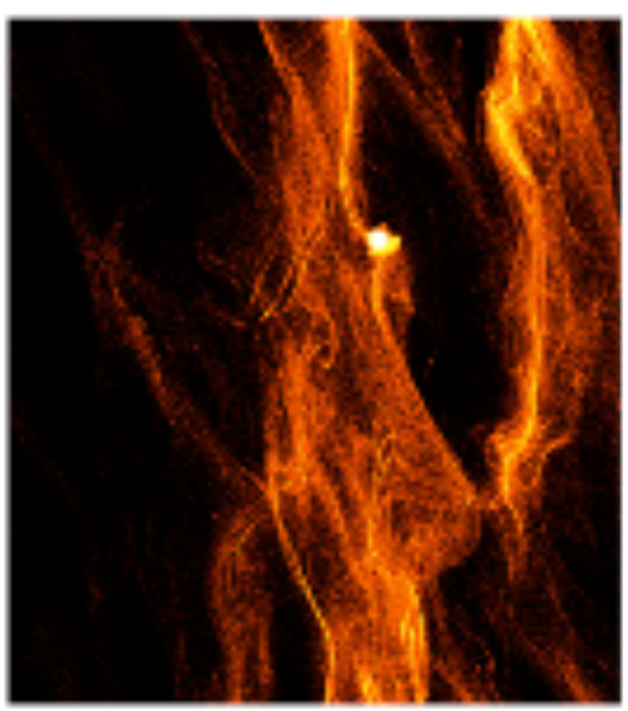
Preliminary conclusion: Turbulence *prevents* self gravitational formation

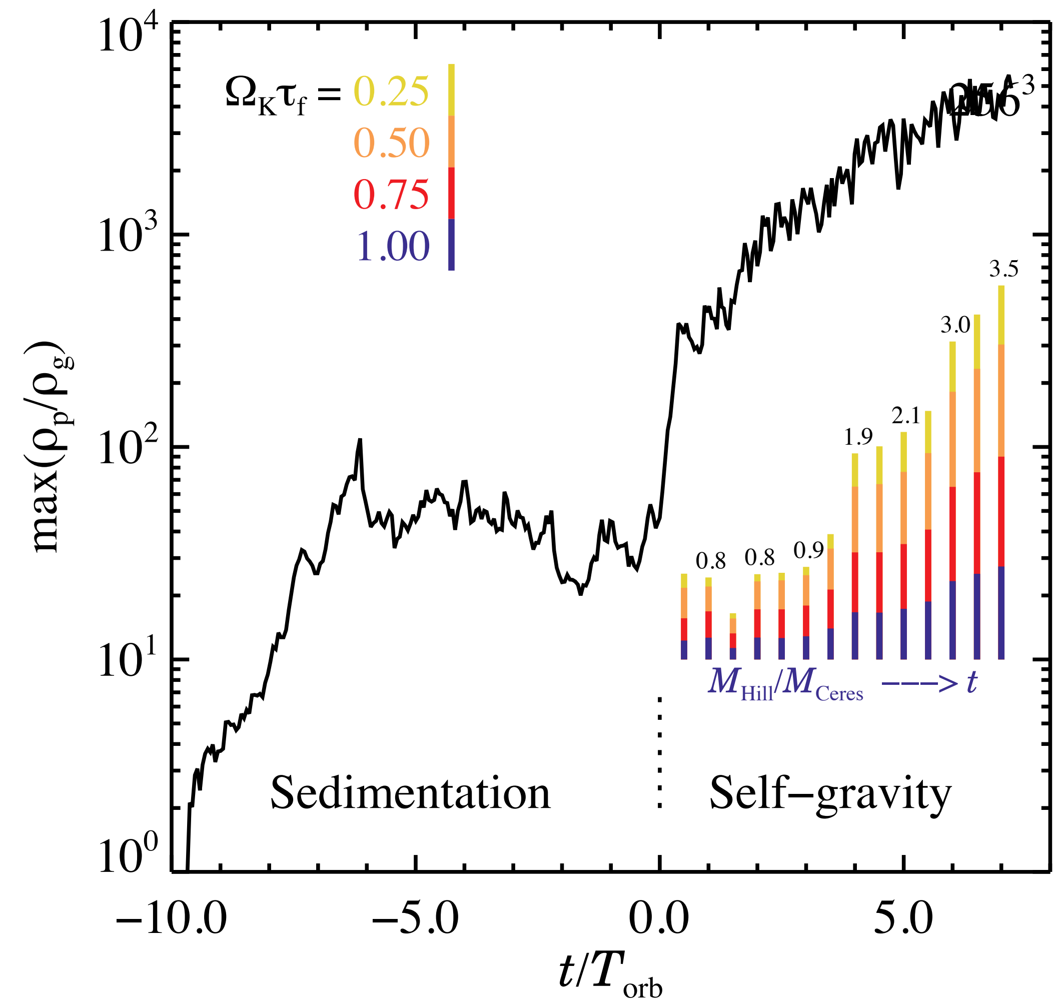
New picture: Gravoturbulent planetesimal formation

Dust trapped *locally* in transient gas vortices in a turbulent disk or concentrated by the streaming instability can eventually become gravitationally bound.

Klahr & Johansen 2008

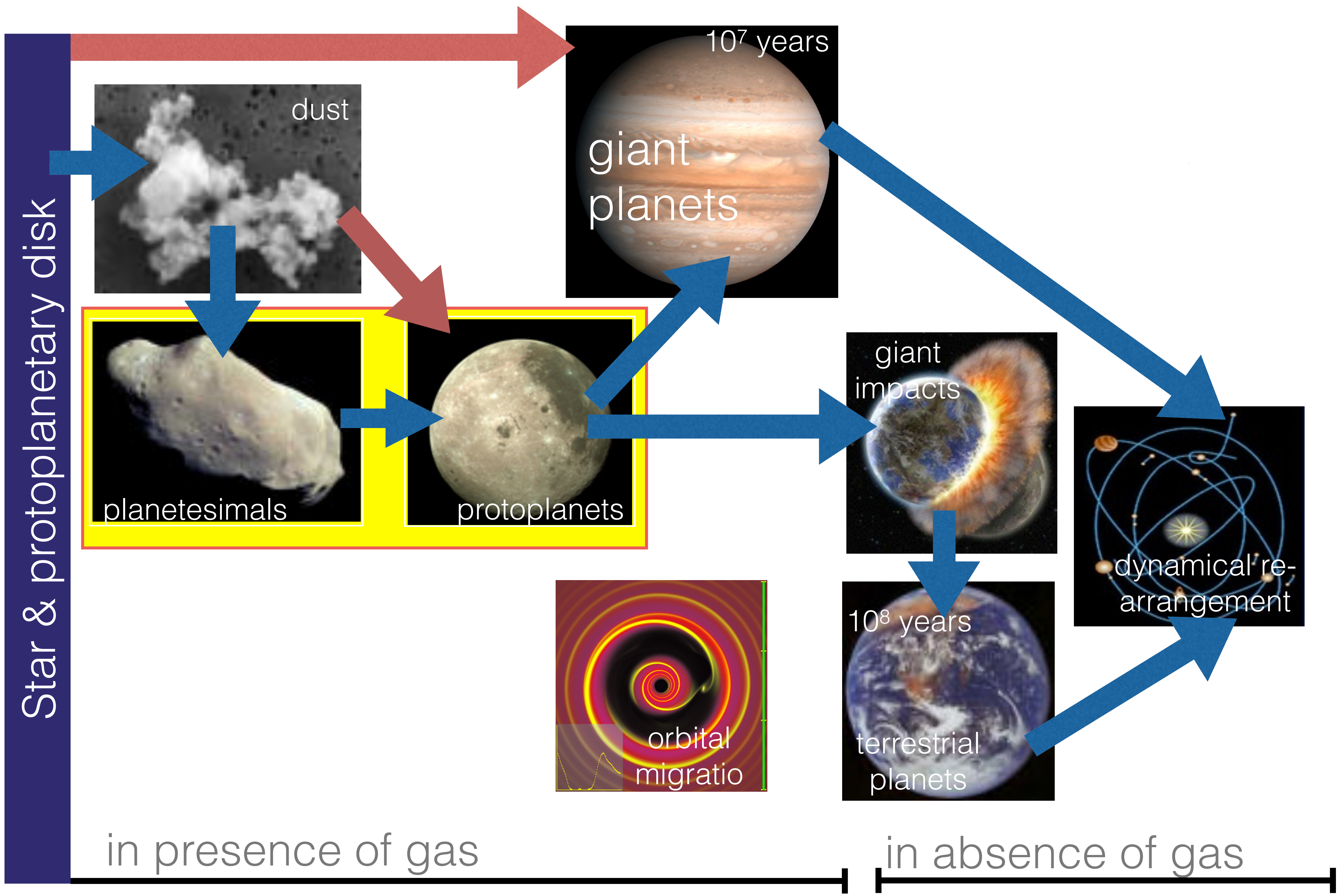






Turbulence aided growth might proceed from pebbles directly to intermediate-sized (100-1000 km) objects.

3. From planetesimals to protoplanets



Growth from ~km to protoplanets (~1000 km)

Growth in this size range:

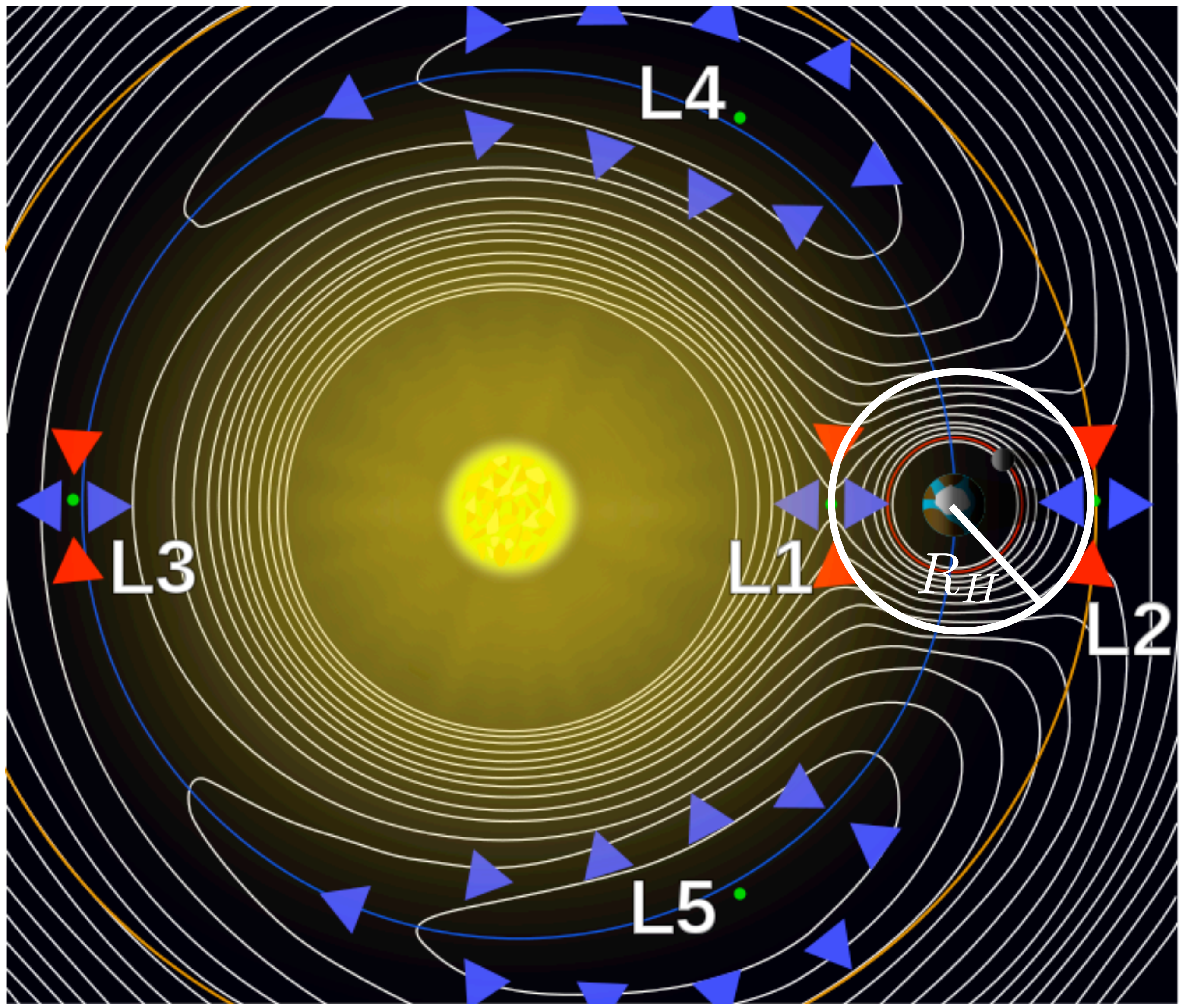
- via two body collision (collisional growth).
- Compared to the earlier stages, gravity is now dominant
- But gas drag still plays a role

Still, the growth from ~km sizes planetesimals to ~1000 km sized protoplanets is still difficult to understand:

- Initial conditions poorly known: how do the first planetesimals form?
- Huge number of planetesimals to follow (no direct integration of Newtons law of gravity): 10 M_{Earth} > 10^8 rocky bodies with R=30 km
- Highly non-linear with complex feed-back mechanisms
  - growing bodies play an increasing role in the dynamics
- Non-trivial impact physics: shock waves, multi-phase fluid, fracturing

Background: Hill sphere

- Idealized system: Star - Planet on circular orbit - massless planetesimal
- Energy & momentum conservation: separate (in the rotating coordinate system) regions which are accessible to the massless particle (Jacobi integral).



Important consequence:

Hill sphere: region where planet gravity dominant over stellar gravity. Between the Lagrangian points L_1 and L_2.

It is a measure of the gravitational reach of a planet.

Background: Hill sphere

Estimate: equate orbital frequency of an orbit around the planet with orbital frequency of an orbit around the star:

$$\left(\frac{Gm}{R_H^3}\right)^{1/2} \simeq \left(\frac{GM}{a^3}\right)^{1/2} \simeq \Omega$$

This leads to a similar result as the exact derivation:

$$R_H = \left(\frac{m}{3M}\right)^{1/3} a$$

The width of the feeding zone of a planet: a few times R_H

$$w_{feed} = \tilde{b} R_H \qquad \tilde{b} = 5 - 10$$

Examples:

Earth:	$a = 1\text{AU}$	$m = 6 \times 10^{24}\text{kg}$	$R_H = 0.014\text{AU}$
Jupiter:	$a = 5.2\text{AU}$	$m = 1.9 \times 10^{27}\text{kg}$	$R_H = 0.51\text{AU}$
Neptune:	$a = 30.14\text{AU}$	$m = 1.03 \times 10^{26}\text{kg}$	$R_H = 1.12\text{AU}$

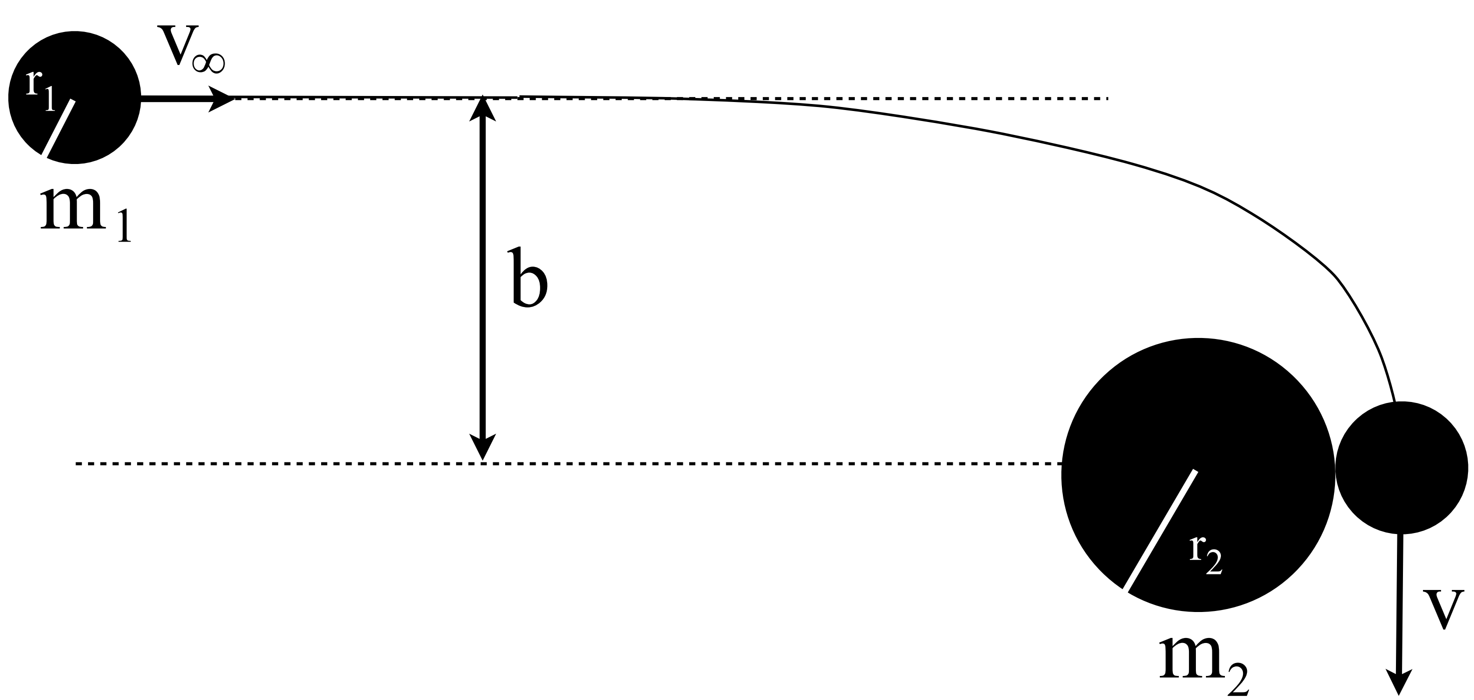
3.1 Focussing factor

Gravitational focussing: 2 body

Billiard game: collisional cross section=geometrical cross section

$$\sigma = \sigma_{geo} = \pi(r_1 + r_2)^2$$

Gravity: increase of the collisional cross section over the geometrical one (gravitational focussing).



$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$1)\ \text{Conservation of energy}:\ E = \underbrace{\frac{1}{2}\mu v_\infty^2}_{\text{at } \infty} = \underbrace{\frac{1}{2}\mu v^2 - G\frac{m_1 m_2}{r_1 + r_2}}_{\text{at closest approach}}$$

$$2)\ \text{Conservation of angular momentum}:\ J = \underbrace{\mu b v_\infty}_{\text{at } \infty} = \underbrace{\mu(r_1 + r_2)v}_{\text{at closest approach}} \longrightarrow v = \frac{b v_\infty}{r_1 + r_2}$$

Gravitational focussing: 2 body

Combining gives

$$b^2 = (r_1 + r_2)^2 \left(1 + \frac{v_{esc}^2}{v_\infty^2}\right)$$

with the escape velocity given as

$$v_{esc} = \sqrt{\frac{2G(m_1 + m_2)}{r_1 + r_2}}$$

This means that the collisional cross-section σ is given as:

$$\sigma = \pi r^2 = \pi (r_1 + r_2)^2 \left(1 + \left(\frac{v_{esc}}{v_\infty}\right)^2\right)$$

geometrical cross-section | gravitational focusing factor F_g

Focussing factor: proportional to square of the escape to random velocity.
Random velocity: excess over the velocity on a circular orbit.

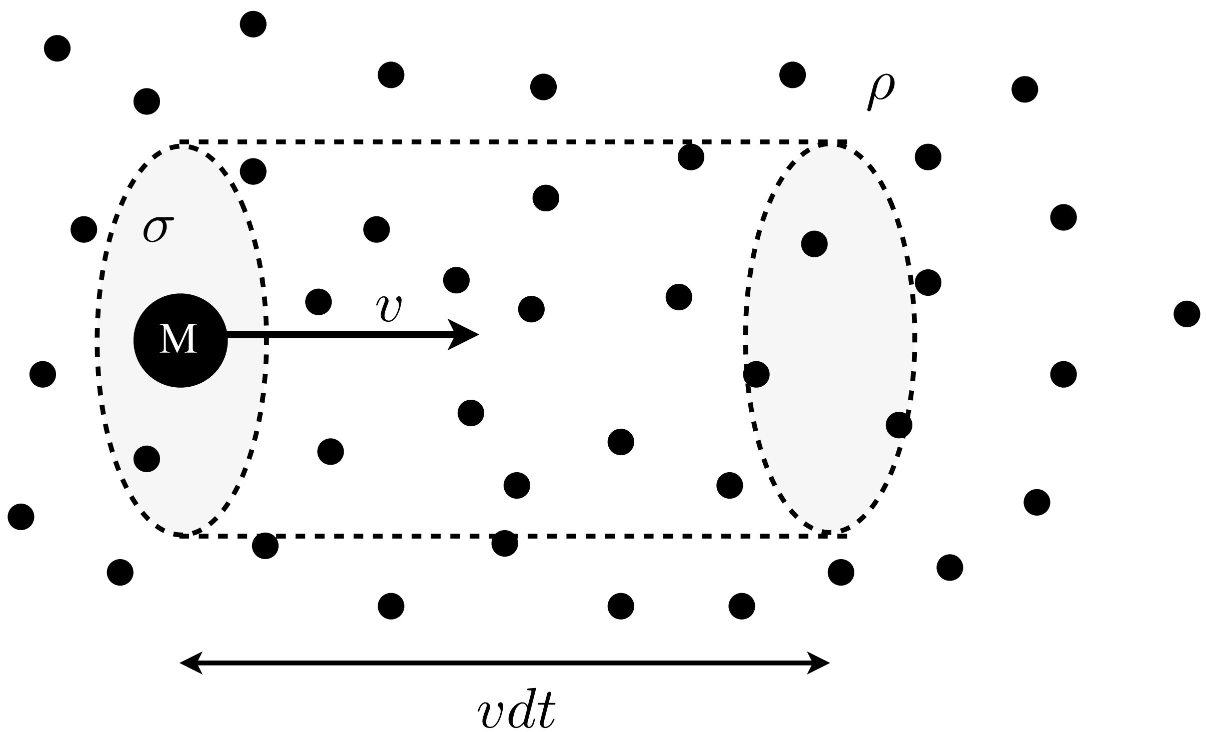
In honor of V. Safronov, a Russian scientist who was the first to develop this collisional accretion scenario, one often uses the so called Safronov number

$$\Theta = \left(\frac{v_{esc}}{v_\infty}\right)^2 \qquad \rightarrow \sigma = \sigma_{geo}(1 + 2\Theta)$$

3.2 Growth rate

Mass growth rate

Scenario: one big body accreting from small background planetesimals.



Growth rate: cylinder swept per time

$$dm_p = \rho_s \sigma v_\infty dt$$

$$\frac{dm_p}{dt} \simeq \rho_s v \sigma = \rho_s v_\infty \left(\pi r^2 \left(1 + \frac{v_{esc}^2}{v_\infty^2} \right) \right)$$

Using $\rho_s = \frac{\Sigma_s}{2H_s}$ and estimating planetesimal vertical scale height as

$\frac{H_s}{a} \approx \frac{v_\infty}{v_K} = \frac{v_\infty}{a\Omega}$ we have $\frac{dm_p}{dt} = \frac{1}{2}\Sigma_s \Omega \pi r^2 \left(1 + \frac{v_{esc}^2}{v_\infty^2} \right)$

For an isotropic velocity distribution one finally finds:

$$\frac{dm_p}{dt} = \frac{\sqrt{3}}{2}\Sigma_s \Omega \pi r^2 \left(1 + \frac{v_{esc}^2}{v_\infty^2} \right) = \frac{\sqrt{3}}{2}\Sigma_s \Omega \pi r^2 F_g$$

Mass growth rate II

$$\frac{dm_p}{dt} = \frac{\sqrt{3}}{2}\Sigma_s\Omega\pi r^2\left(1+\frac{v_{esc}^2}{v_\infty^2}\right) = \frac{\sqrt{3}}{2}\Sigma_s\Omega\pi r^2 F_g$$

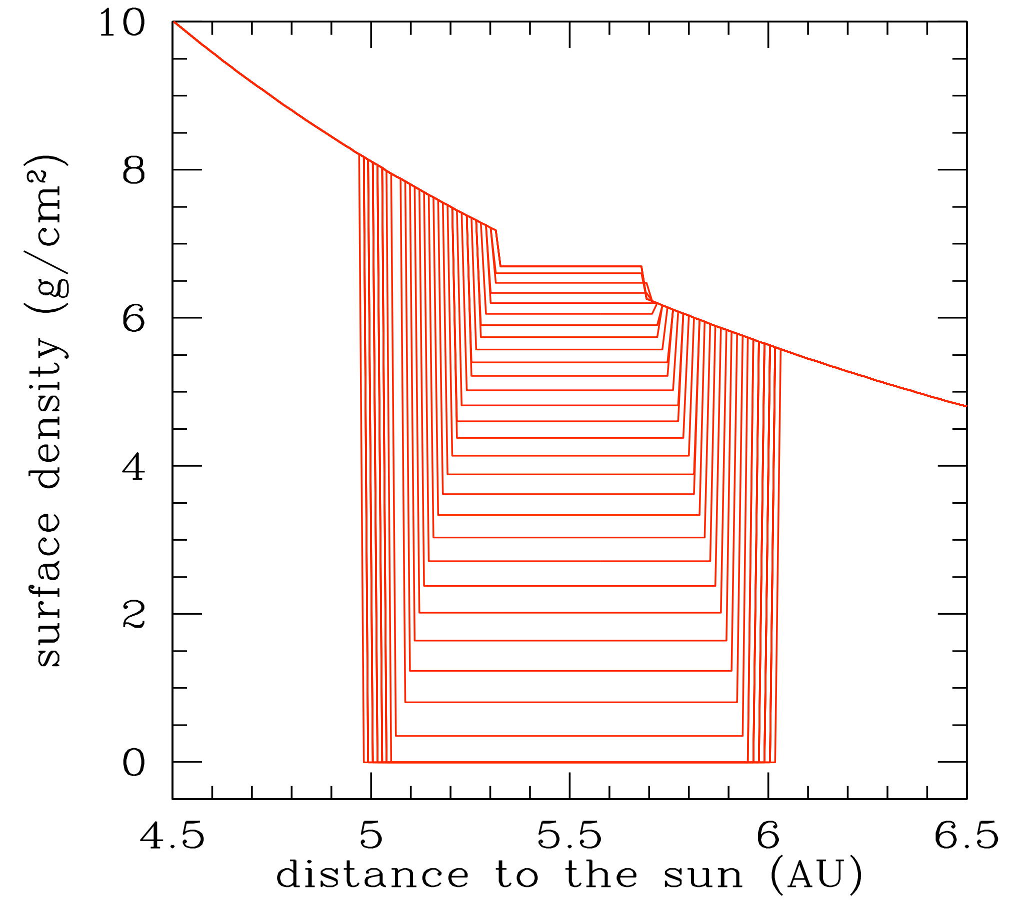
Notes:

- the velocity dispersion (random velocities) of planetesimals is the key factor.
- the growth rate is larger in disks with larger planetesimal surface densities.
- $\Sigma_s\Omega$ generally decrease with distance: planets grow slower at large distance

Decrease of planetesimal surface density

Protoplanet growth=>decrease of surface density of planetesimals. For accretion from a feeding zone with spatially constant planetesimal surface density for a planet with semimajor axis a

$$\frac{d\Sigma_s}{dt} = -\frac{(3M_*)^{1/3}}{6\pi a^2\tilde{b}_{max}m_p^{1/3}}\frac{dm_p}{dt}$$



3.3 Isolation mass

Isolation mass

Embryo grows by accreting planetesimals: empties it surroundings.
At the same time extends its gravitational reach (Hill radius): new planetesimals available to accrete.

The mass of the embryo accreting from an annulus is approximately

$$M = 2\pi a 2\Delta a \Sigma_p(a)$$

The width of the annulus is given by the feeding zone

$$\Delta a = w_{feed} = \widetilde{b} R_H$$

Since the mass of reachable planetesimals grows slower than linearly, the growing embryo will eventually become starved of planetesimals and reach a maximum mass, the so-called isolation mass.

We obtain the value by solving

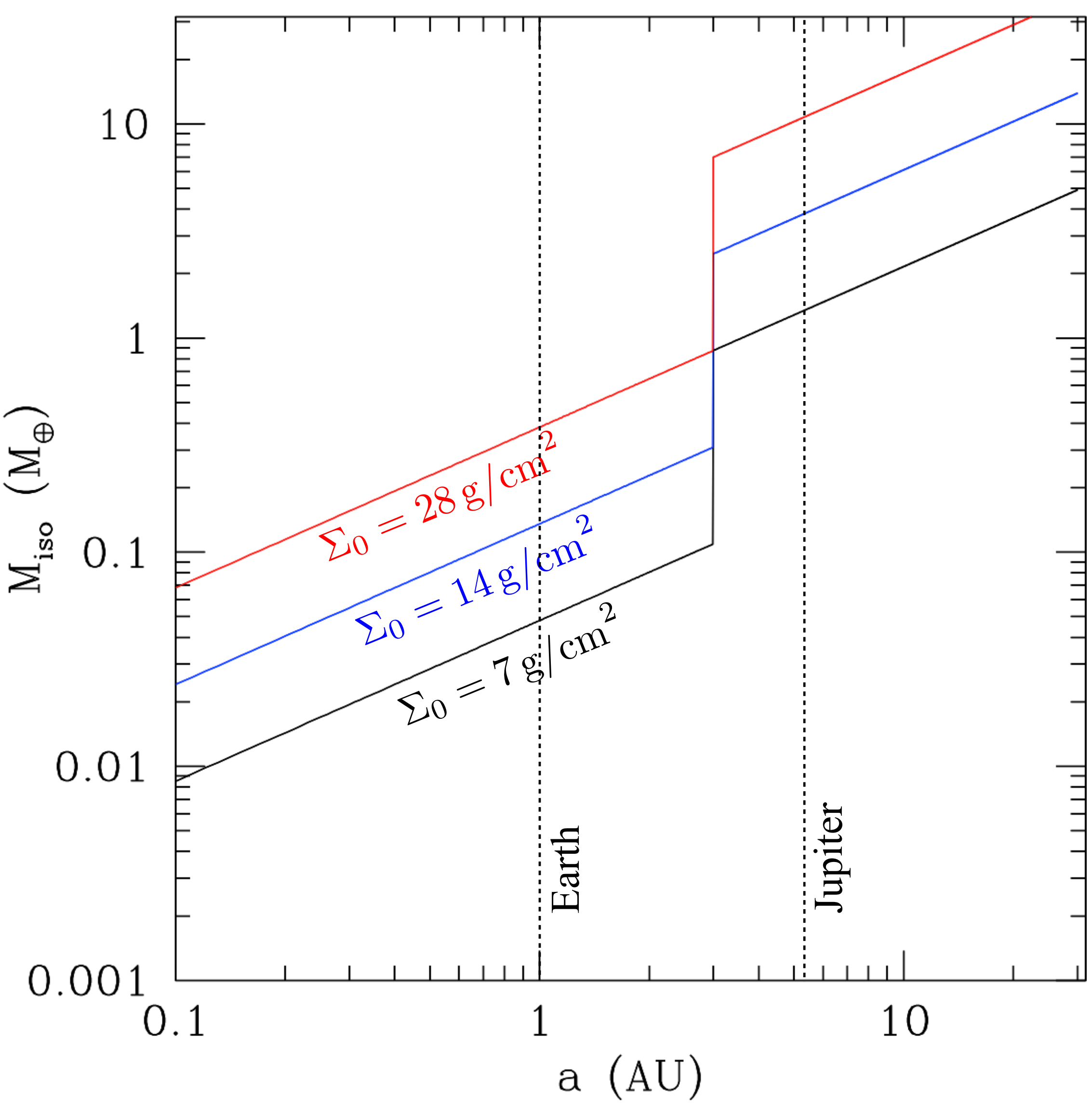
$$M_{iso} = 2\pi a 2\tilde{b} R_H \Sigma_p(a) = 4\pi a^2 \tilde{b} \Sigma_p(a) \left(\frac{M_{iso}}{3M_*} \right)^{1/3}$$

Isolation mass II

This yields

$$M_{iso} = \frac{\left(4\pi\tilde{b}a^2\Sigma_p(a)\right)^{3/2}}{(3M_*)^{1/2}}$$

For Σ falling slower than a^{-2} : M_{iso} increase with distance.



- For MMSN: M_{iso} ≅ 0.05 M_{earth} at 1 AU
  M_{iso} ≅ 1.4 M_{earth} at 5.2 AU
- Embryos must coalesce beyond M_{iso} to form terrestrial planets in inner solar system
- Difficult to form bodies of 10 Earth mass in the Jupiter region unless Σ > 3 MMSN.
- M_{iso} maximal for *in situ* accretion on a *circular* orbit.
  - Orbital migration changes the game
  - Dust/Pebble/Planetesimal drift also.
  - Eccentricity too. But must excite...

3.4 Growth regimes

Runaway growth

- First stage of collisional growth of planetesimals to protoplanets

Runaway growth mechanism

0)spontaneous formation of one body (slightly) more massive than the other

1)equipartition of energy: e and i of the big body small.

2)e and i of small bodies (in the early stage) not affected/increased.

3)the relative velocity between the big and the small body becomes small.

4)at the same time, v_{esc} of the big body increase due to its increase in mass.

5)F_g of the big body thus becomes $$F_g = \left(1 + \frac{v_{esc}^2}{v_\infty^2}\right) \gg 1$$

The small bodies have in comparison a much smaller F_g.

6)the runaway body grows faster than the planetesimals, consuming all planetesimals in the feeding zone (in principle). It decouples from the mass distribution of the small ones.

A clearly a strongly nonlinear process.

Runaway growth II

For the focussing factor we have: $$F_g = \left(1 + \frac{v_{esc}^2}{v^2}\right) \simeq \frac{v_{esc}^2}{v^2} = \frac{1}{v^2}\frac{2GM}{R}$$

For the mass accretion rate this means $$\frac{dM}{dt} = \pi G \frac{\Sigma_p \Omega}{v^2} M R \propto R^4$$

or in relative terms

$$\frac{1}{M}\frac{dM}{dt} \propto M^{1/3}$$

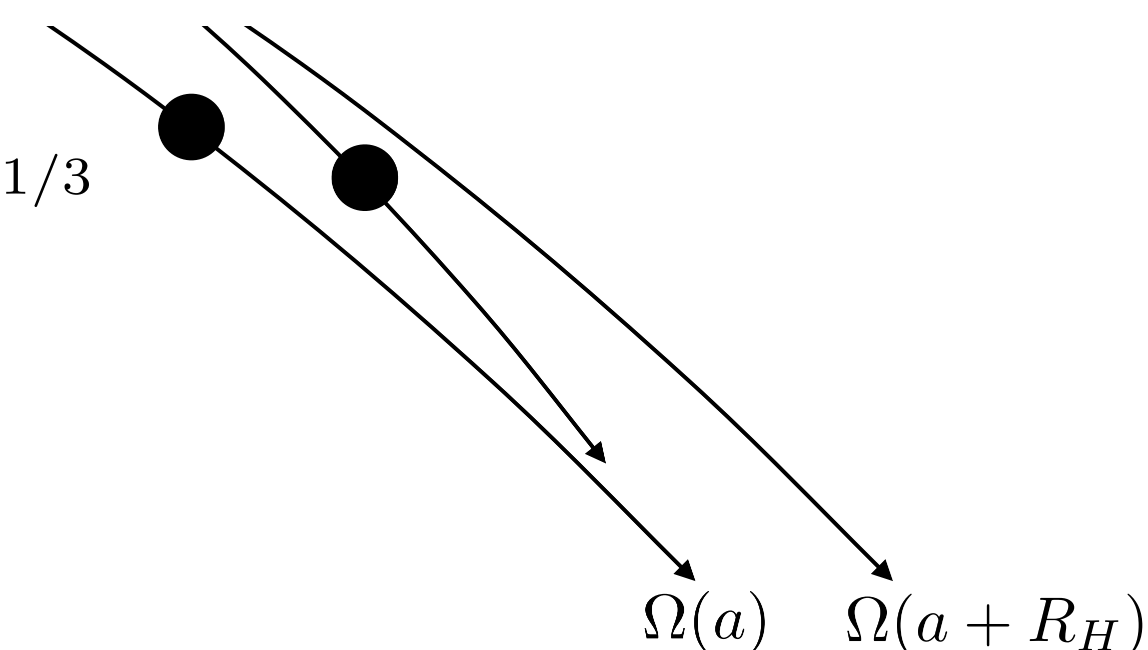
The bigger the body, the faster it grows!

How fast can it get (3 body effect)?

$$v = \Omega R_H$$

$$\frac{v_{esc}^2}{v^2} = \frac{2GM}{R}\frac{1}{\Omega^2 R_H^2} \approx 4.16\,\frac{a}{R}\left(\frac{M}{M_\odot}\right)^{1/3}$$

$$\longrightarrow \frac{v_{esc}^2}{v^2} \approx \text{few } 10^3$$



Oligarchic growth

- Second stage of collisional growth of planetesimals to protoplanets

When bodies have grown to a certain mass (~0.01 M_{earth}), growth mode changes to oligarchic. Big bodies are now called oligarchs.

Initially, planetesimal disk not affected by the presence of the bigger protoplanets: runaway. Later however,

- runaway bodies become the main scatterer.
- It “heats” up (increases) the random velocities of the small bodies.

Clearly, reduces the gravitational focussing factor

$$F_g = \left(1 + \frac{v_{esc}^2}{v_\infty^2}\right)$$

As a result, more massive bodies grow more slowly than the less massive ones (similar to orderly growth, cf below), but protoplanets still grow faster than planetesimals in their surroundings (similar to runaway growth).

Oligarchic growth II

In the oligarchic regime, the growth of the velocity dispersion is dominated by the big body, and focusing is strong.

$$\frac{dM}{dt} = \frac{1}{2}\Sigma_p \Omega \pi R^2 \left(1 + \frac{v_{esc}^2}{v^2}\right) \propto M^{4/3}\left(e^2+i^2\right)^{-1}$$

The processes that affect e and i are: $v \approx \sqrt{e^2+i^2}v_K$

- Scattering of small bodies by large body: e, i ↗
- Large mass: Dynamical friction with small planetesimals: e, i ↘
- gas drag (leading to equilibrium for the planetesimals): all e, i ↘

Numerical experiments show that: $v \propto M^{1/3}$

With this we have $\frac{dM}{dt} \propto M^{4/3}\left(e^2+i^2\right)^{-1} \propto M^{2/3}$

The relative growth rate is

$$\frac{1}{M}\frac{dM}{dt} \propto \frac{1}{M^{1/3}}$$

i.e. slowing down with increasing mass. Growth proceeds towards a set of similar mass embryos.. (from where the name “oligarchy”).

Orderly growth

Once the gaseous nebula dispersed (after ~10 Myrs), and all planetesimals have been accreted into oligarchs:

- no mechanisms (gas damping, viscous friction) to damp the random velocities of the big bodies
- Gravitational scattering increases the random velocities to $v \sim v_{esc}$, meaning that F_g becomes ~ 1.

The collisional cross section is thus reduced to the geometrical cross section. Growth in this regime is very slow.

With $F_g = 1$, the master equation becomes

$$\frac{dM}{dt} = \frac{\sqrt{3}}{2} \Sigma_p \Omega \pi R^2$$

or in relative terms

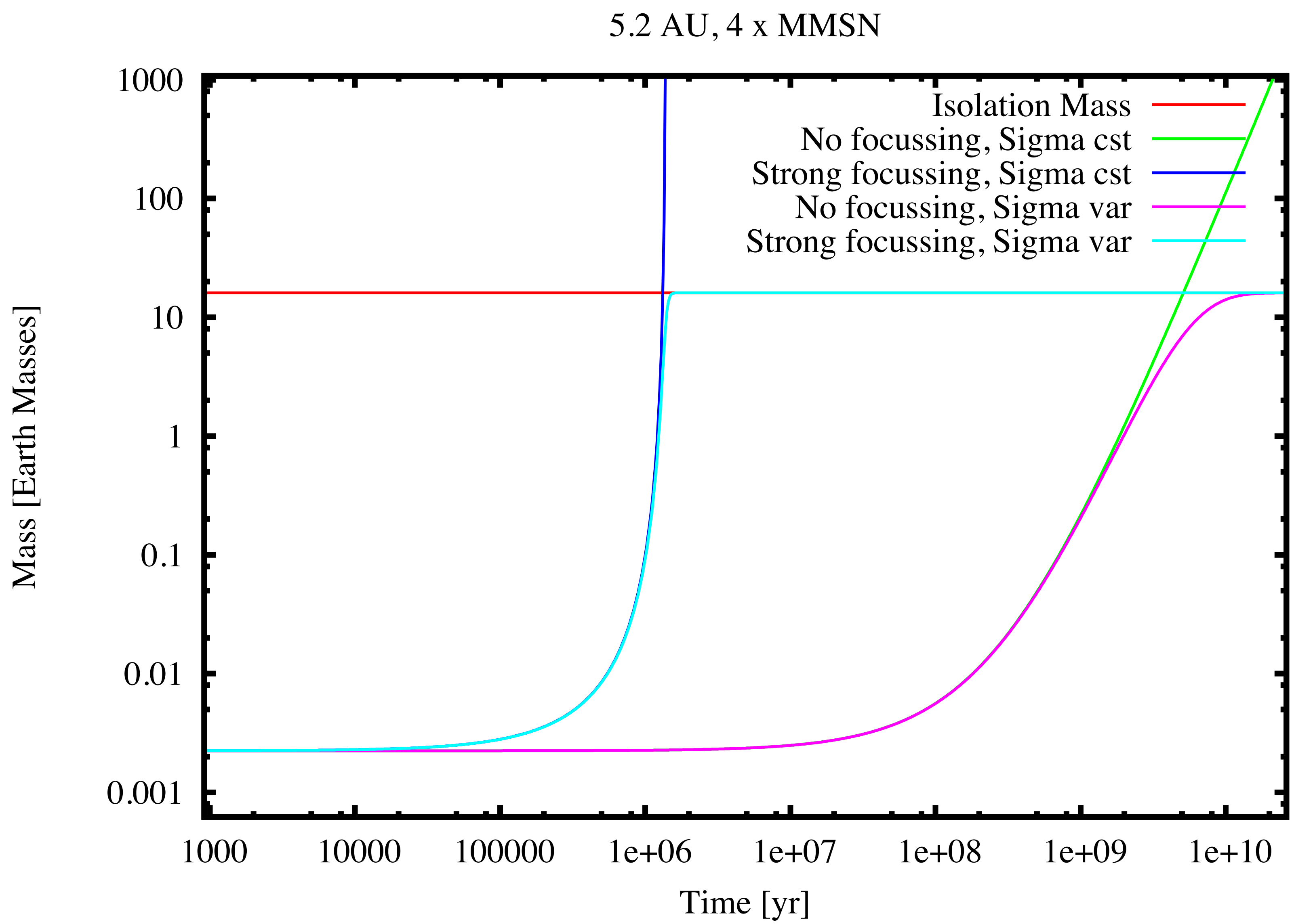
$$\frac{1}{M}\frac{dM}{dt} \propto \frac{1}{M^{1/3}}$$

The growth rate decreases with increasing mass as in the oligarchic regime. However, F_g is much smaller than in the oligarchic regime.

Orderly growth

Orderly growth is the final regime for planet growth, at least in the inner solar system.

Example: 5.2 AU, 4x MMSN



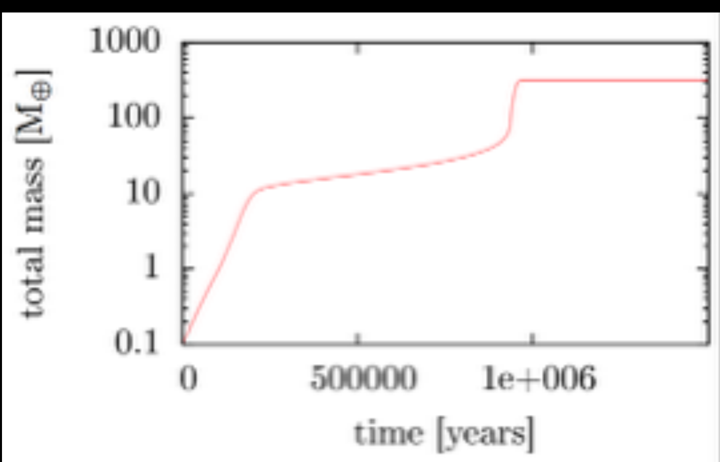
Note

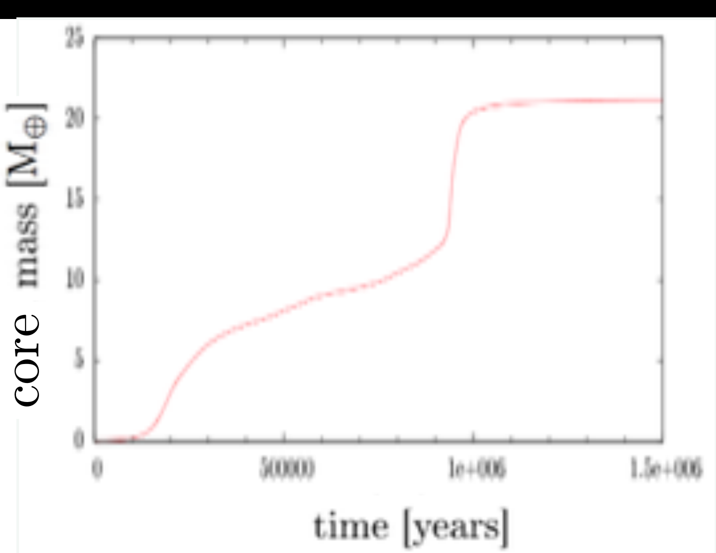
-In runaway, isolation mass reached in ~10^6 yrs

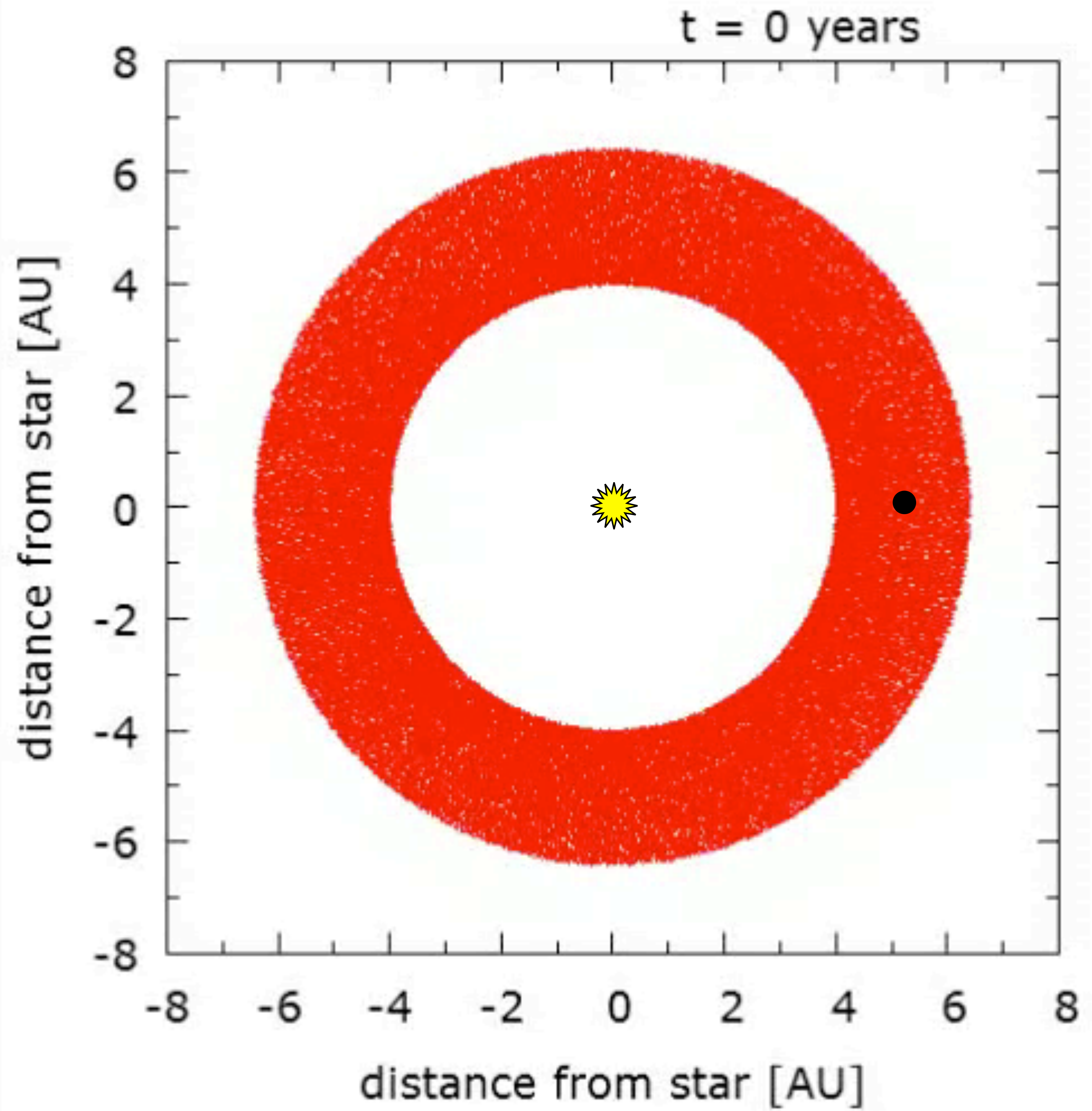
-In orderly growth, isolation mass reached in ~10^{10} yrs

N-Body simulation

- Star, planetesimal swarm & growing planet at 5.2 AU
- Corrotating coord. system
- Planet also accretes gas
- Rapid gas accretion at about 0.9 Myr

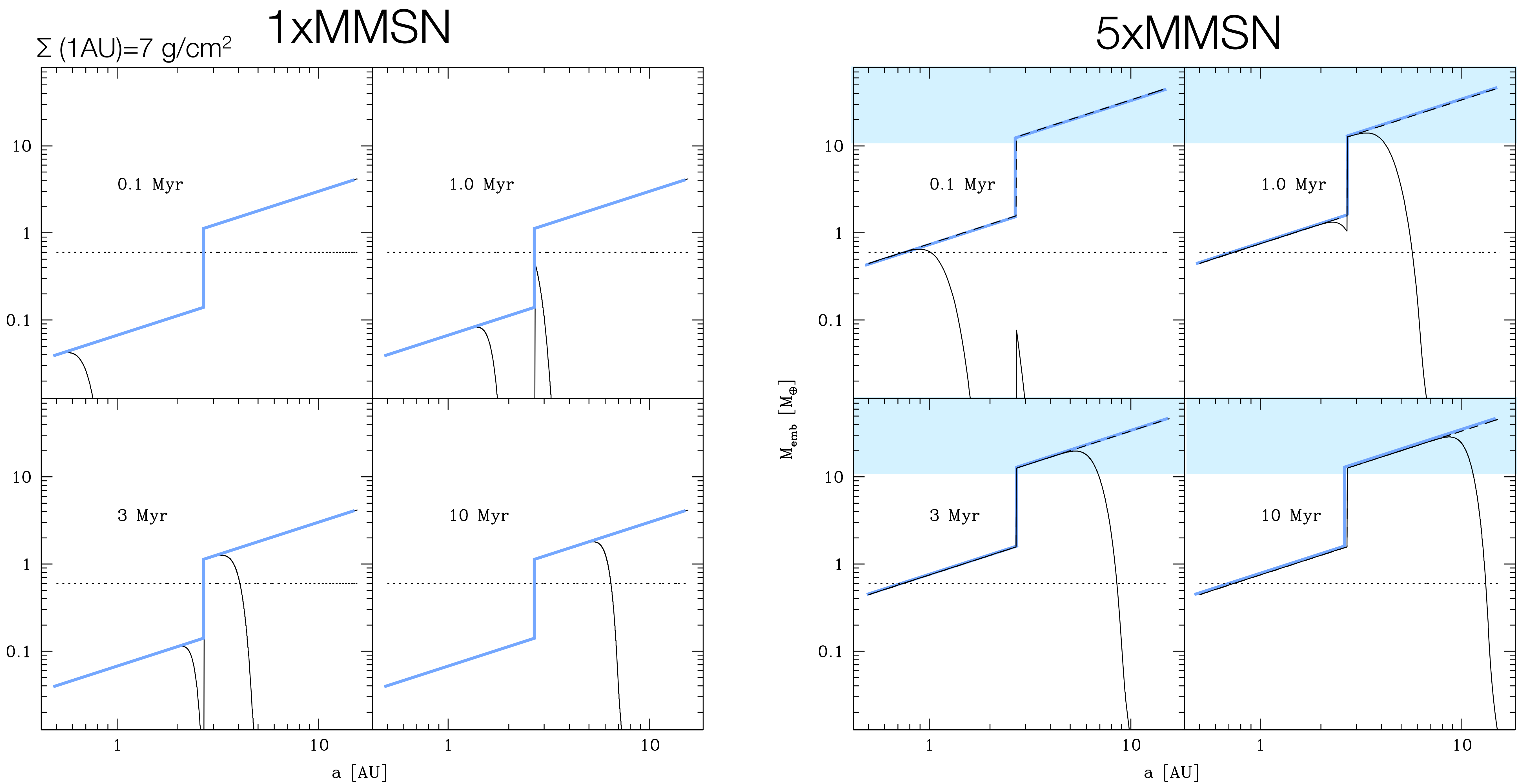






3.5 Growth as a function of semimajor axis

Growth as function of semimajor axis



Low random velocities: needs small planetesimals, full 3 body F_g

- Growth faster at small distances. Annulus of growth moves outwards.
- But stops at smaller (isolation) masses. No giant planet *in situ.*
- Quick *and* massive: Beyond the iceline (here @ 2.7 AU).
- Higher Σ: Protoplanets more massive & quicker: giant planet cores > 10 M_e.

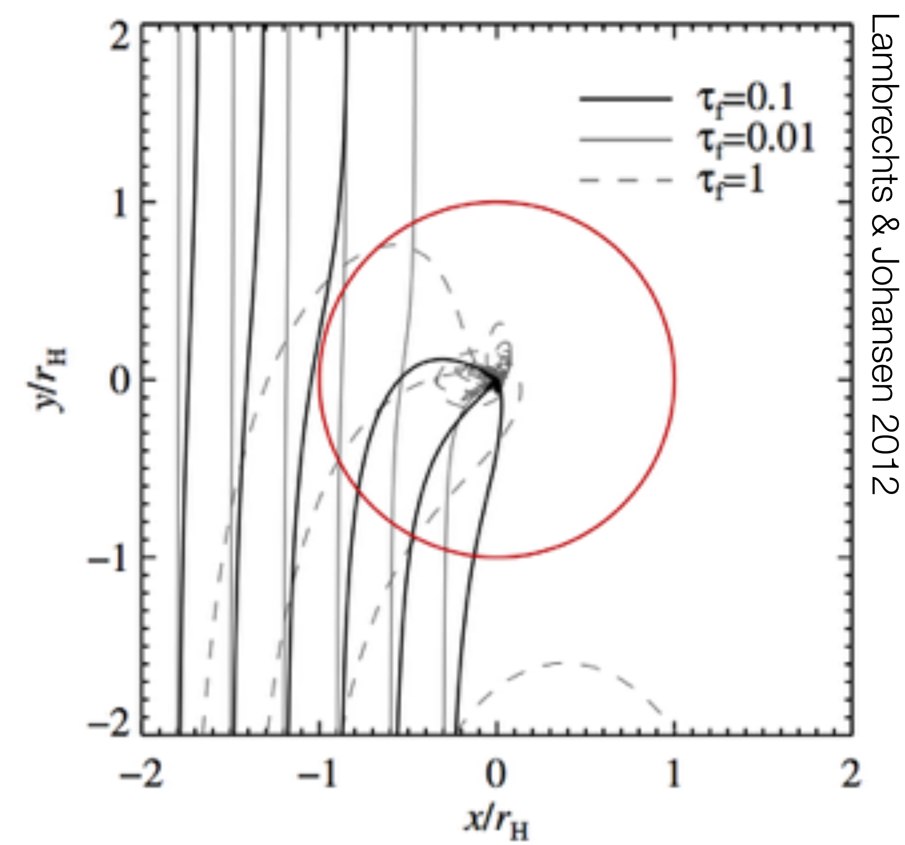
New vision: pebble accretion

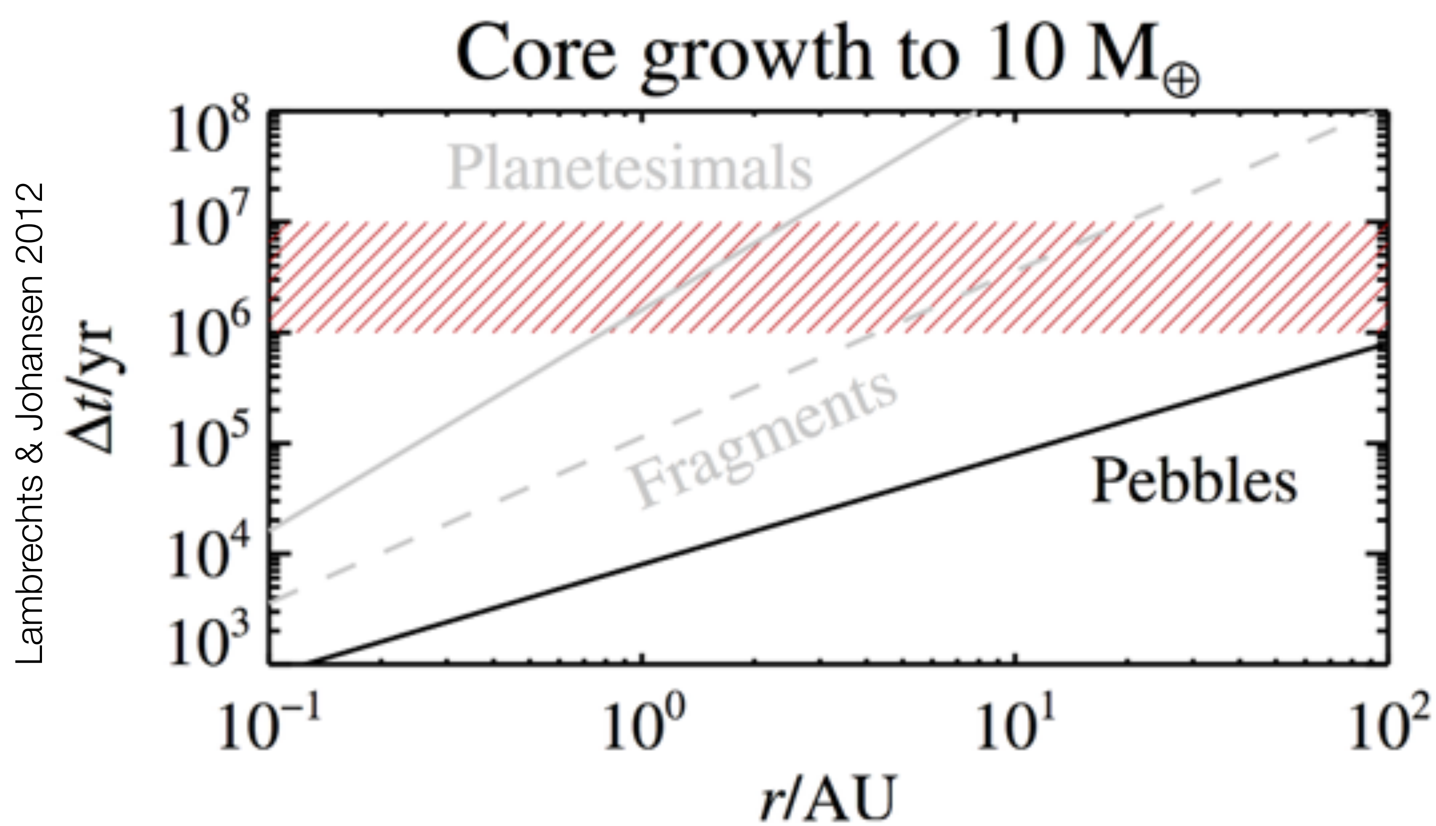
- Growth by accretion of pebbles instead of planetesimals

- Accretion rate: gravity *and* gas drag

$$\frac{dM}{dt} = 2R_H \Sigma_P v_{\rm H}$$

for bodies with $t_{enc} \approx t_{friction}$ (1-100 cm)





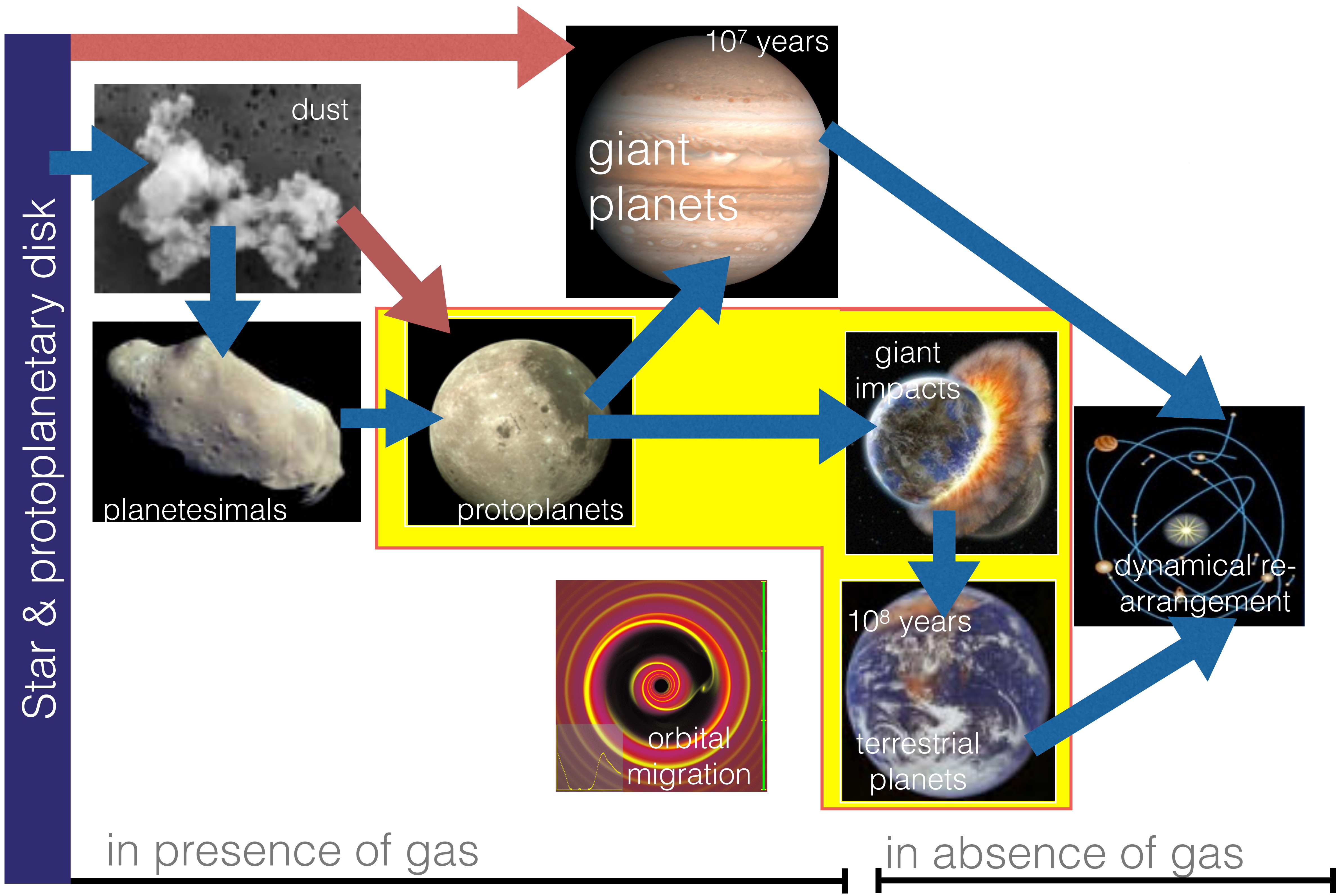
Growth is a factor

$$\frac{R_{\rm H}}{R_{\rm core}} \approx \begin{matrix} 30\text{-}10^3 \text{ (5 AU)} \\ 10^2\text{-}10^4 \text{ (50 AU)} \end{matrix}$$

faster than planetesimals

- Need to have a big starting body to have pebble accretion going…

4. Terrestrial planet formation



Terrestrial planet formation

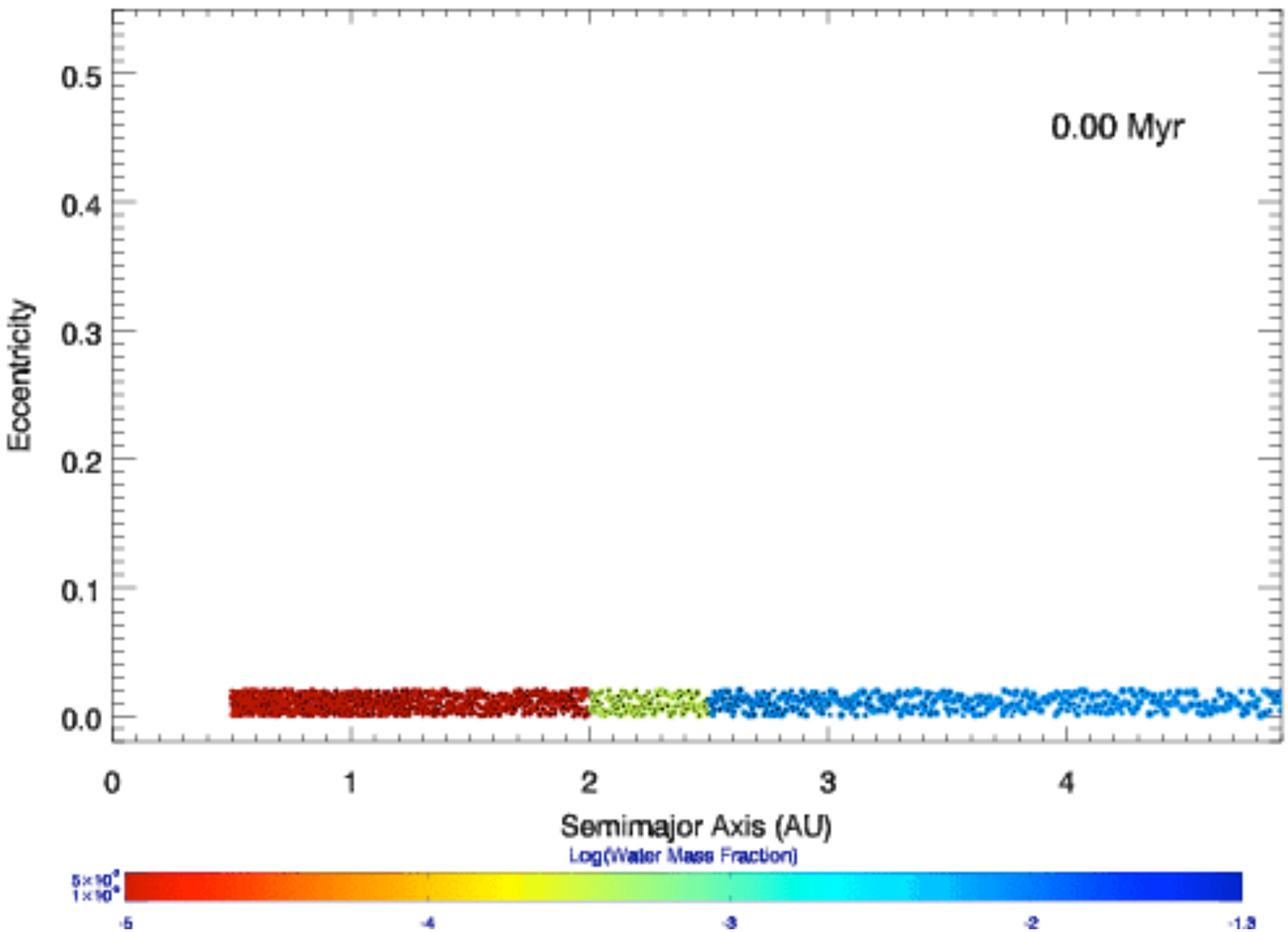
- Once damping influence of the gas disk gone, eccentricity grows, and growth from M_{iso} (oligarchs) with 0.01 - 0.1 M_{Earth} to final masses by giant impacts starts.
- Evolution until long time stable configuration is reached (sufficient mutual distances in term of Hill spheres).

- Constraints (for the solar system):
  1. the orbits, in particular the small eccentricities (Earth: 0.03)
  2. the masse, in particular Mars' small mass
  3. the formation time of Earth from isotope dating (50-100 Myr)
  4. the bulk structure of the asteroid belt (no big bodies)
  5. Earth' relatively large water content (mass fraction 10^{-3})
  6. influence from Jupiter & Saturn

- Method: N body simulation.

Simulation of the inner Solar System

Time evolution of 1885 embryos with Jupiter at 5.2 AU present from t=0.
MMSN surface density.



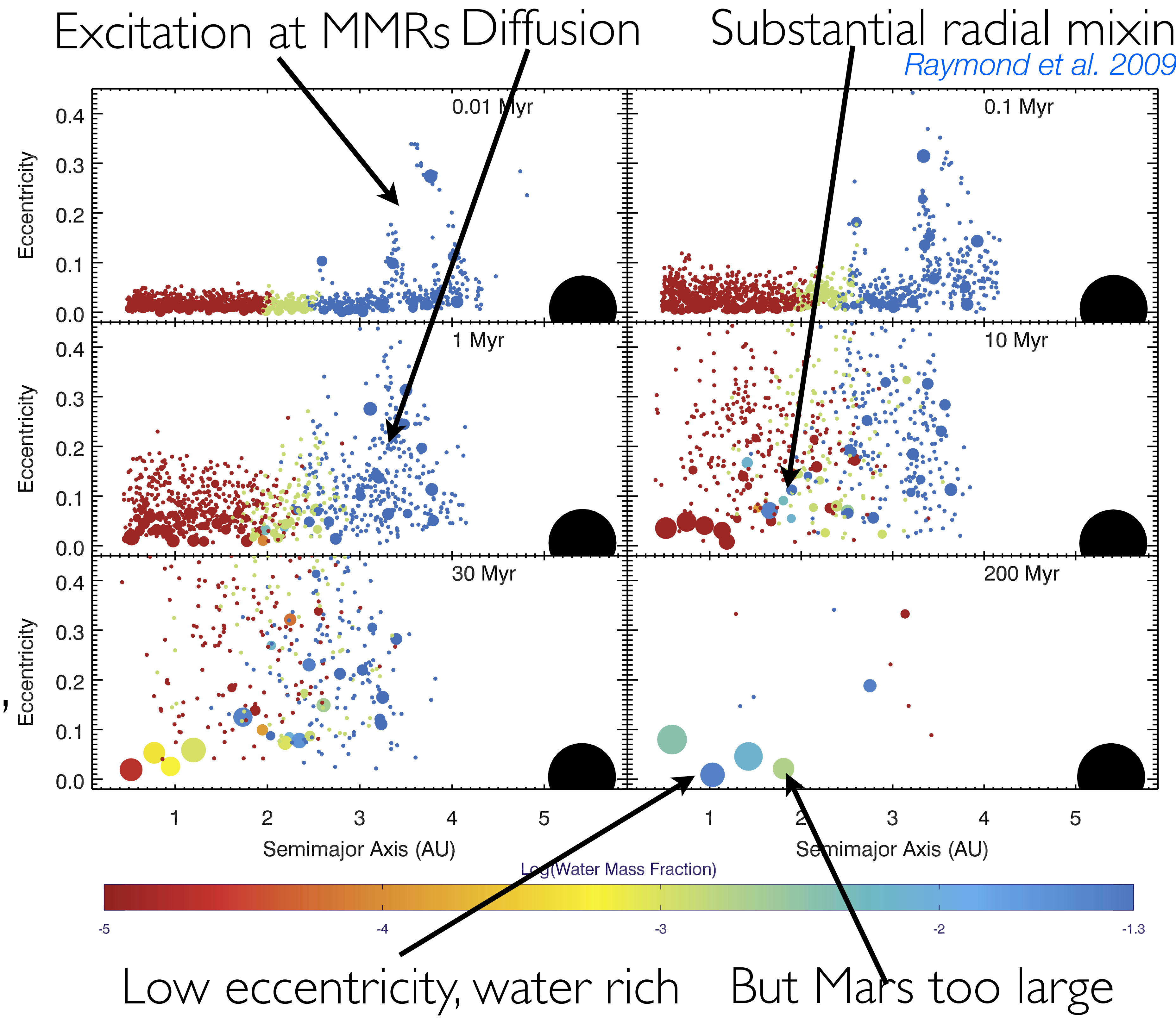
- lasts of order 200 Myr
- considerable mixing
- delivery of water
- giant collisions

Raymond, Quinn & Lunine 2006

The color of each particle represents its water content, and the dark inner circle represents the relative size of its iron core.

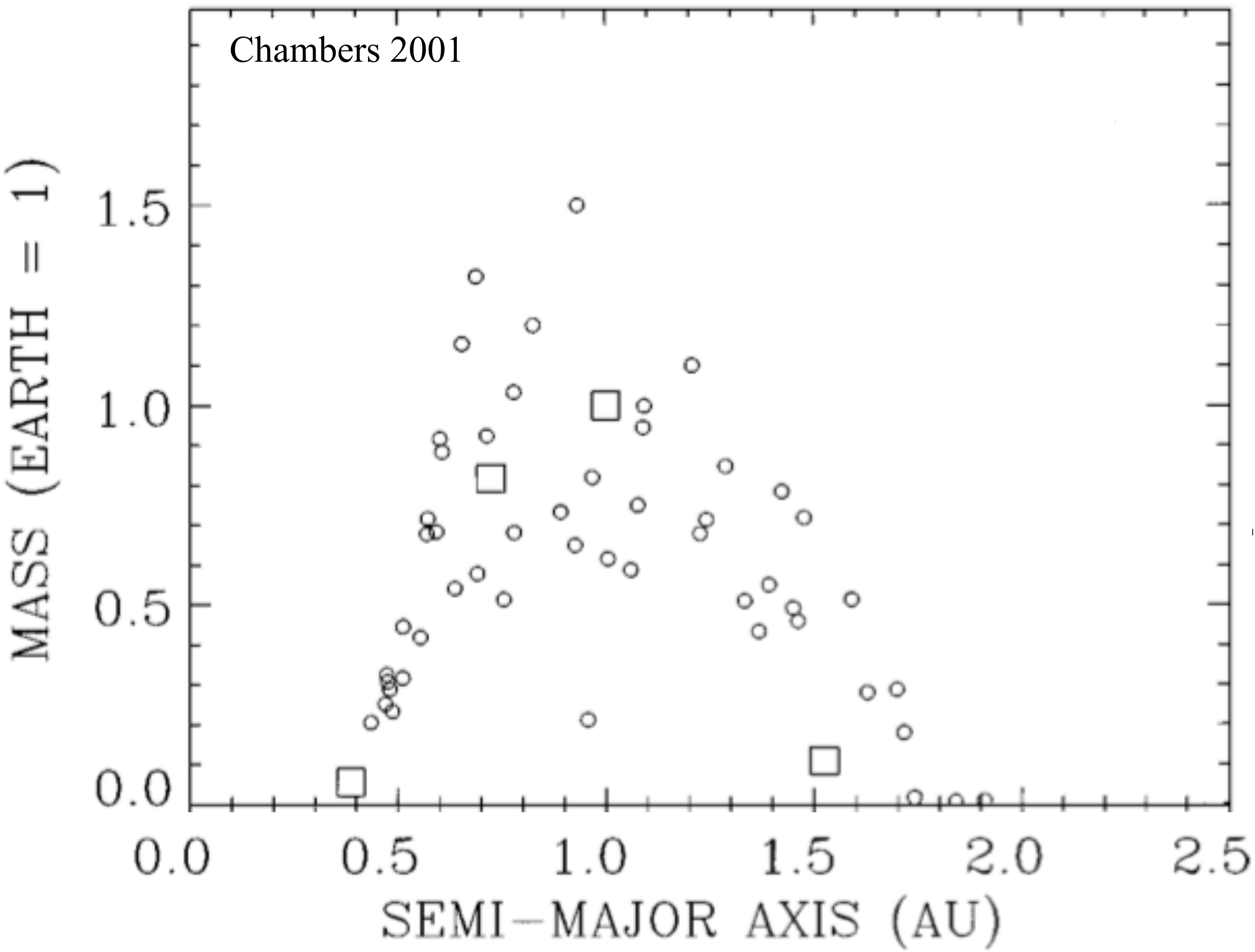
Solar system: classical models

- Solid disk extends to about 4 AU
- 4 terrestrial planets with masses between 0.6-1.8 M_{Earth}
- M, t_{form}, ecc. and water content ok
- But Mars to massive, and 3 addit. embryos



Giant planets?
When where?

Solar system: classical models: Mars problem

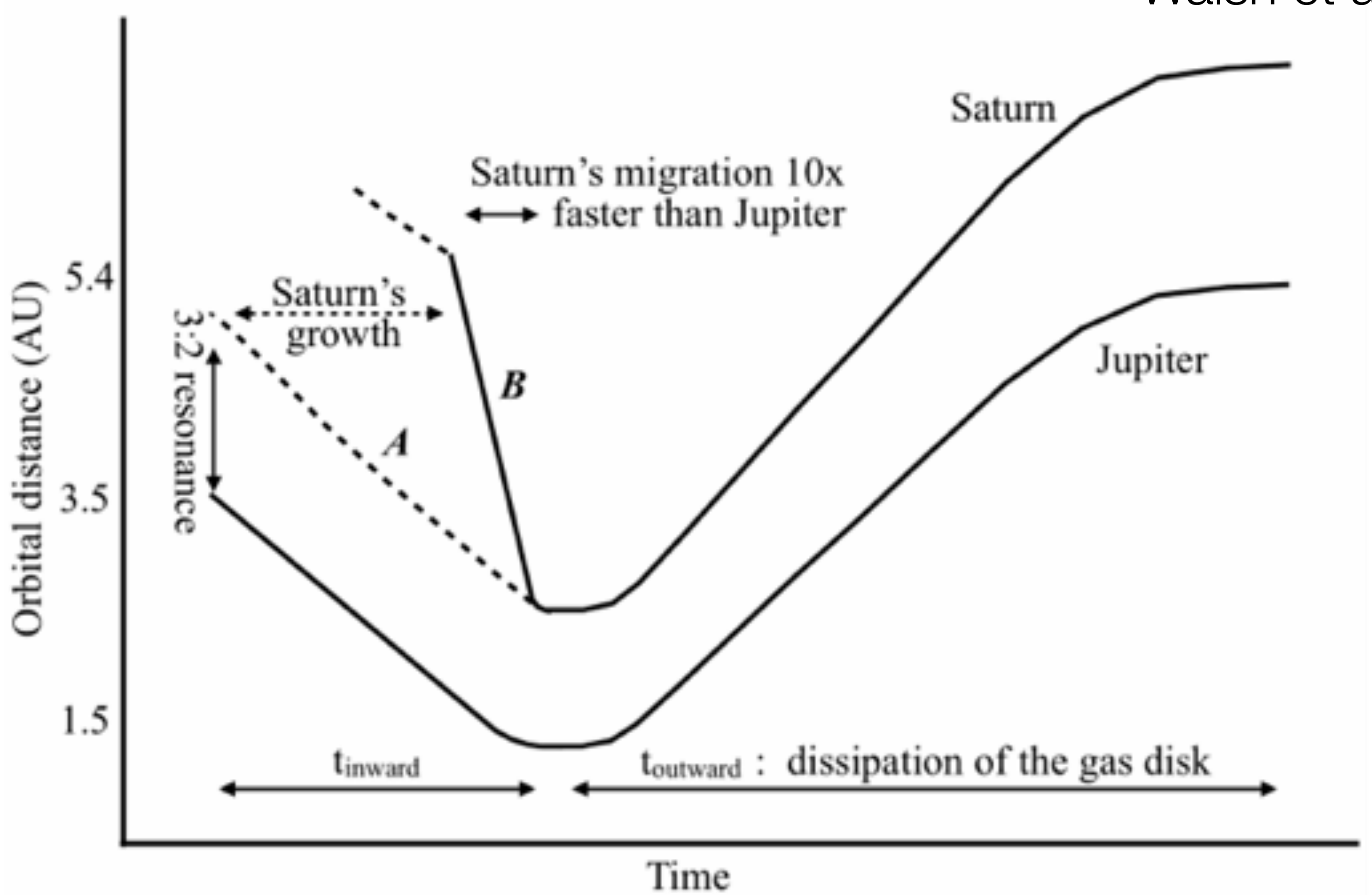


In classical models Mars' mass is too large by a factor of 5–10 and embryos are often stranded in the asteroid belt.

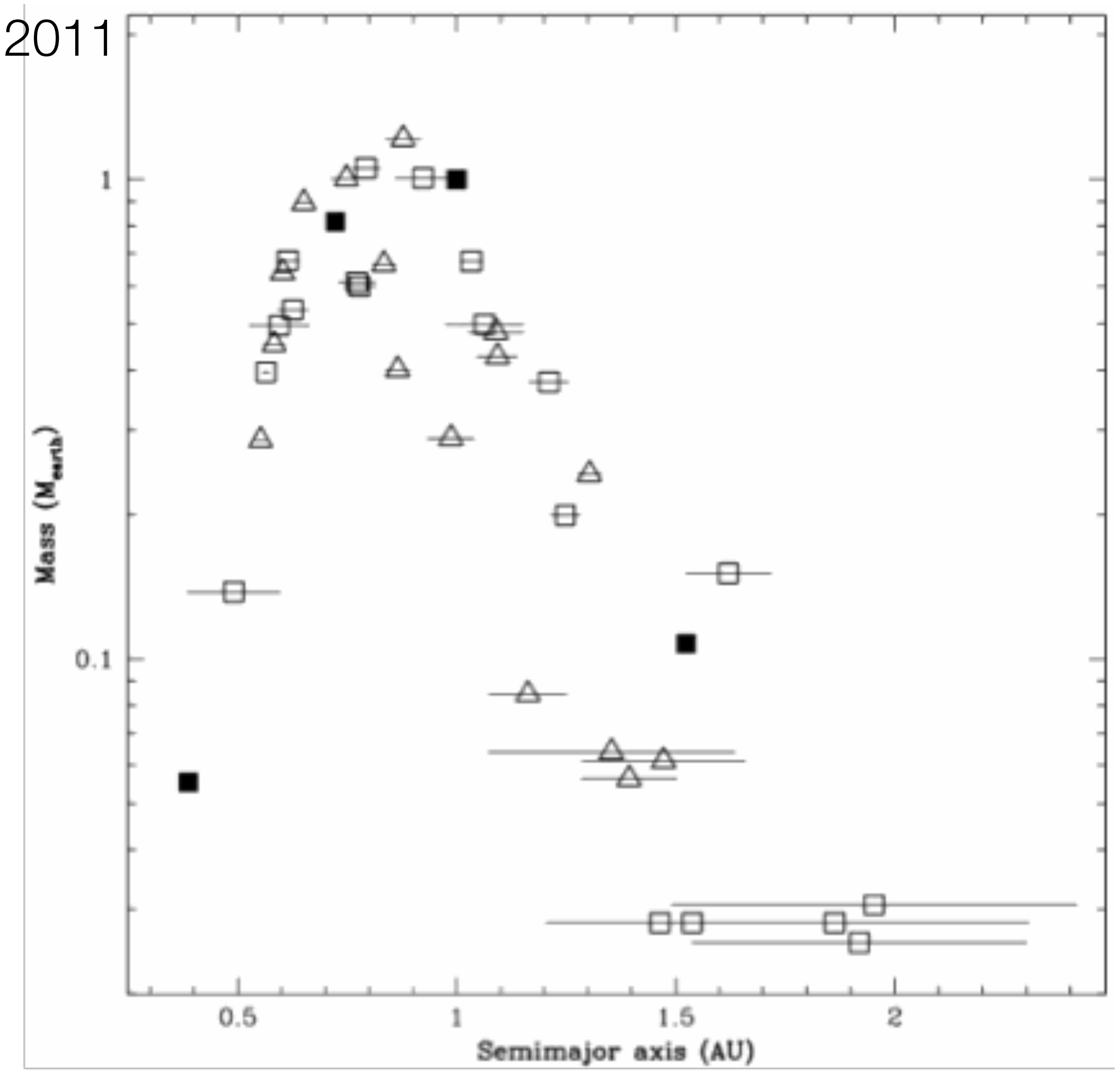
A way out is to (arbitrarily) cut the disk of particles at about 1 AU (Hansen 2008). Mars then diffuses out of the zone with other embryos and planetesimals and remains at a low mass.

But what could cause this cut? Migration traps, or the “Grand Tack”.

Solar system formation: grand tack model



Walsh et al. 2011



Jupiter migrates in to 1.5 AU, get in 2:3 MMR with Saturn. The two "tack" and migrate outward. The grand tack models explains

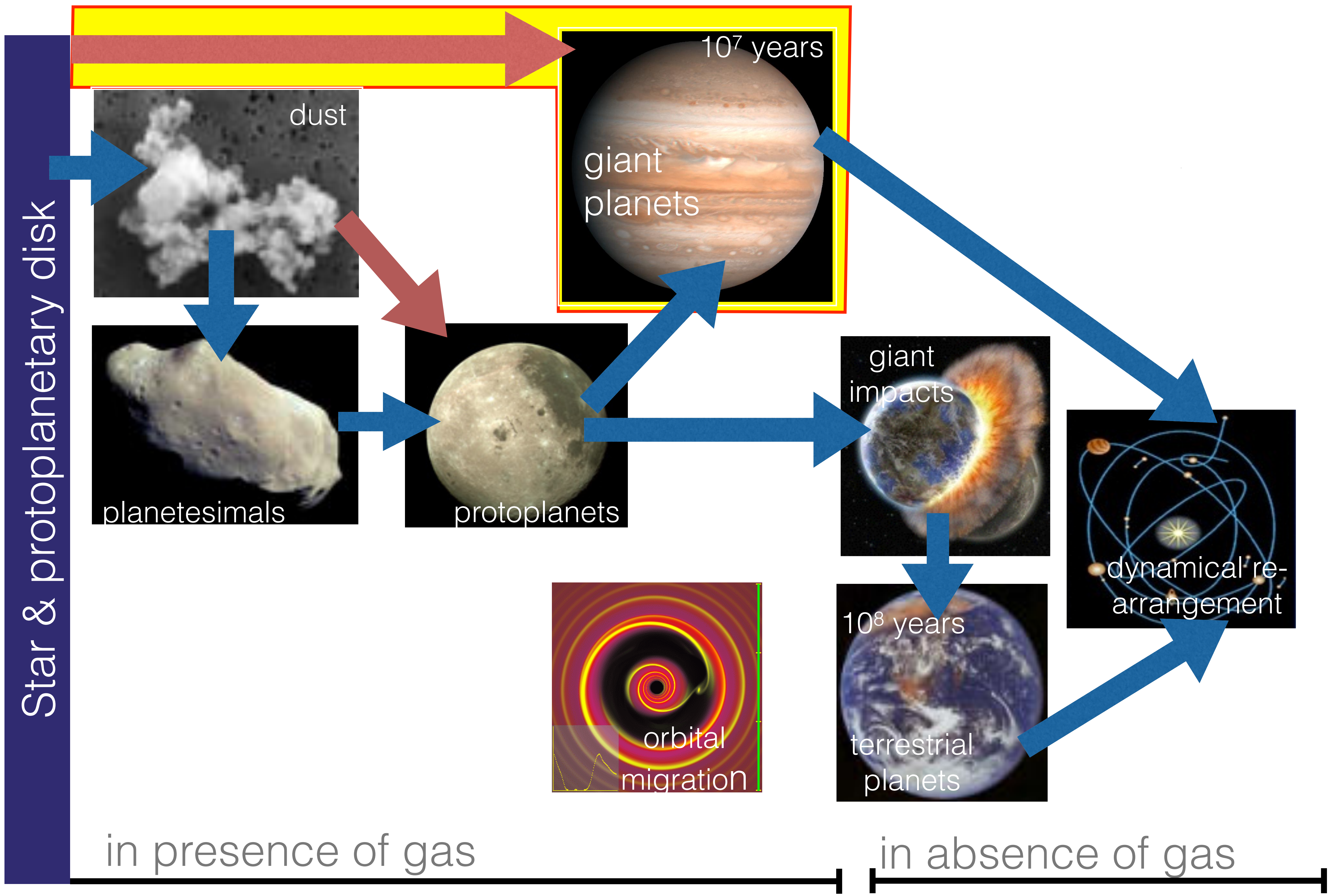
- -Mars' low mass and short formation timescale
- -structure of the asteroid belt (C and S type asteroid)
- -provides initial conditions for the later dynamical evolution (Nice model)

Lecture II
Giant planet formation and orbital migration

Lecture 2 overview

1. Giant planet formation by gravitational instability
2. Giant planet formation by core accretion
    2.1 Gas accretion
    2.2 Critical mass
    2.3 Jupiter in situ formation
3. Orbital migration
    3.1 Impulse approximation
    3.2 Gap formation
    3.3 Migration timescales

1. Giant planet formation: Gravitational instability



Gravitational instability model

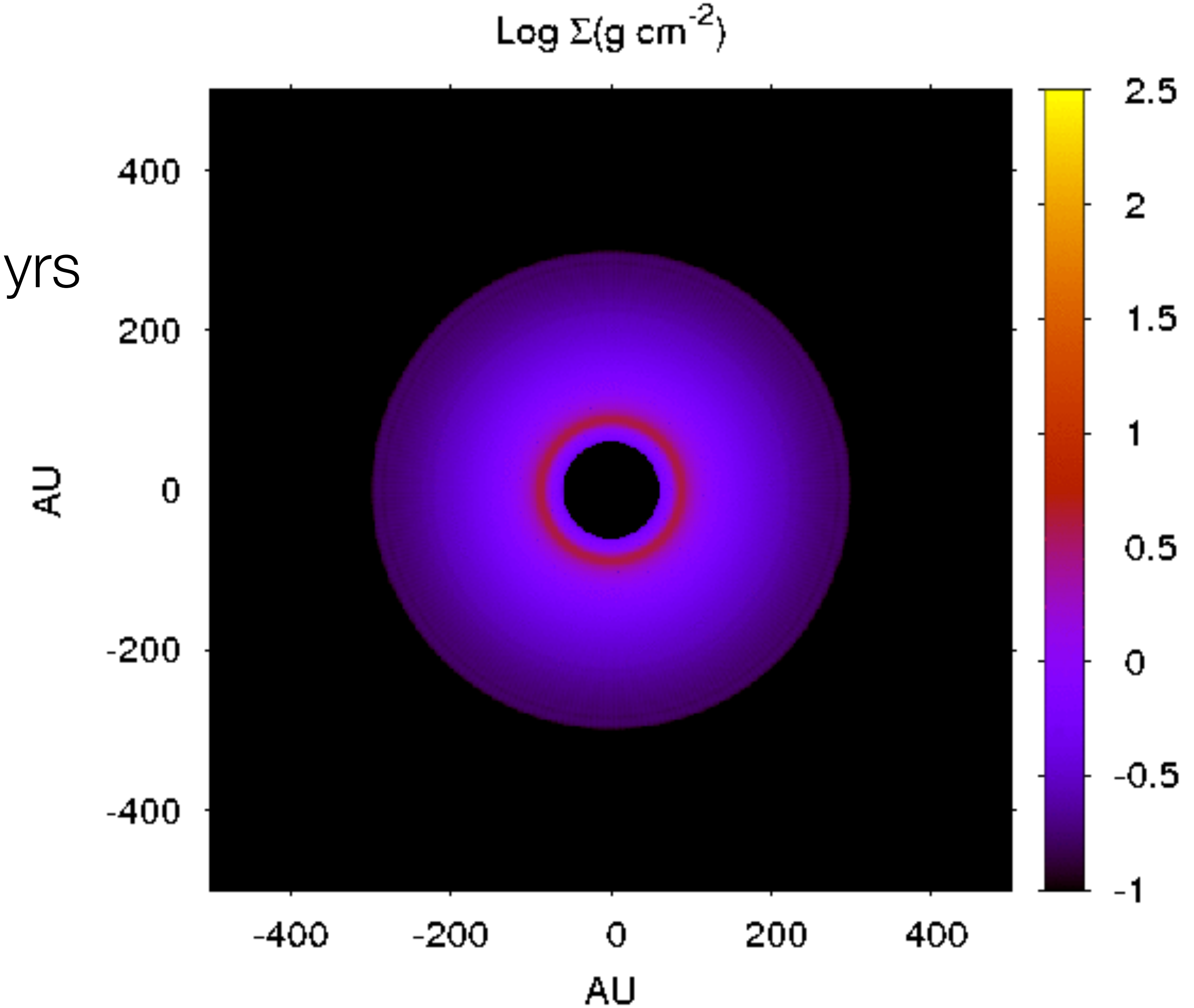
Self-gravitational collapse of a large disk gas patch. Also called direct collapse model.

Top-down process

Formation timescale ~ 1000 yrs

Occurs at large radii

Outcome of process unclear



When does this occur?

Find out with a classical linear stability analysis of a self-gravitating uniformly rotating fluid disk of zero thickness.

Stability of an uniformingly rotating sheet

Stability of a self-gravitating fluid disk or sheet of zero thickness. Constant surface density Σ_0 and temperature T. The sheet is in the z=0 plane and rotating with constant angular velocity $\Omega=\Omega_\mathbf{z}$. Governing equations (mass conservation, Euler, Poisson eqs.) in the rotating frame of reference are:

$$(1)\ \frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{v}) = 0$$

$$\overrightarrow{\Omega} = (0, 0, \Omega)$$

$$a_{cor} = 2(\overrightarrow{v} \times \overrightarrow{\Omega})$$

$$(2)\ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\Sigma}\nabla p - \nabla \phi - 2(\mathbf{\Omega} \times \mathbf{v}) + \Omega^2(x\mathbf{e_x} + y\mathbf{e_y})$$

$$a_{cent} = -\overrightarrow{\Omega} \times (\overrightarrow{\Omega} \times \overrightarrow{r})$$

$$(3)\ \Delta \phi = 4\pi G \Sigma \delta(z)$$

(mass is in the z plane)

$\Delta = \Sigma \frac{\partial^2}{\partial x_i^2}$ (Laplace operator)

Because the sheet is assumed to be isothermal, the vertically integrated pressure is given by:

$$p = p(\Sigma) = c^2 \Sigma$$

In the unperturbed state, we assume an equilibrium solution given by:

$$\Sigma = \Sigma_0;\ \ \mathbf{v} = 0;\ \ p = p_0 = c^2\Sigma_0 \ \overset{(2)}{\longrightarrow} \nabla \phi_0 = \Omega^2(x\mathbf{e_x} + y\mathbf{e_y});\ \ \Delta\phi_0 = 4\pi G \Sigma_0 \delta(z)$$

(rot. frame!)

Stability of an uniformingly rotating sheet II

We now introduce small perturbations in the equilibrium quantities:

$$\Sigma(x,y,t) = \Sigma_0 + \epsilon\Sigma_1(x,y,t);\ \mathbf{v}(x,y,t) = \epsilon\mathbf{v_1}(x,y,t);\ \ldots\ ;\ \epsilon \ll 1$$

We keep only the terms linear in ε. We obtain the linearized equations for the evolution of the perturbations:

$$(4)\ \frac{\partial\Sigma_1}{\partial t} + \Sigma_0\nabla\cdot(\mathbf{v_1}) = 0$$

$$(5)\ \frac{\partial\mathbf{v_1}}{\partial t} = -\frac{c^2}{\Sigma_0}\nabla\Sigma_1 - \nabla\phi_1 - 2(\mathbf{\Omega}\times\mathbf{v_1})$$

$$(6)\ \Delta\phi_1 = 4\pi G\Sigma_1\delta(z)$$

We now look for solutions of the type:

$$\omega = \text{angular frequency} = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{\lambda} = \text{wave number}$$

$$\Sigma_1(x,y,t) = \Sigma_a e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\mathbf{v_1}(x,y,t) = (v_{ax}\mathbf{e_x} + v_{ay}\mathbf{e_y})e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\phi_1(x,y,t) = \phi_0 e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

Stability of an uniformingly rotating sheet III

Without loss of generality, we chose the x-axis to be parallel to the propagation of the perturbation k, i.e. $\mathbf{k} = k\mathbf{e_x}$

Poisson equation: Outside the sheet, we must have $\Delta\phi_1 = 0$ whereas in the z=0 plane we have the solution given above. Only function that satisfies these constraints and that vanishes at infinity is given by:

$$\phi_1 = \frac{2\pi G\Sigma_a}{|k|} e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

This solution substituted back into the linearized equation yields:

$$(7) \quad -i\omega\Sigma_a = -ik\Sigma_0 v_{ax}$$

$$(8) \quad -i\omega v_{ax} = \frac{c^2 ik\Sigma_a}{\Sigma_0} + \frac{2\pi Gi\Sigma_a k}{|k|} + 2\Omega v_{ay}$$

$$(9) \quad -i\omega v_{ay} = -2\Omega v_{ax}$$

This set of equations can be written in form of a matrix. It has a non trivial solution only when

$$\omega^2 = 4\Omega^2 - 2\pi G\Sigma_0|k| + k^2c^2 \geq 0$$

Dispersion relation for the uniformingly rotating sheet.

Stability of an uniformingly rotating sheet IV

Dispersion relation for the uniformingly rotating sheet.

$$\overrightarrow{C} e^{i(\overrightarrow{k} \cdot \overrightarrow{x} - \omega t)}$$

$$\omega^2 = 4\Omega^2 - 2\pi G \Sigma_0 |k| + k^2 c^2$$

$$\omega = \text{angular frequency} = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{\lambda} = \text{wave number}$$

$$p = p(\Sigma) = c^2 \Sigma$$

What does this equation mean? Ideas?

-If $\omega^2>0$, we have finite oscillations: stable disk

This happens if the positive terms involving Ω and c^2 dominate.

-If $\omega^2<0$, the perturbations will grow exponentially in time: unstable disk!

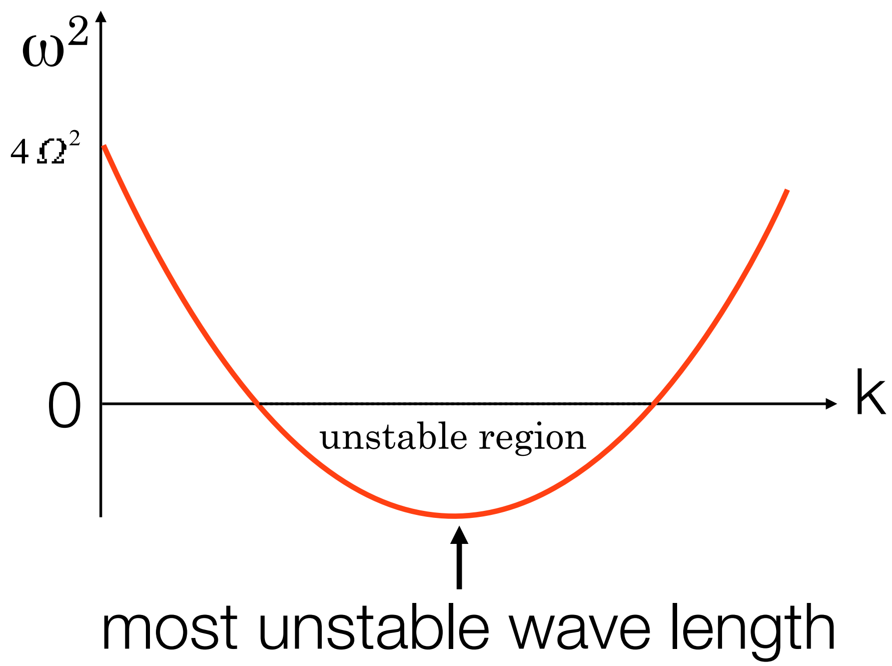
This happens if the negative term with Σ_0 dominantes.

Note: - long wavelengths (small k) are stabilized by rotation

- short wavelength (large k) are stabilized by pressure

The same criterion also applies for spiral galaxies.

Stability of an uniformingly rotating sheet V



Overall stability is achieved if $\omega(k)^2 \geq 0$ everywhere, i.e. the minimum -determined by setting the derivative equal zero - must still be positive. This condition yields the condition necessary for stability of the uniformly rotating sheet, the so called Toomre criterion (Toomre, 1964).

$$Q = \frac{2c\Omega}{\pi G \Sigma_0} > 1$$

stability criterion for the uniformly rotating sheet: cold, slowly rotating, massive disks are unstable

In hydrodynamic simulations: spiral waves form at Q~1.5

Cooling criterion

The Toomre criterion says when the disk forms spiral density waves.

In order for the gas to also *fragment in bound clumps* a second criterion must be fulfilled: the gas must cool sufficiently fast. Otherwise the clump gets sheared apart (Gammie 2001):

$$t_{\rm cool}\Omega \lesssim \beta_{\rm crit} \approx 3 \quad \text{i.e.} \quad \frac{t_{\rm cool}}{t_{\rm orb}} \lesssim \frac{1}{2}$$

If Q<1.5, but Ω_K t_{cool}>3: only spiral waves form, but no fragments

-efficient angular momentum transport

-disk heats up, mass decreases: disk gets marginally stable

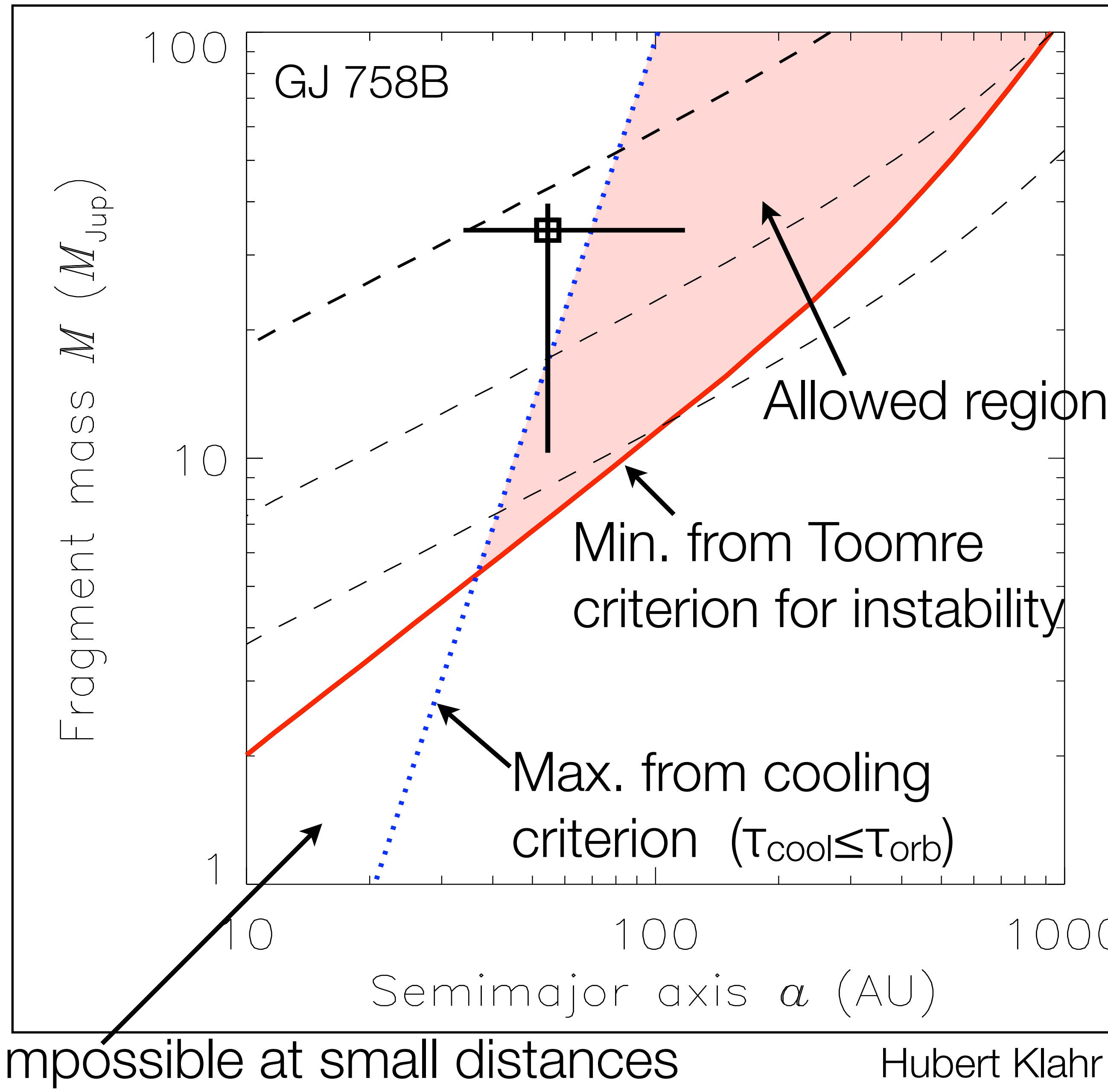
-instability should be a short phase

Regions of gravitational instability

$$Q = \frac{c_s \Omega_K}{\pi G \Sigma} \lesssim 1.5$$

Toomre (1964)

$$\& \quad \frac{t_{\rm cool}}{t_{\rm orb}} \lesssim \frac{1}{2}$$



Hubert Klahr

but at small distances

$c_s \searrow \implies Q \searrow$

$c_s \searrow \implies \tau_{\rm cool} \nearrow$

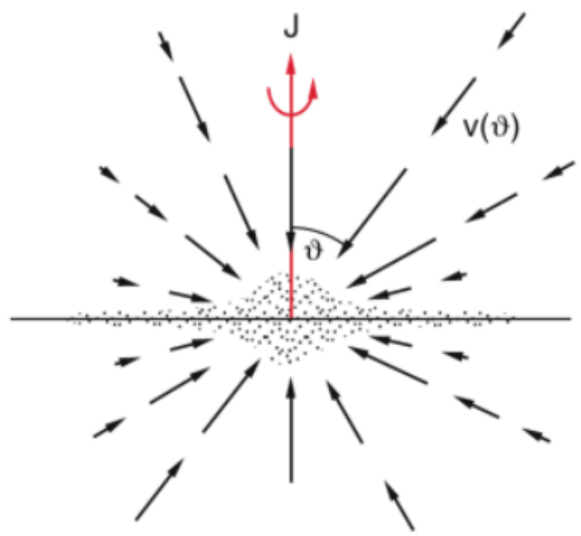
- Unlikely inside ~30 AU
- Needs massive disks

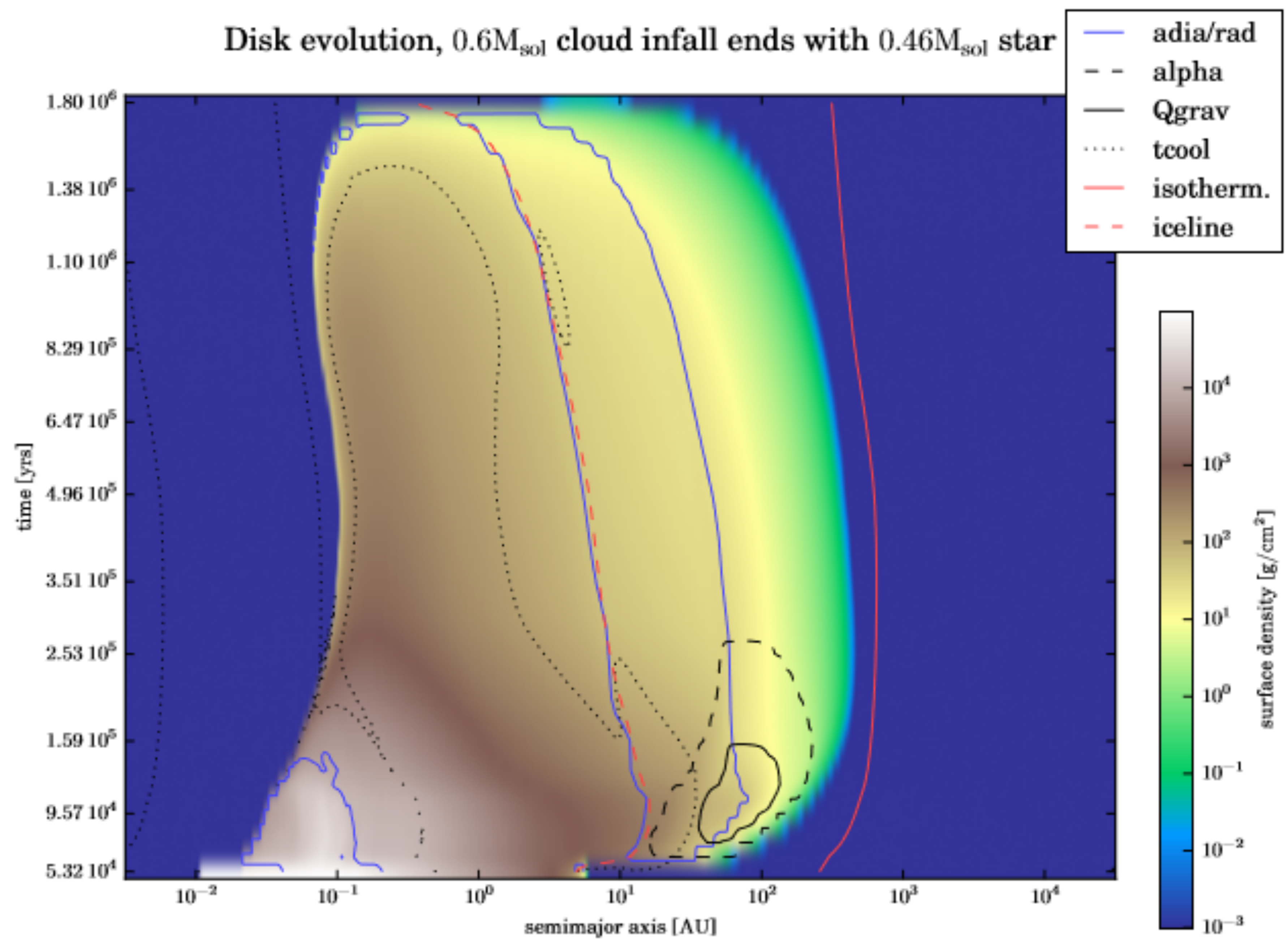
Early hydrodynamic models assumed (incorrectly) isothermal conditions (immediate cooling): artificial formation of clumps

Rafikov 2005

New vision: GI during disk infall

Loading by infalling gas from collapsing cloud can drive the disk into instability

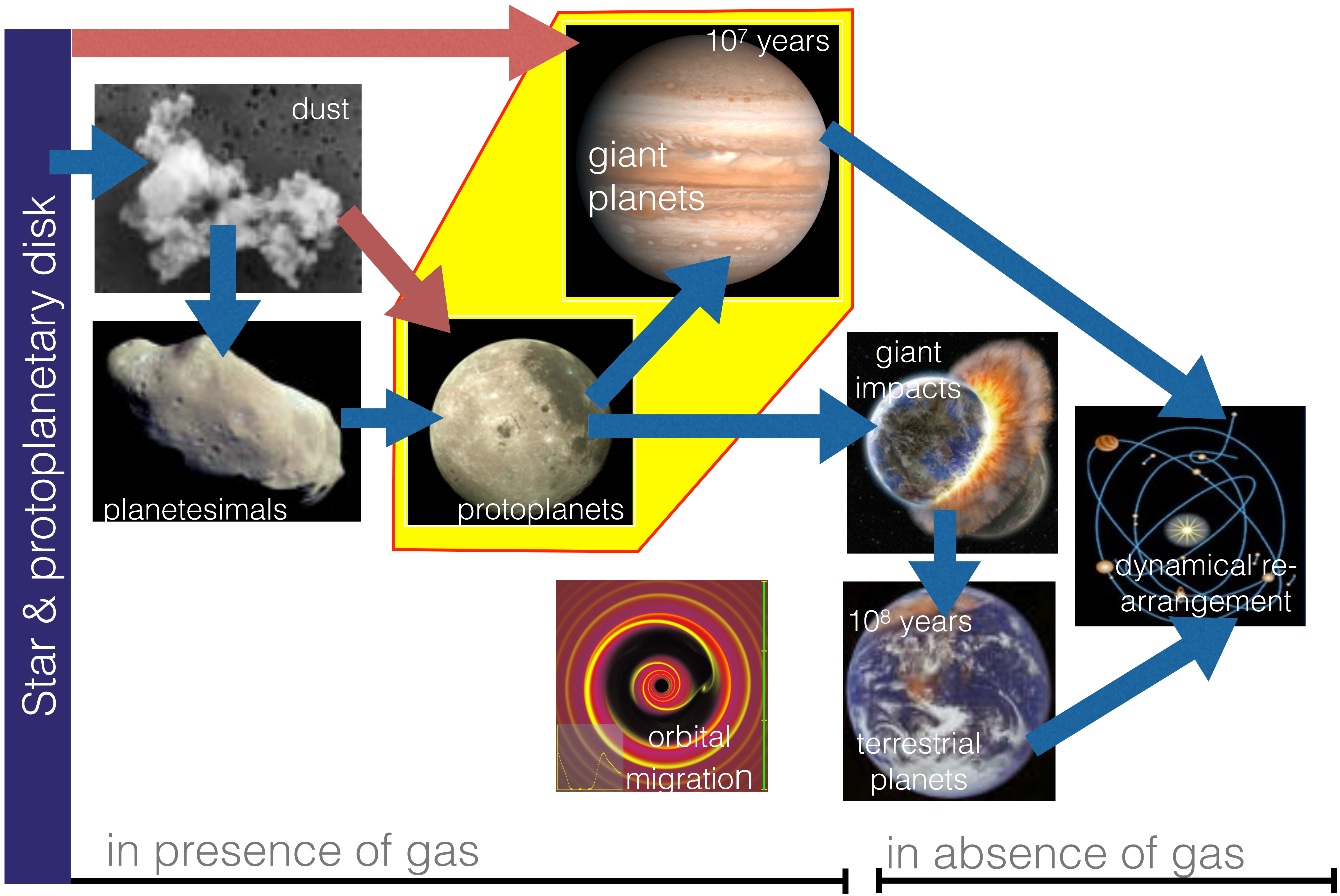




- But instead of planets BD or companions stars may form…
- Or everything falls into the star due to migration…
- No consensus so far

Dittkrist et al. in prep.

2. Giant planet formation: Core accretion



Outcome of the sequential growth process (last lecture)

Inner solar system

Many small 0.01 to 0.1 M_{Earth} protoplanets.
During the presence of the gas disk, growth stalled at this mass, as gas damping hinders development of high eccentricities (i.e. mutual collision between these bodies).

Outer solar system

A few 1 to 10 M_{Earth} protoplanets.
If formed quickly and massive enough ($M>$ca 10 M_{Earth}), potential to accrete gas to form a giant planet.

Core accretion paradigm

Giant planets (such as Jupiter or Saturn): 90 - 95 % gas (H_2 and He)
Thus, must form during disk lifetime (3-10 Myrs).

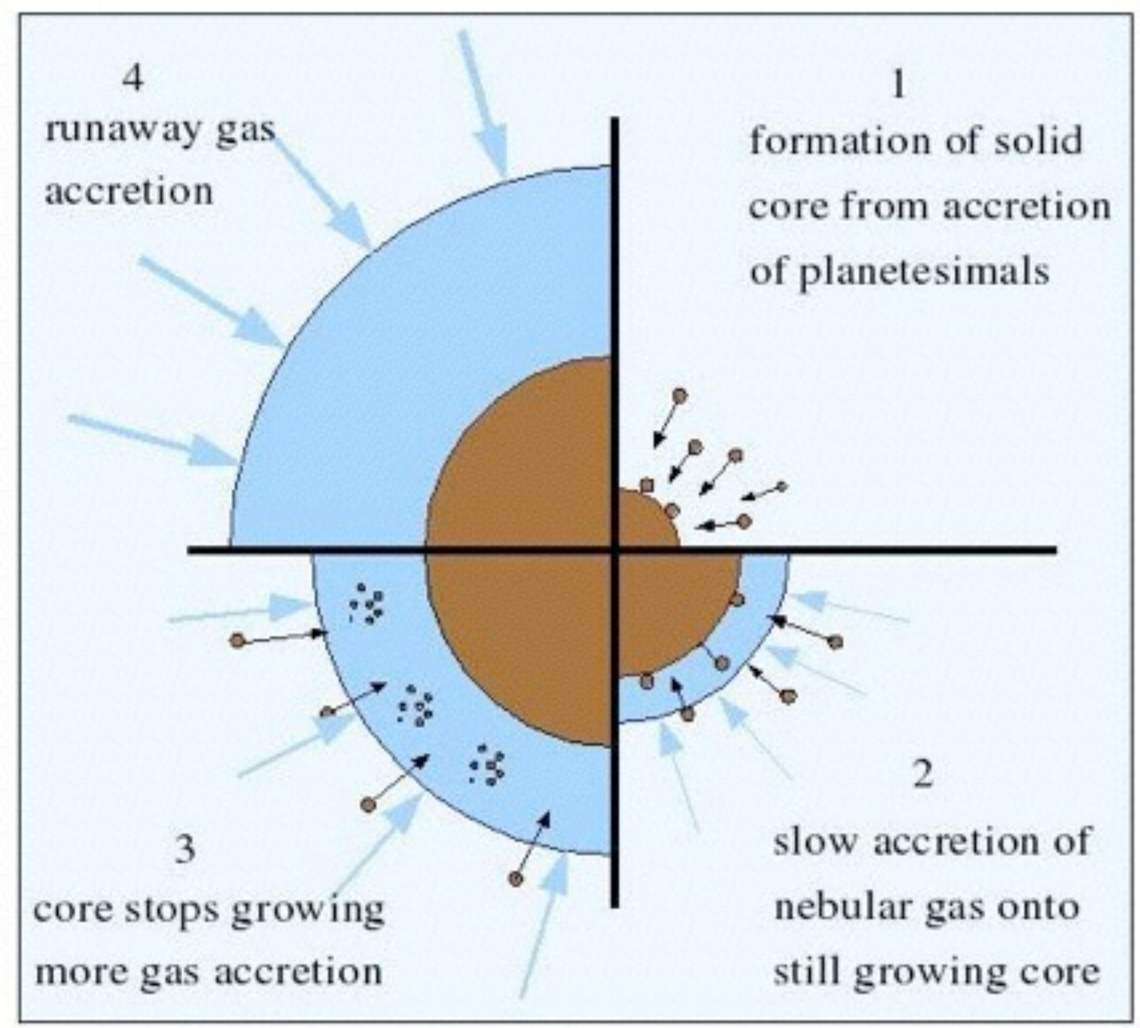
The competing giant planet models
- direct gravitational collapse (very fast, but other issues)
- core accretion: may take long

Core accretion or nucleated instability

Perri & Cameron 1974; Mizuno et al 1978; Mizuno 1980; Bodenheimer & Pollack 1986; Pollack et al 1996; Alibert et al 2005

Two steps:

1. formation of a critical solid core (>10 M_e)
2. fast runaway gas accretion



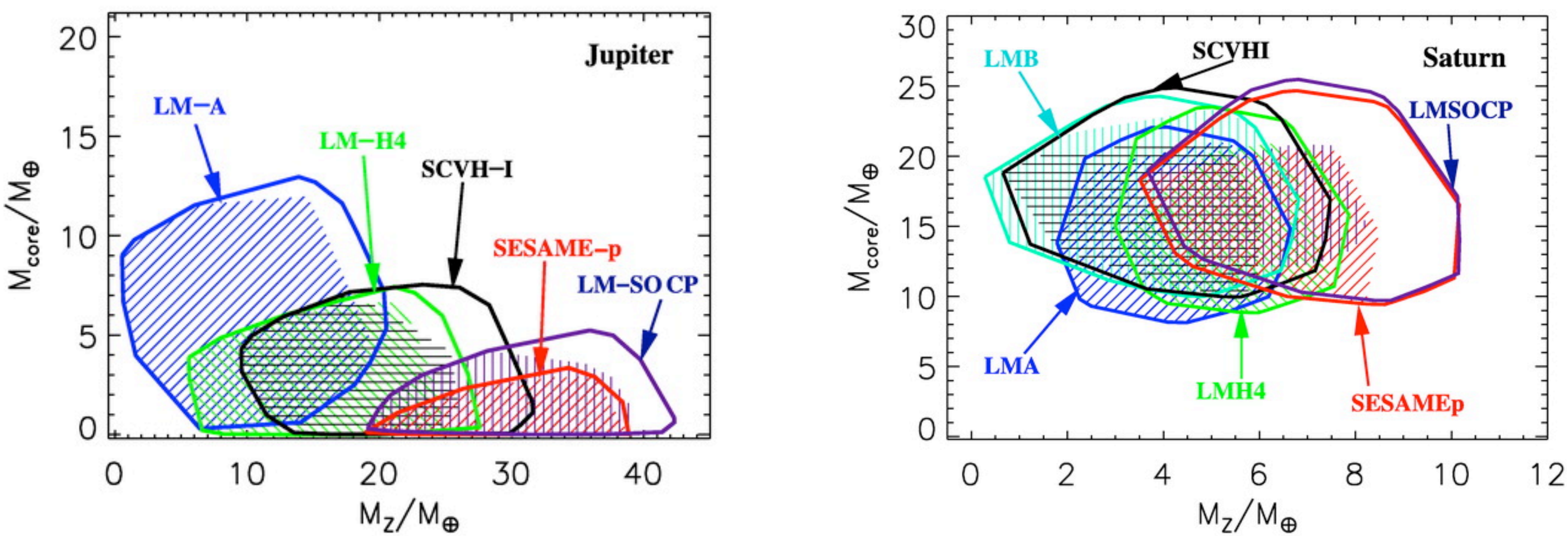
Basic requirement:
A critical core must form before the gas disappears. Not trivial!

Constraints from Jupiter and Saturn

Internal structure of the giant planets is obtained through modeling. Adjust heavy element content so as to meet observations (mass, radius, gravitational moments, surface abundance, jovian seismology)

Jupiter: enriched 1.5-6 times solar.
Saturn: enriched 6-14 times solar).
But: large uncertainties, from the EOS.

Region	Jupiter	Saturn
Core	0–10	6–17
Molecular region	1.6–6.1	2.8–8.8
Metallic region	0.7–34	0–17
Total (core + envelope)	11–42	19–31



Saumon & Guillot 2004

This is regarded as a indication that core accretion lead to the formation of Jupiter and Saturn. Recently it was however found that direct collapse can also lead (under certain circumstances) to enriched planets.

2.1 Gas accretion

Mass growth

Growth of the core: accretion of planetesimals (oligarchic) as in Lecture I

Growth of the envelope (gas)

1D, radial structure equations as for stars:

(1) $\frac{dm}{dr} = 4\pi r^2 \rho$	$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho$ (2)	Mass conservation Hydrostat. equilibrium
(3) $\frac{dl}{dr} = 4\pi r^2 \rho \left(\epsilon - T\frac{\partial S}{\partial t}\right)$	$\frac{dT}{dr} = \frac{T}{P}\frac{dP}{dr}\nabla$ (4)	Energy conservation Energy transport

$$\nabla = \frac{d\ln T}{d\ln P} = \min(\nabla_{\rm ad}, \nabla_{\rm rad}) \quad \nabla_{\rm rad} = \frac{3}{64\pi\sigma G}\frac{\kappa l P}{T^4 m}$$

Notable difference to stars:

-no nuclear fusion

-but: impacting planetesimals. Dominant source of energy early on.

Gas accretion rate given by ability to radiate away energy (T_{KH}):

liberated gravitational potential energy-> radiate away (cool)->contract->empty space inside Hill sphere->gas flows in from nebula (accretion)

2.2 Critical mass

Analytical toy model

Solve simplified structure equations (Stevenson 1982). One finds: For too massive cores, no envelope in hydrostatic equilibrium exists (critical core mass): rapid gas accretion must ensue.

Derivation of the critical core mass with a toy model

Core mass M_{core} , core radius R_{core}, gaseous envelope of mass M_{env}.
Luminosity from accretion of planetesimals onto the core only

$$L = \frac{GM_{\rm core}\dot{M}_{\rm core}}{R_{\rm core}} \quad (1)$$

Energy transport by radiative diffusion only (no convection)

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)}{r^2}\rho \quad (2)$$

$$\frac{L}{4\pi r^2} = -\frac{16}{3}\frac{\sigma T^3}{\kappa_R \rho}\frac{\mathrm{d}T}{\mathrm{d}r} \quad (3)$$

simplified structure equation

We can combine these equations into

$$\frac{\mathrm{d}T}{\mathrm{d}P} = \frac{3\kappa_R L}{64\pi\sigma GMT^3} \quad (4)$$

Analytical toy model II

Separate the variables to integrate making the approximation M(r) ≈ M_t (the total mass) and taking L and also κ_R to be constants (!)

$$\int_{T_{\rm disk}}^{T} T^3 dT = \frac{3\kappa_R L}{64\pi\sigma G M_t}\int_{P_{\rm disk}}^{P} dP. \tag{5}$$

Well inside the planet, $T^4 \gg T_{disk}^4$ and $P \gg P_{disk}$, so approximately

$$T^4 \simeq \frac{3}{16\pi}\frac{\kappa_R L}{\sigma G M_t} P. \tag{6}$$

So called "radiative zero" solution. Replace P in eq. (6) with ideal gas EOS

$$P = \frac{k_B}{\mu m_p}\rho T, \tag{7}$$

giving us an expression for T^3. Put back into equation (3) and trivially integrate again with respect to r to obtain the temperature as fct. of radius

$$T \simeq \left(\frac{\mu m_p}{k_B}\right)\frac{G M_t}{4r} \tag{8}$$

and, with eq. (6) and (7), also the density as function of radius.

$$\rho \simeq \frac{64\pi\sigma}{3\kappa_R L}\left(\frac{\mu m_p G M_t}{4 k_B}\right)^4 \frac{1}{r^3} \tag{9}$$

Analytical toy model III

With this density profile the mass of the envelope is obtained easily

$$\begin{aligned} M_{\rm env} &= \int_{R_{\rm core}}^{R_{\rm out}} 4\pi r^2 \rho(r) dr \\ &= \frac{256\pi^2\sigma}{3\kappa_R L} \left(\frac{\mu m_p G M_t}{4 k_B} \right)^4 \ln \left(\frac{R_{\rm out}}{R_{\rm core}} \right) \end{aligned}$$

This is an implicit relation between the total and envelope mass. For the core mass we can of course write

$$M_{\rm core} = M_t - M_{\rm env}$$

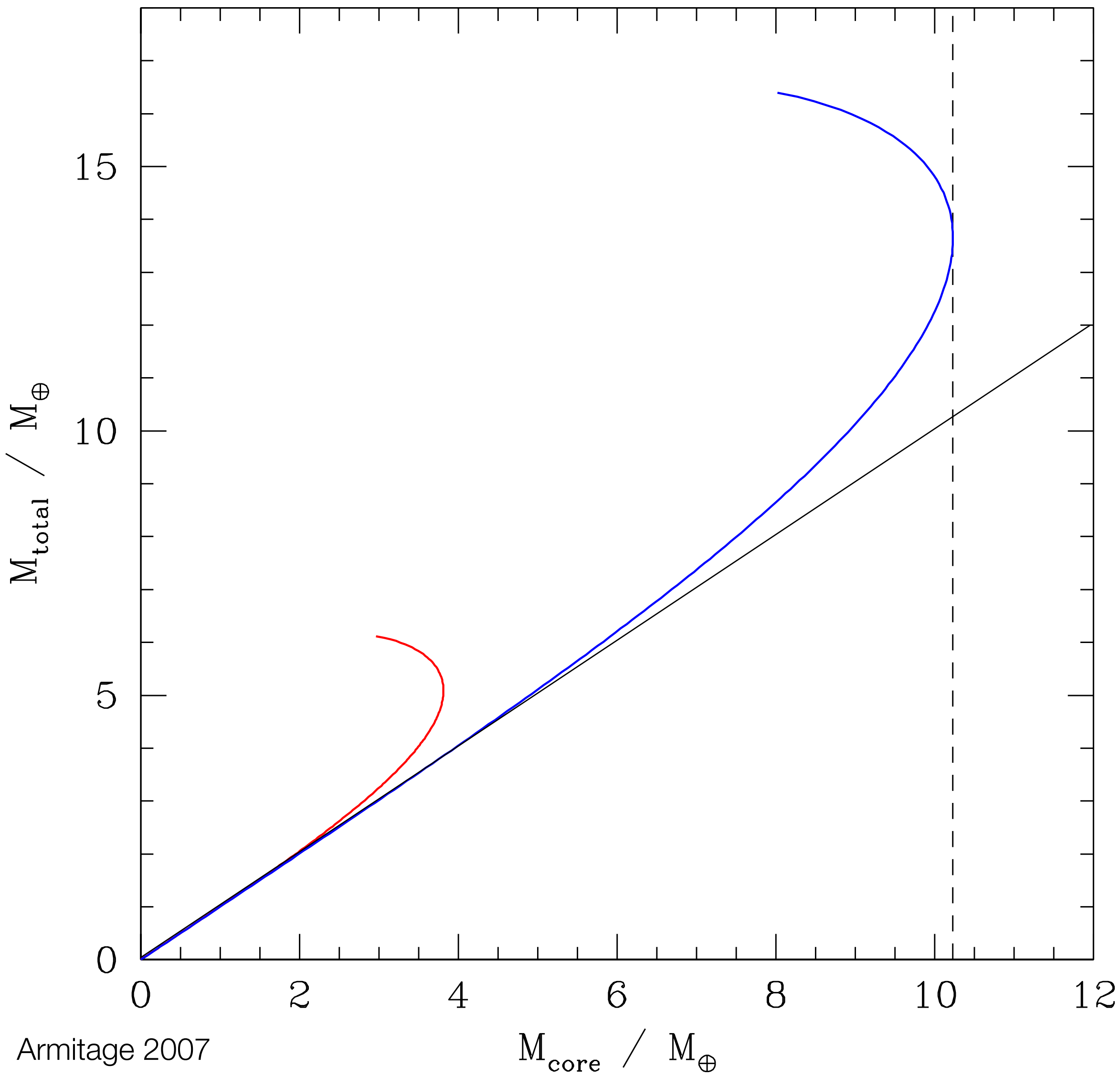
Finally we find an implicit core mass - total mass relation (C=quasi-constant)

$$M_{\rm core} = M_t - \left(\frac{C}{\kappa_R \dot{M}_{\rm core}} \right) \frac{M_t^4}{M_{\rm core}^{2/3}}$$

What does this equation mean?

Analytical toy model III

$$M_{\rm core} = M_t - \left(\frac{C}{\kappa_R \dot{M}_{\rm core}}\right) \frac{M_t^4}{M_{\rm core}^{2/3}}$$



Armitage 2007

Blue: high planetesimal accretion rate
red: low planetesimal accretion rate
black: no envelope

1) $M_{\rm env}$ increases with $M_{\rm core}$
2) Dashed: critical core mass beyond which no solution exists (~10 $M_{\rm earth}$)

Large accretion rate or opacity: high $M_{\rm crit}$

Physical interpretation:
Core mass above > critical mass: no hydrostatic equilibrium in the envelope. Gravity wins over pressure.

Rather:
1) the envelope has to contract (generating luminosity in this way to counteract gravity)
2) further gas will fall in as fast as gravitational potential energy can be radiated (runaway).

2.3 Jupiter in situ formation

Classical models

Compared to the early (toy) models, the classical models (in particular Pollack et al. 1996) calculate

- the core accretion rate self-consistently. Accretion occurs from a feeding zone with a width depending on the planet's mass. As the core grows, the planetesimal surface density decreases.

- full structure equations with realistic EOS and opacities

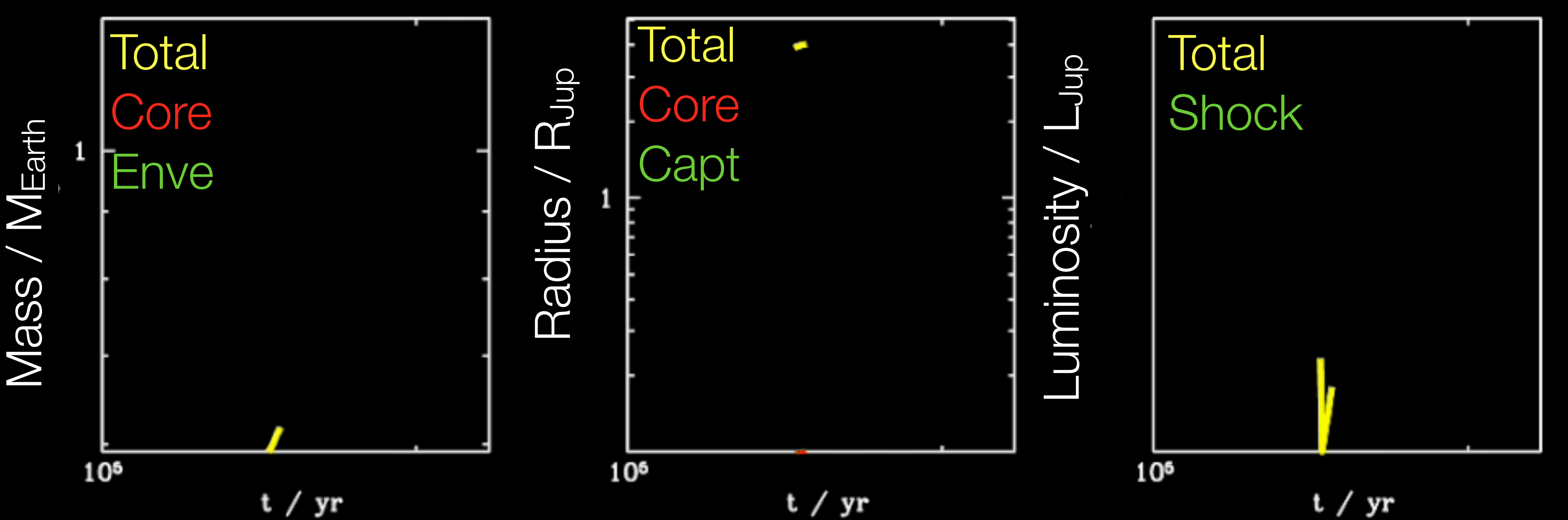
- real evolutionary sequences (i.e. they include the TdS/dt term)

They however still assume that:

  - the protoplanetary disk giving the boundary conditions is static in time.
  - the formation occurs in situ (no migration).

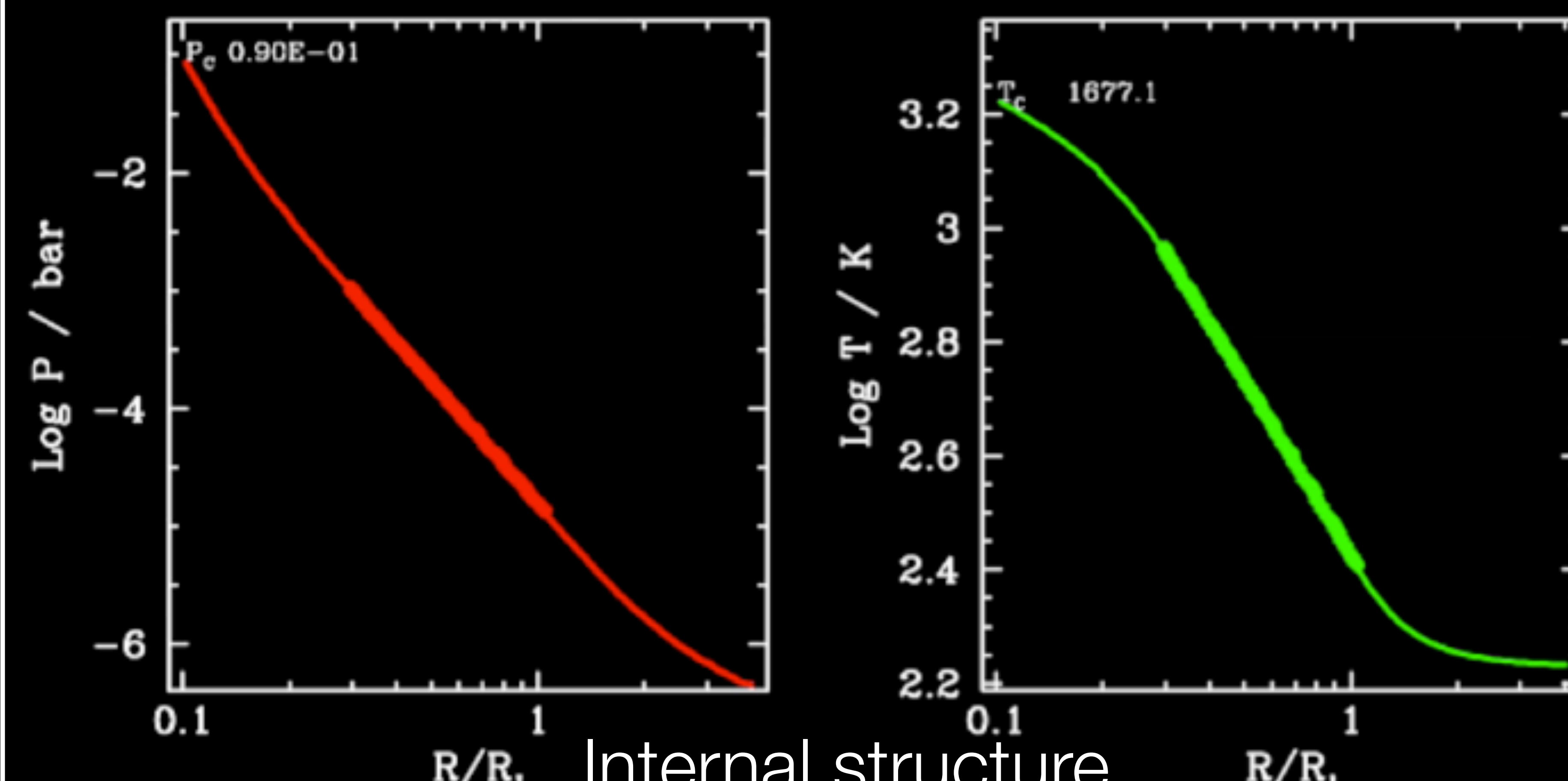
Jupiter: entire "life"

Time 1.48E+05 yrs



Numerical Data

$M_{tot}/M_\oplus$	0.618
$M_{core}/M_\oplus$	0.618
$M_{env}/M_\oplus$	0.000
R/R_J	4.03
L/L_J	1.21E+02
dM_{core}/dt	6.99E−06

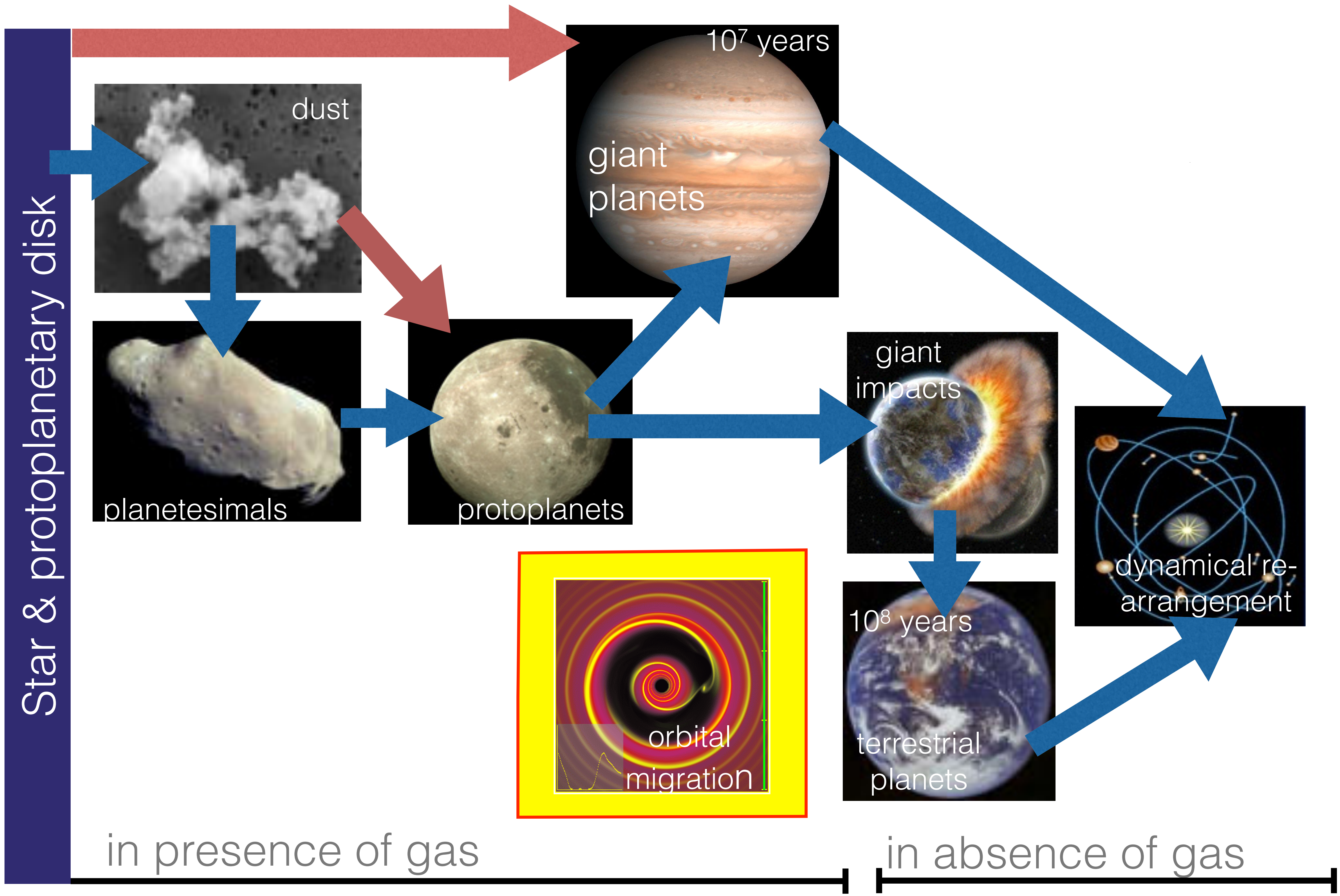


Internal structure

Summary on giant planet formation

- The core accretion model is a relatively mature model that can reproduce many observational constraints, in particular in the context of population synthesis models.
- It however relies on a rapid accretion of a massive core which is not fully understood.
- Active areas of research regarding the core accretion model are the effects of the opacity and of the composition of the envelope, and the consequences of hydrodynamic, multidimensional models instead of classical quasi-static 1D model.
- In the gravitational instability model, many fundamental mechanism are in contrast not yet understood.
- There is currently no consensus whether this model leads to the formation of gas giant planets. If yes, then they are probably massive and found at large orbital distances like the HR 8799 planets.

3. *Orbital migration*



Orbital migration

Last lecture: giant planets should form in a region outside the iceline, i.e. at ~3-5 AU. Solar System: Giant planets at such distances and further out: good confirmation of this theory.

The detection of the first extrasolar planet by Mayor and Queloz in 1995, which was a giant planet at an orbital distance of just 0.05 AU was therefore for many a major surprise.

ApJ, 241, 425 (October 1, 1980)

DISK-SATELLITE INTERACTIONS

PETER GOLDREICH
California Institute of Technology
AND
SCOTT TREMAINE
Institute for Advanced Study, Princeton, New Jersey
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ABSTRACT

We calculate the rate at which angular momentum and energy are transferred between a disk and a satellite which orbit the same central mass. A satellite which moves on a circular orbit exerts a torque on the disk only in the immediate vicinity of its Lindblad resonances. The direction of angular momentum transport is outward, from disk material inside the satellite's orbit to the satellite and from the satellite to disk material outside its orbit. A satellite with an eccentric orbit exerts a torque on the disk at corotation resonances as well as at Lindblad resonances. The angular momentum and energy transfer at Lindblad resonances tends to increase the satellite's orbit eccentricity whereas the transfer at corotation resonances tends to decrease it. In a Keplerian disk, to lowest order in eccentricity and in the absence of nonlinear effects, the corotation resonances dominate by a slight margin and the eccentricity damps. However, if the strongest corotation resonances saturate due to particle trapping, then the eccentricity grows.

We present an illustrative application of our results to the interaction between Jupiter and the protoplanetary disk. The angular momentum transfer is shown to be so rapid that substantial changes in both the structure of the disk and the orbit of Jupiter must have taken place on a time scale of a few thousand years.

It let to the revision of the standard picture of planet formation (~in situ formation)

Insight that orbital migration represents a key aspect of the theory which must be included.

Ironically, migration was discovered 15 years before the first exoplanet by theoretical considerations.

Basic mechanism

Planet interacts gravitationally with the disk => density waves

Density waves react back on the planet => torque Γ_{tot}

Torque change the planet's angular momentum J_p

$$\frac{dJ_p}{dt} = \Gamma_{tot}$$

with $$J_\mathrm{p} = M_p r_p v_k = M_p r_p^2 \Omega_k = M_p \sqrt{G M_\star r_p}$$

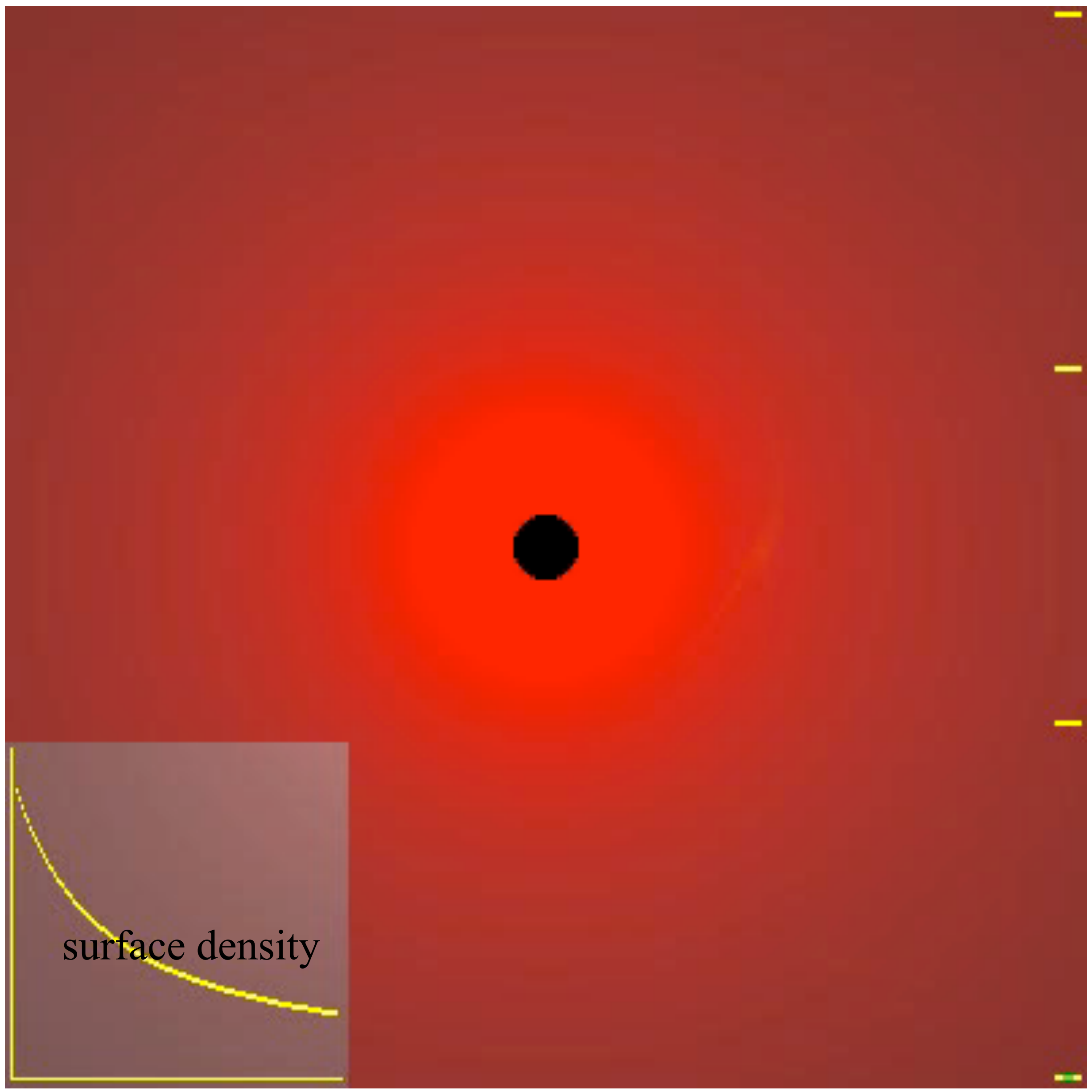
From which we obtain the migration rate:

$$\frac{dr_p}{dt} = 2 r_p \frac{\Gamma_{tot}}{J_p}$$

Depending upon the sign of the torque the migration can proceed inwards or outward.

Basic types

- for low mass planets the density waves propagate through the disk
- for larger mass planets, a gap opens in the disk



Simulations by P. Armitage

Type I migration

migration mode of low mass planets, no gap

Type II migration

migration mode of large mass planets, with gap

The movie shows the transition by ramping up the planet mass.

Inertial and rotating frame

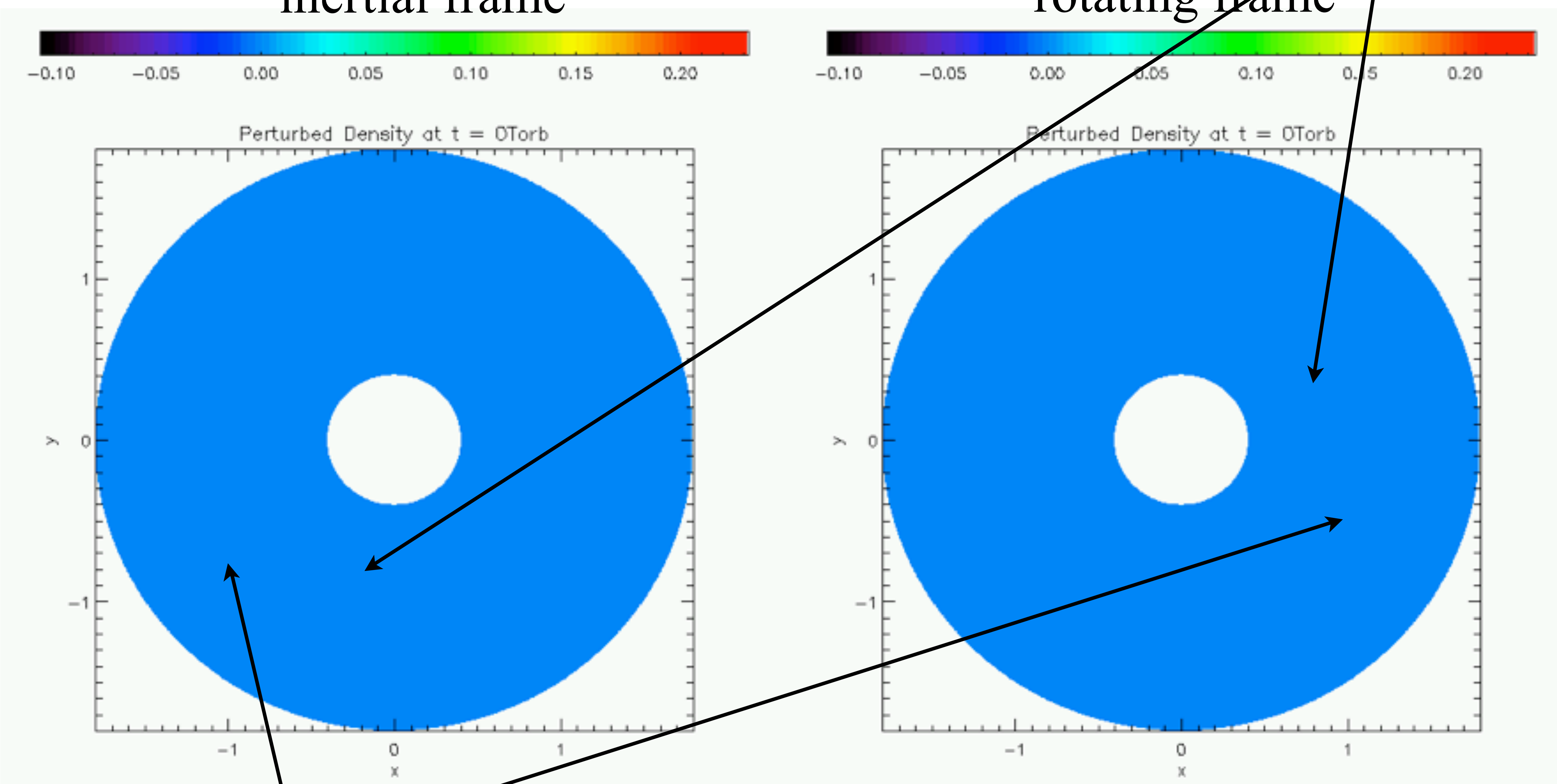
Basic mechanism of angular momentum exchange:

- heading density enhancement => pulls the planet forward => outward migration
- trailing density enhancement=> pulls the planet backwards => inward migration

forward pull: Outwards migration

inertial frame

rotating frame



backward pull: Inwards migration

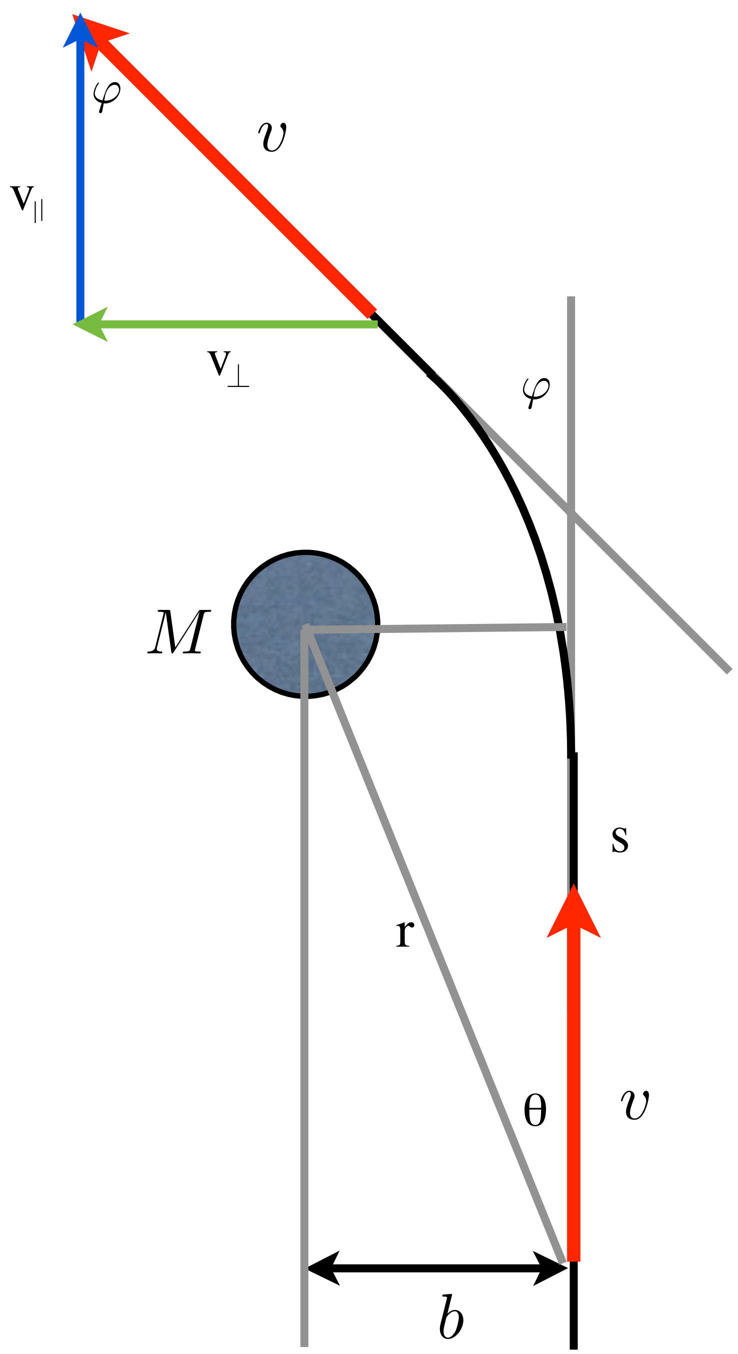
Simulations by C. Baruteau

3.1 Impulse approximation

Impulse approximation

A simple approach (Lin & Papaloizou 1979) to calculate the torque.

- gravitational interaction between planet and gas parcel flowing past
- neglect that in a corrotating frame (around the sun)
- two body approximation



Derive first the expression for the gravitational deflection angle φ for the case of a body of mass m, initial relative velocity v and an impact parameter b encountering a big body with mass M.

The force perpendicular to the initial velocity means for $v_{\perp}$

$$F_{\perp} = m\frac{dv_{\perp}}{dt} \Longrightarrow \quad v_{\perp} = \int_{-\infty}^{\infty} dv_{\perp} = \frac{1}{m}\int_{-\infty}^{\infty} F_{\perp} dt$$

From the geometry of the encounter

$$F_{\perp} = F\sin\theta = F\left(\frac{b}{r}\right) = \left(\frac{G\mathcal{M}m}{r^2}\right)\left(\frac{b}{r}\right)$$

Impulse approximation II

For small angles, we can use the Born approximation, where for the total velocity $v_{init} \approx v_{final} \approx v$

$$v_{\|} dt = v dt = ds \implies dt = \frac{ds}{v}$$

Thus $$v_{\perp} = \frac{1}{m} \int_{-\infty}^{\infty} F_{\perp} dt = \frac{2}{m} \int_{0}^{\infty} \left(\frac{G\mathcal{M}m}{r^2} \right) \left(\frac{b}{r} \right) \left(\frac{1}{v} \right) ds$$

Since from geometry $r = (s^2 + b^2)^{1/2}$

$$v_{\perp} = \frac{2G\mathcal{M}}{v} \int_{0}^{\infty} \frac{b}{(s^2 + b^2)^{3/2}} ds = \frac{2G\mathcal{M}}{v} \int_{0}^{\infty} \frac{ds/b}{(1 + (s/b)^2)^{3/2}}$$

Defining $x = s/b$ allows to evaluate the integral to find traversal velocity

$$v_{\perp} = \frac{2G\mathcal{M}}{vb} \int_{0}^{\infty} \frac{dx}{(1 + x^2)^{3/2}} = \frac{2G\mathcal{M}}{vb} \cdot \left. \frac{x}{(1 + x^2)^{1/2}} \right|_{0}^{\infty}$$

$$= \frac{2G\mathcal{M}}{vb}$$

For the (small) angle we have $\varphi = v_{\perp}/v$ from geometry thus we find for the angle

$$\boxed{\varphi = \frac{2G\mathcal{M}}{v^2 b}}$$

Impulse approximation III

Use our results form the previous page to calculate the momentum exchange.

Associate velocity v of the body with mass m with the velocity difference between a gas parcel and the planet and define:

$$v \doteq \Delta v \quad (\Delta v = v_{gas} - v_p) \qquad v_\perp \doteq \delta v_\perp \qquad v_{||} \doteq \delta v_{||}$$

The change in the perpendicular component of the velocity is thus given as before by:

$$|\delta v_\perp| = \frac{2GM_p}{b\Delta v}$$

This velocity change occurs radially: no angular momentum change. But two body encounter: conserves energy: change in the perpendicular component also implies a change in the parallel component $\delta v_{||}$.

From the conservation of energy (and Pythagoras) we have

$$\Delta v^2 = |\delta v_\perp|^2 + (\Delta v - \delta v_{||})^2$$

Evaluating this, and neglecting the quadratic term in $\delta v_{||}$ (small deflection)

$$\delta v_{||} \simeq \frac{1}{2\Delta v}\left(\frac{2GM_p}{b\Delta v}\right)^2$$ Change in parallel velocity

Impulse approximation IV

Change of angular momentum of the gas parcel associated with $\delta v_{||}$ must be balanced by the opposite change of angular momentum of the planet. For a planet with a semi-major axis *a*, this implies a change in specific angular momentum:

$$\Delta j = a\, \delta v_{||} = \frac{2G^2 M_{\rm p}^2 a}{b^2 \Delta v^3}$$

Net differential torque

Gas exterior to the planet: overtaken by the planet=>angular momentum loss for the planet => gain for the gas.

Gas interior to the planet: overtakes the planet=>angular momentum gain for the planet => loss for the gas.

The net direction of migration thus depends on the difference between the interior and exterior torque.

Impulse approximation V

To compute this net torque, integrate the single particle torque over all gas in the disk. Consider a small annulus outside the orbit of the planet at distance a. The mass in the interval (*b;b+db*) is $dm \approx 2\pi a \Sigma db$

The net torque will be the sum of all the torques (inside and outside) and will depend on the exact structure of the disk.

If the planet has an orbital frequency Ω_p and the gas has Ω, the gas parcel suffers impulses separated by

$$\Delta t = \frac{2\pi}{|\Omega - \Omega_p|}$$

For small displacements b<<a, a first order expansion of the angular frequencies yields:

$$|\Omega - \Omega_p| \simeq \left|\frac{d\Omega_p}{da}\right| b \simeq \frac{3\Omega_p}{2a} b$$

The total temporal change of the angular momentum of the planet must be the integral over the angular momentum transfer of all interacting gas parcels per unit time:

$$\frac{dJ}{dt} = -\int \frac{\Delta j \, dm}{\Delta t}$$

Eliminate Δv by assuming Keplerian orbits and a first order expansion

$$\Delta v \simeq |\Omega'_{\rm p}| ab = (3/2)\Omega_{\rm p} b$$

Impulse approximation VI

Substituting yields

$$\frac{dJ}{dt} = -\int_0^\infty \frac{8G^2 M_\mathrm{p}^2 \Sigma a}{9\Omega_\mathrm{p}^2 b^4} db$$

This integral diverges at the inner boundary, but if we specify some minimum impact parameter $b_{min}>0$, we easily find (for a constant surface density)

$$\Gamma_{tot} = \frac{dJ}{dt} = -\frac{8G^2 M_\mathrm{p}^2 \Sigma a}{27\Omega_\mathrm{p}^2 b_\mathrm{min}^3}$$

Values of b_{min} are between the Hill radius (for low-mass planets) and the disc scale-height H (for massive planets). Then, one finds a torque which agrees approximately with that obtained from more detailed analyses:

- the torque scales with the surface density of the disk
- the torque scales with the square of the planet mass
- the migration timescale varies with planet mass as $\tau_{mig} = \frac{J}{dJ/dt} \propto \frac{1}{M_p}$

For fixed disk conditions, more massive planets migrate faster.

3.2 Gap formation

Gap opening

Gas inside the planet looses angular momentum and moves inwards while gas outside gains angular momentum and moves outwards. For this mechanism to result in the opening of a gap, two conditions have to be met.

Condition I (thermal condition):

Hills sphere of a planet >= the disk scale height. Otherwise the disc accretes past the planet away from the disc midplane.

$$r_H = r_p \left(\frac{M_p}{3M_*} \right)^{1/3} \geq H$$

Which implies a mass ratio planet/star of:

$$q = \frac{M_p}{M_*} \geq 3 \left(\frac{H}{r} \right)_p^3 = 3h_p^3$$

Typically the disk aspect ratio is $h \approx 0.05$ and $q \geq 1.25 \cdot 10^{-4}$ corresponding to $M > 0.13\ M_{Jupiter}$.

Gap opening II

Condition II (viscous condition)**:**

Viscous effect must not be able to close the gap. This can be expressed by the condition:

$$\tau_{close} \geq \tau_{open}$$

In terms of torque, this condition is written

$$\left(\frac{dJ}{dt}\right)_{LR} \geq \left(\frac{dJ}{dt}\right)_{visc}$$

Or recalling previous expressions:

$$\frac{8}{27}\frac{G^2 M_p^2 r_p \Sigma}{9\Omega_p^2 b_{min}^3} \geq 3\pi\nu\Sigma r_p^2 \Omega_p$$

With $\nu = \alpha c_s H,$ and $b_{min} = R_H$ we get:

$$q \geq \frac{243\pi}{8}\alpha h^2$$

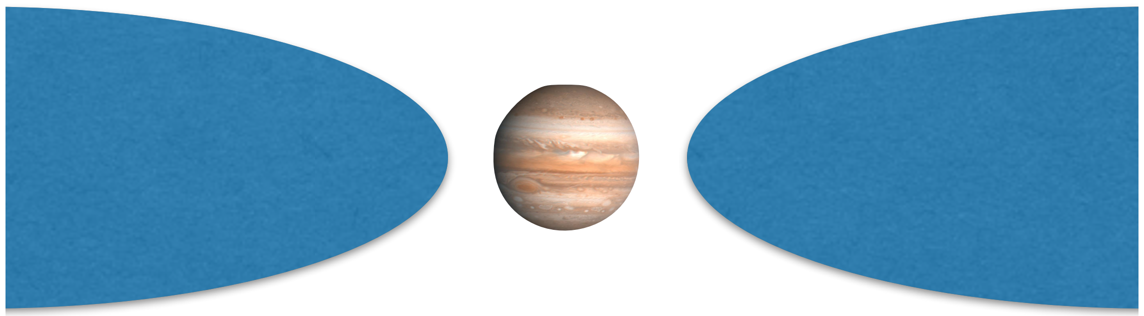
Typically h≈0.05, $\alpha = 10^{-2}$ so that q ≥ 2.39·10^{-3} corresponding to $M > 2.5\ M_{Jupiter}$

In usual conditions, it is the viscosity criterion that determines the opening of a gap.

3.3 Migration timescales

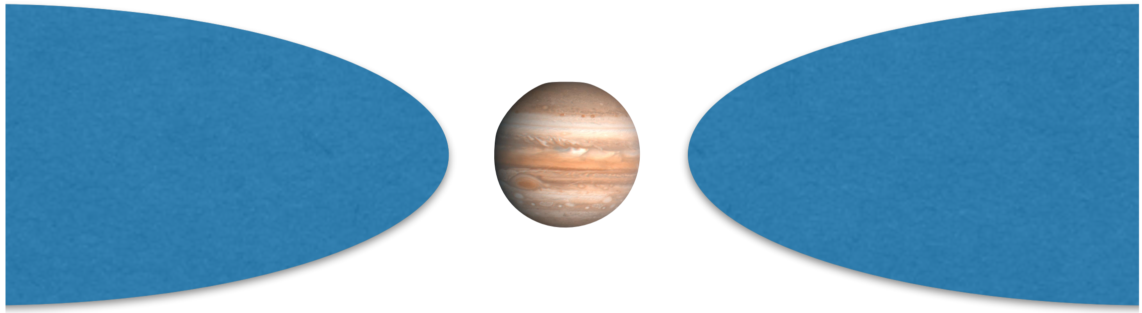
Type II migration

Planet massive enough to open gap: gas is pushed away from the planet and hence the torques diminish. The planet is kept in the middle of the gap



- if it were to be closer to the inner edge, it would gain angular momentum, and it would migrate back outwards, while
- if it were closer to the outer edge, it would lose angular moment and migrate back inwards

Static disk: the planet is also static, no migration.
Real disk: evolving on the viscous timescale. Also the planet's orbit is evolving on this timescale. The reality is more complex: flux across gap



Type II migration

Type II migration timescale

$$\tau_{II} = \frac{r_p^2}{\nu} = \frac{r_p^2}{\alpha c_s H} = \frac{1}{\alpha}\left(\frac{r_p}{H}\right)^2 \Omega_p^{-1}$$

where we have used the fact that the viscosity is given by $\nu = \alpha c_s H$ and the sound speed is approximated by $c_s = H\Omega_p$

Typical numbers: $\alpha=10^{-2}$: ~10^5 yrs, $\alpha=10^{-3}$: ~10^6 yrs

This migration timescale is independent of the mass of the planet and only depends upon the mass of the star and the characteristics of the disk. This simple picture is valid only if the planet is not too massive. One therefore distinguishes two regimes:

Disk dominated type II (B>>1): $\tau_{II} = \tau_{visc}$

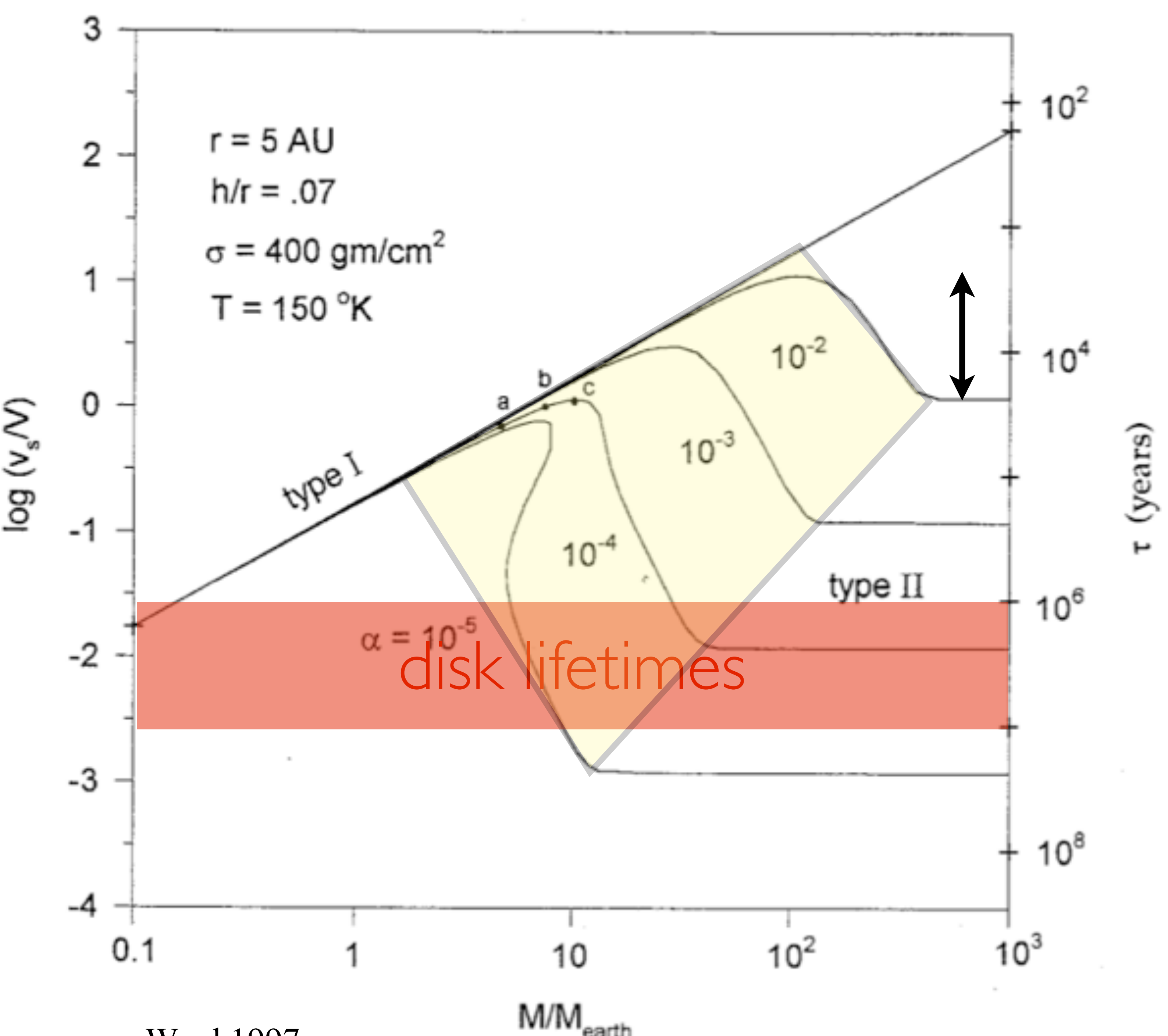
Planet dominated type II (B<<1): $\tau_{II} \sim \tau_{visc} B$

$$B = \frac{3\pi\Sigma_0 R_0^2}{M_p}$$

Clearly, in the planet dominated regime, migration is slower.

Migration timescales: too fast type I

The migration rates predicted for type I migration in a locally isothermal disk can be extremely short: ~10^4 yrs



Ward 1997

Planets seem to migrate so fast that they should all fall into the star within the lifetime of the disk (unless they grow extremely rapid)!

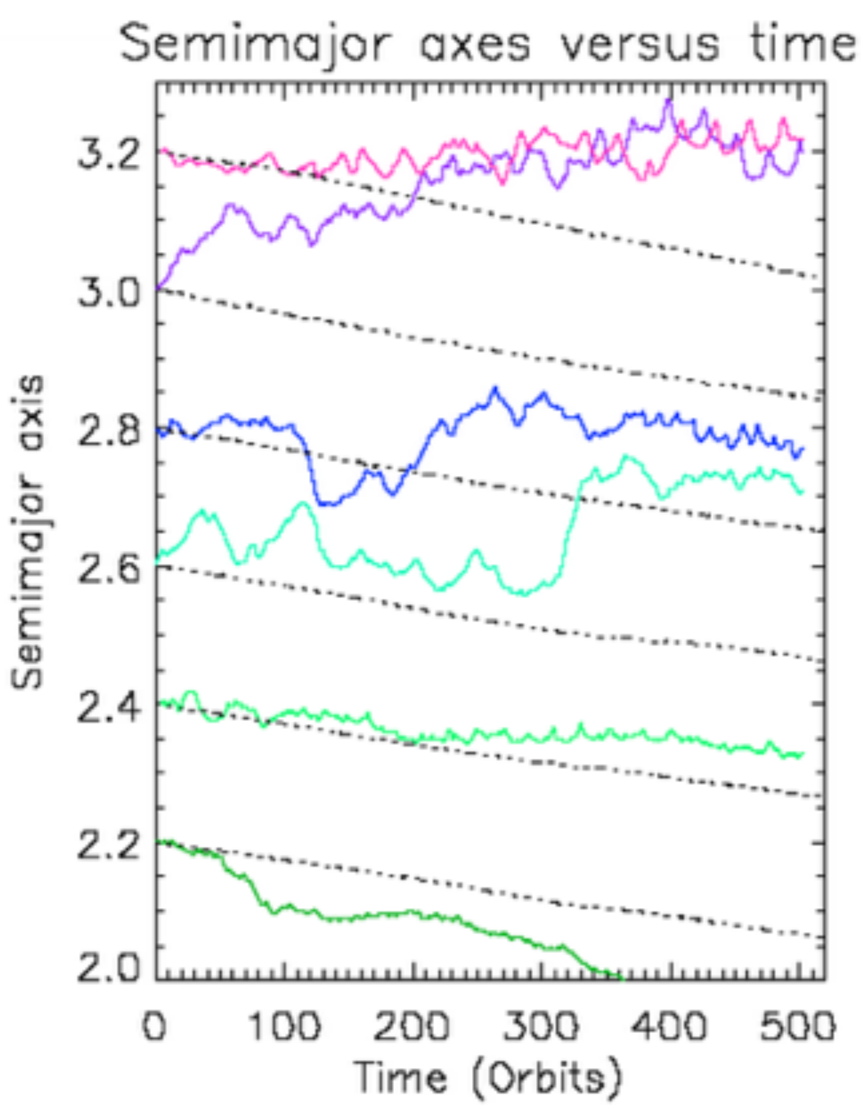
These very short migration timescales represent another major issue in modern planet formation theory.

simple linear theory for isothermal disks cannot be the final word!

Updated type I migration rates

1) Random walk migration in turbulent disks

In such turbulent disks, it is found that for low mass planets, Type I migration is no longer effective due to large fluctuations in the torque. The fluctuations in the torque created by the perturbations in the density can be larger than the mean torque expected for standard Type I migration in a laminar disk.

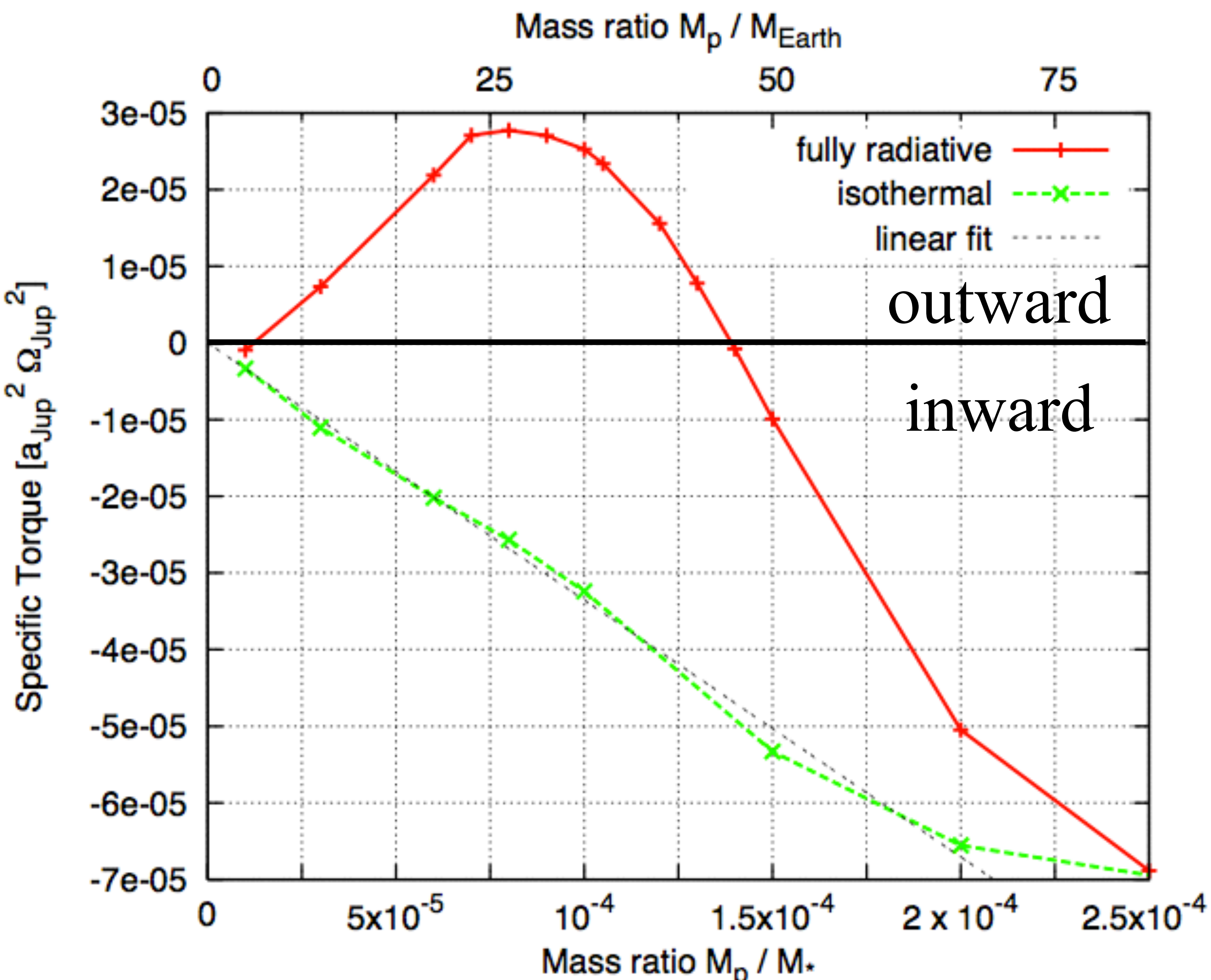


Migration of M=10 M⊕ planets. The planets undergo migration similar to a random walk for the duration of the simulation, with no clear tendency for the planets to migrate inward or outward.

Non-isothermal type I migration

2) Migration in non-isothermal disk

Crida et al. 2006; Baruteau & Masset 2008; Casoli & Masset 2009; Pardekooper et al. 2010; Baruteau & Lin 2010



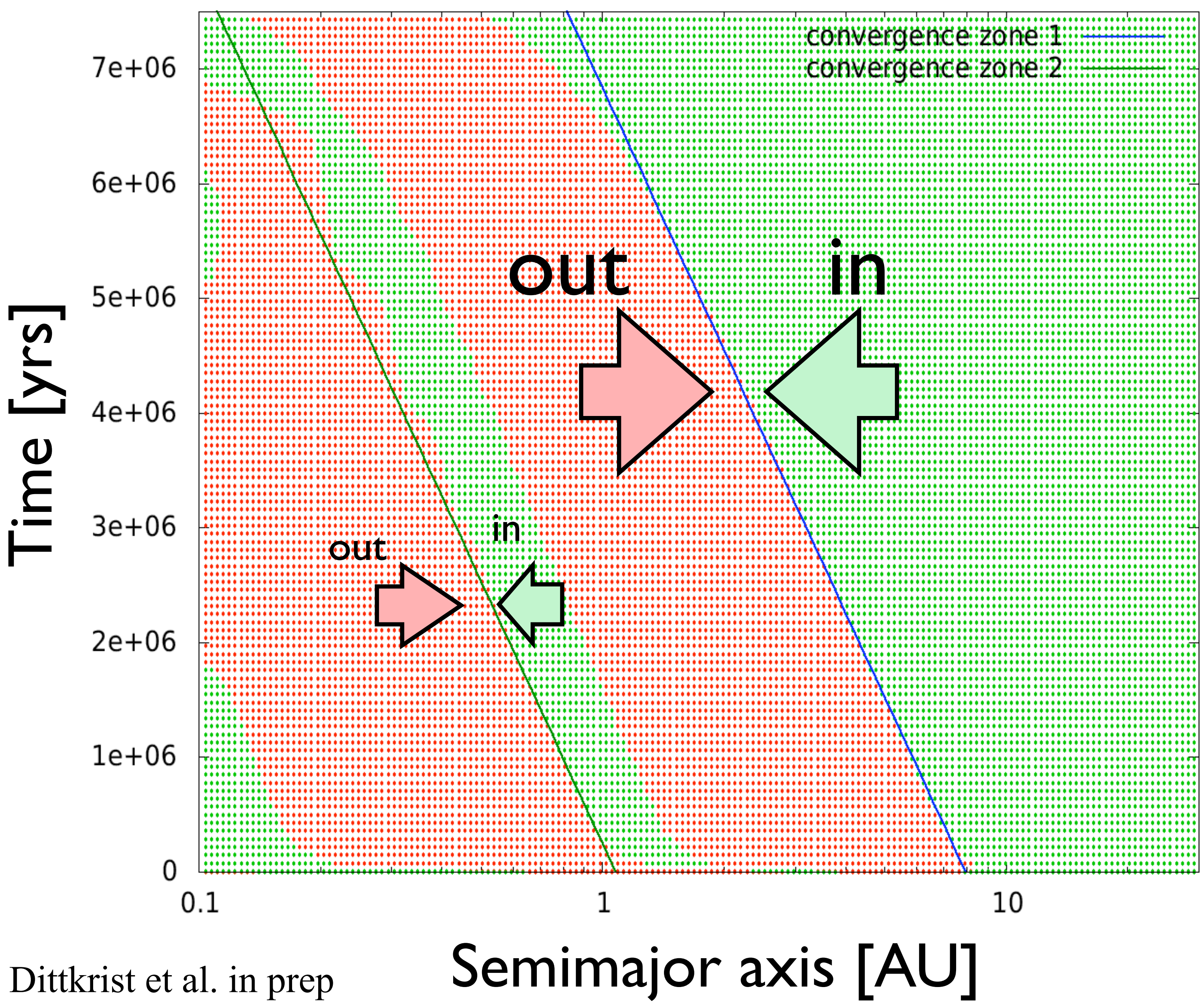
Kley & Crida 2009

An important (and not justified) assumption in the derivation of the classical type I torque: the gas around the planet acts isothermally.

Radiation hydrodynamic simulations treating correctly the energy transport: below a threshold mass, migration is outwards (different gas density distribution around the planet).

Thermodynamics of the disk is essential

Type I convergence zones



Dittkrist et al. in prep

Important consequence of non-isothermal migration: convergence zones (zero torque locations in which planets get trapped).

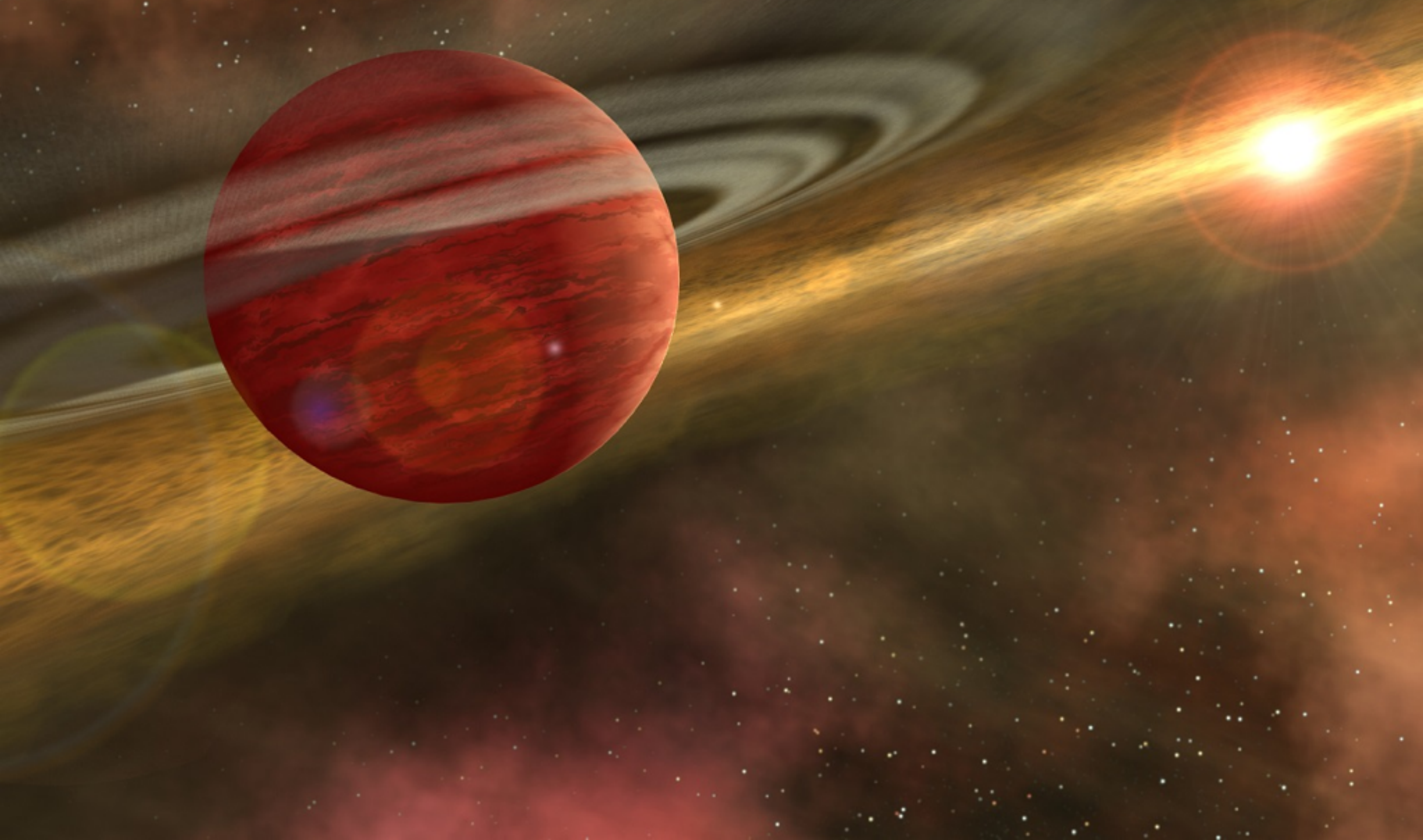
The location of the convergence zone itself moves inward on a viscous timescale. This means that despite being in the type I regime, the planets will move inwards on a much slower viscous timescale, as in type II.

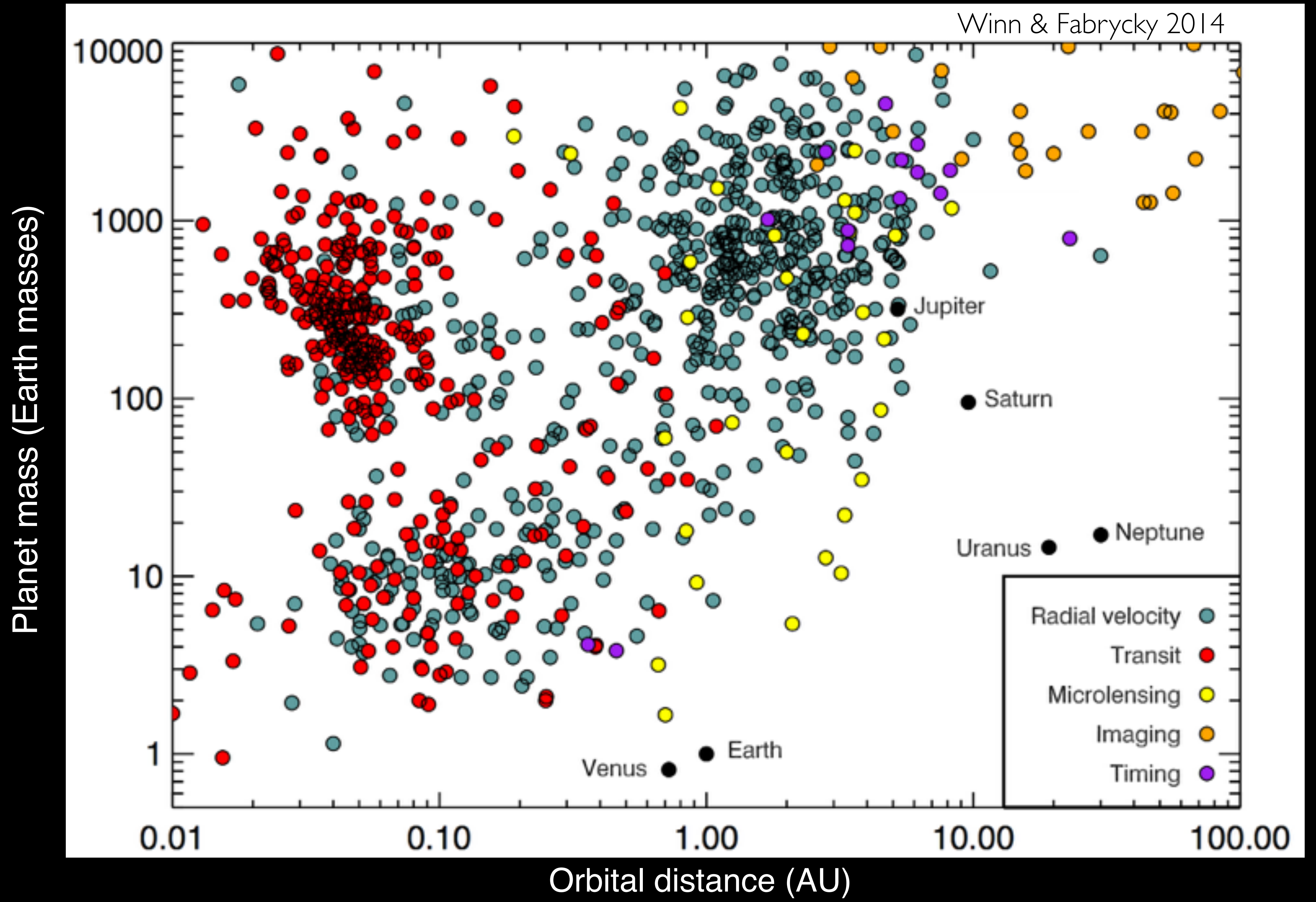
It is tempting to think that these zones are the places to grow massive planets, as they might concentrate many growing protoplanets.

Summary on migration

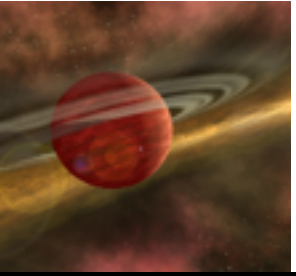
- Disk migration is a natural consequence of the gravitational interaction of the planets with the gas disk
- Computing the migration rate is a complex problem as one is interested in the small difference between positive and negative torques
- Migration timescales can be very short, affecting strongly the architecture of planetary systems
- Migration is generally directed inwards, but recent developments shows a more complex behavior with special planet traps
- Migration is an area of active ongoing research
- There are also other mechanism that can change the semimajor axis of a planet, namely planet/planet scattering or Kozai interaction with distant perturbers combined with tidal circularization
- The discovery of planets in mean motion resonances or of close-in very young companions are strong indications that disk migration happened

Lecture III
Planetary population synthesis





1. Observational motivation



Population synthesis as a tool

Population synthesis is a tool to:

- use all known exoplanets to constrain planet formation models
- test the implications of new theoretical concepts
- provide a link between theory and observations

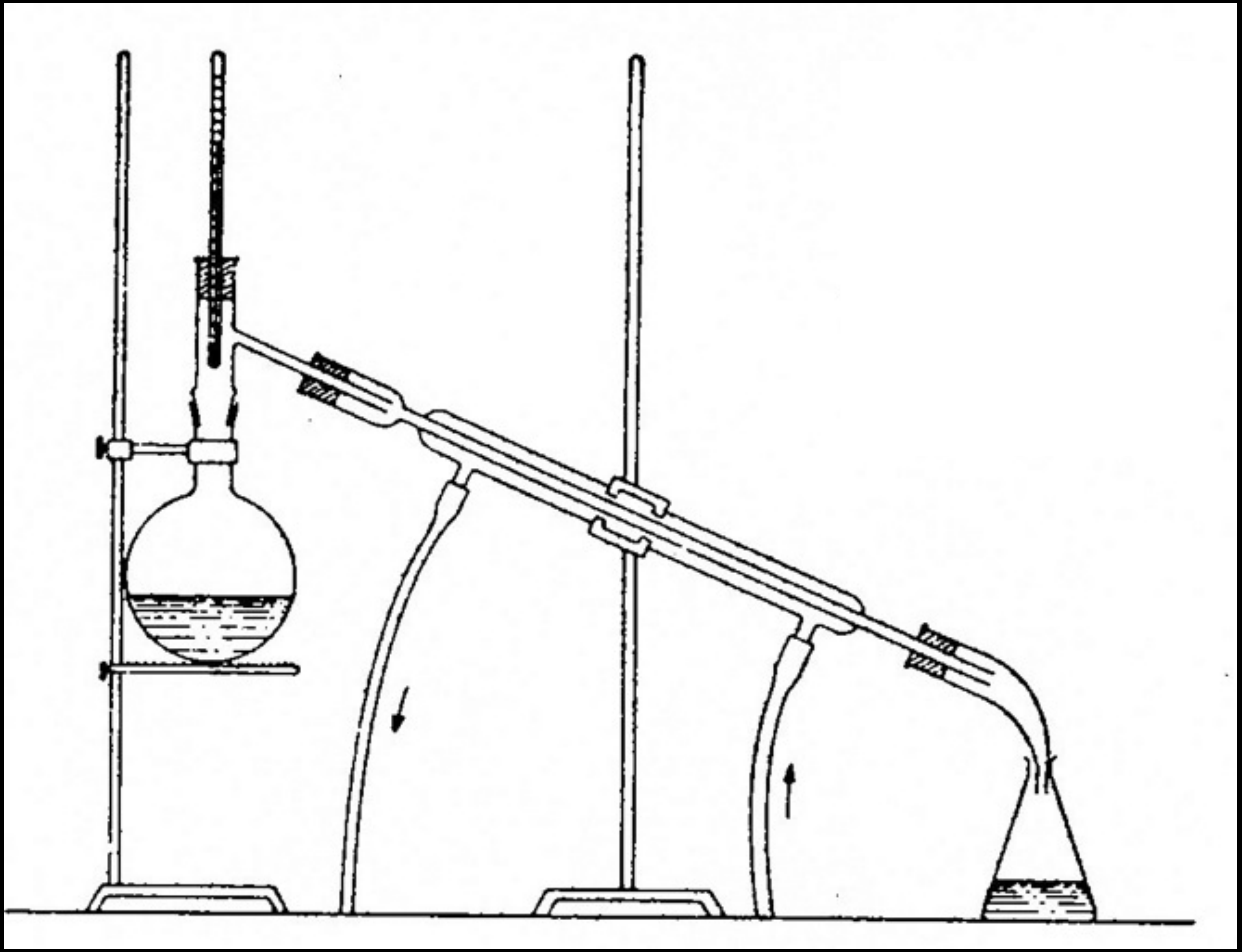
Statistical approach rather than comparing individual systems

- need to compute the formation of many planets
- the approach and the physics must be simplified
- but it must capture the key effects

⇒ builds on all detailed studies of specific physical mechanism, combining them into a global formation & evolution model

- depends on / reflects the general progress of planet formation theory

One learns a lot even if a synthetic population does not match the observed one!

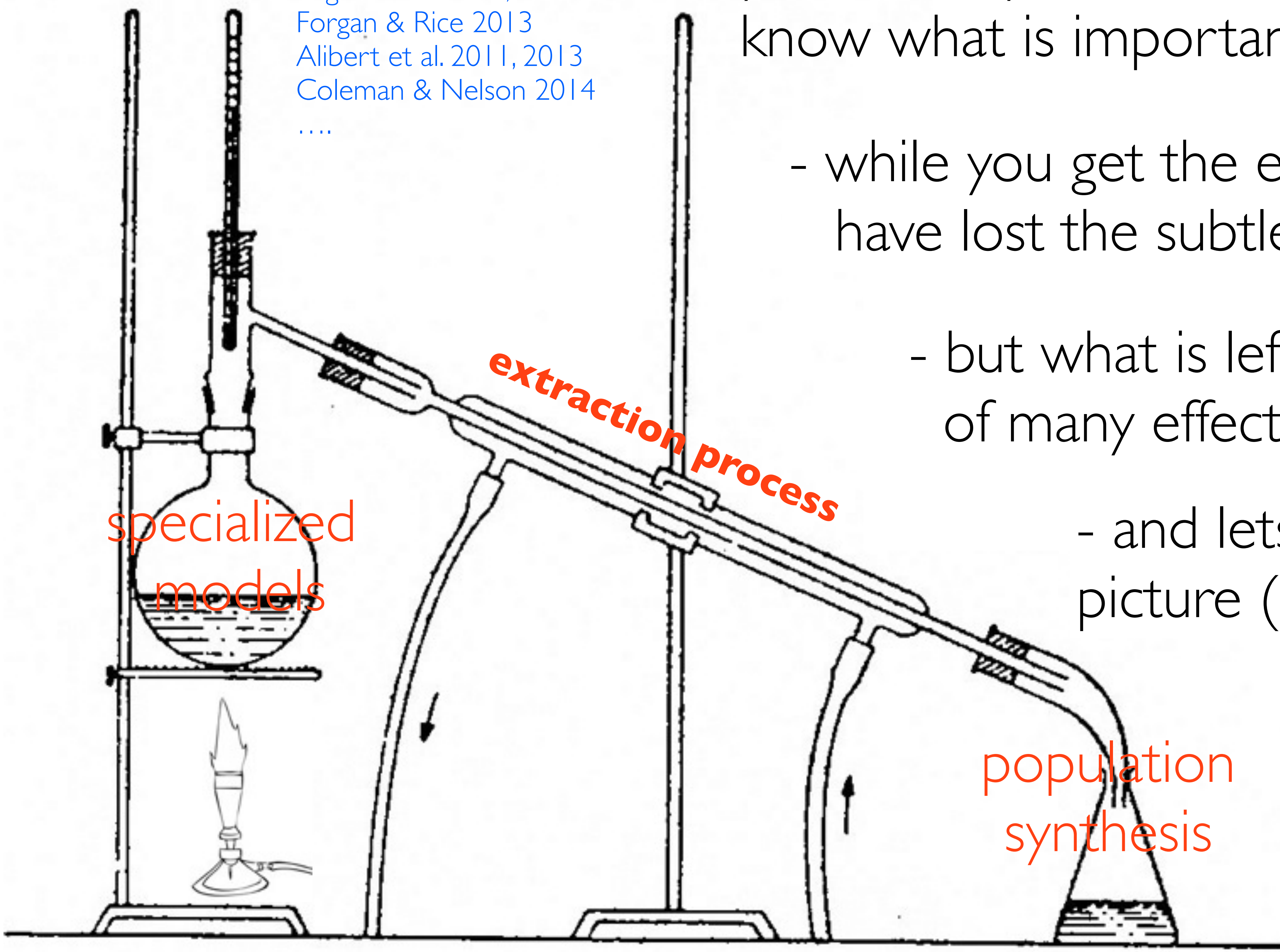


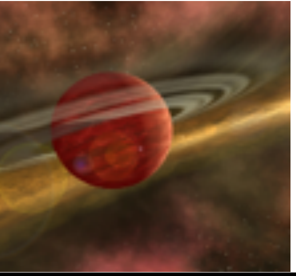
2. Population synthesis principle

The essence of the method

Ida & Lin 2004-2013
Thomes et al. 2008
Mordasini et al. 2009-2015
Miguel et al. 2008, 2009
Forgan & Rice 2013
Alibert et al. 2011, 2013
Coleman & Nelson 2014
....

- you need specialized models to know what is important
- while you get the essence, you have lost the subtlety of the original
- but what is left is a concentrate of many effects
- and lets you see the big picture (hopefully)





Population synthesis work flow

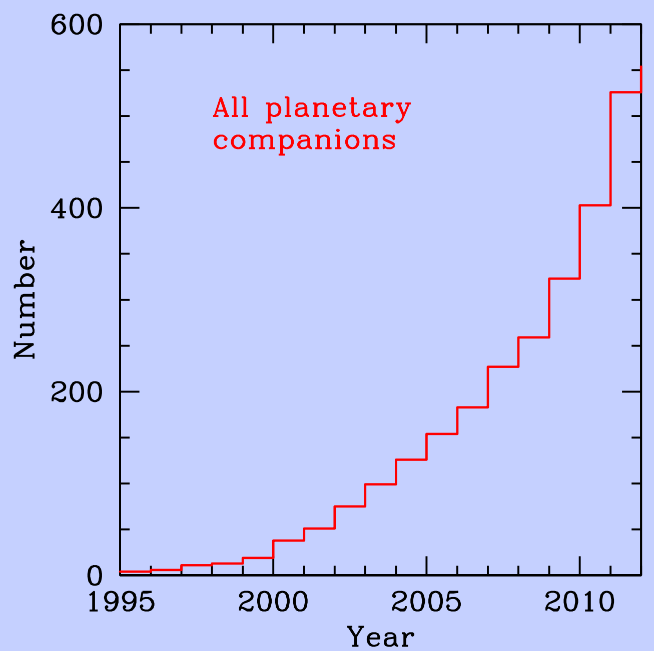
Formation model

Link disk properties ⇒ planet properties

Initial Conditions: Probability distributions & parameters

Disk gas mass
Disk dust mass
Disk lifetime

From observations



Observed population

Draw and compute synthetic planet population

Apply observational detection bias

Comparison:

Observable sub-population
- Distribution of semi-major axis
- Distribution of masses
- Distributions of luminosities
- Distribution of radii

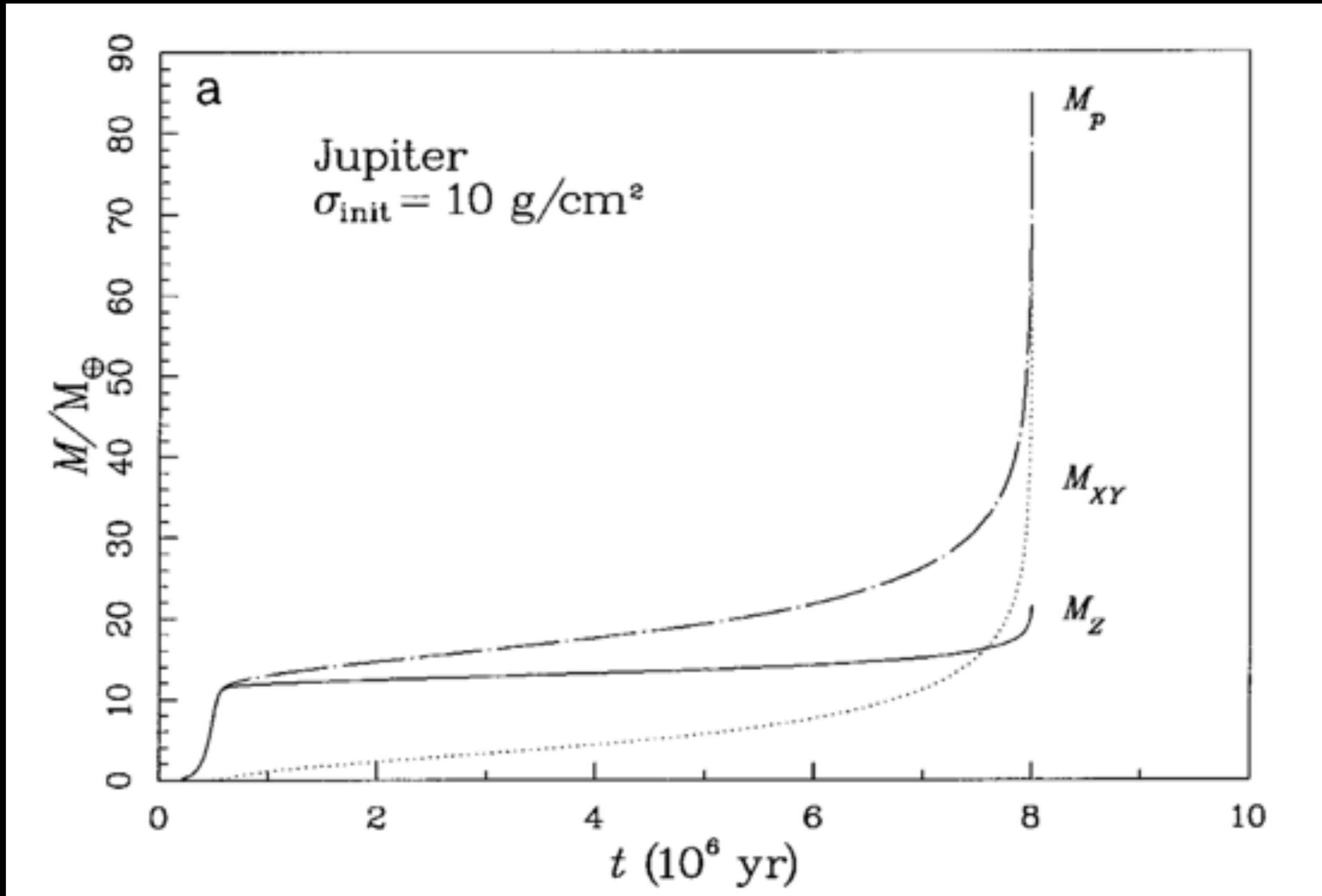
No match: improve, change parameters

Match

Model solution found!

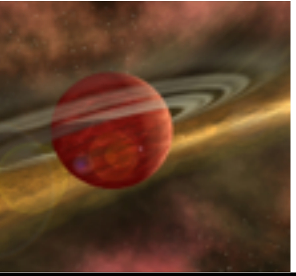
Predictions

(going back to the full synthetic population)



Pollack et al. 1996

3.
Input physics: global models



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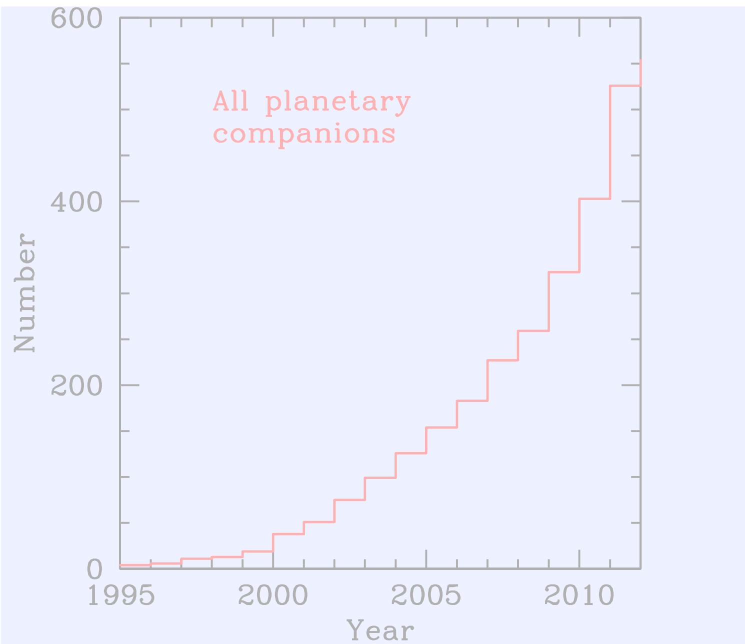


Observed population

Comparison:

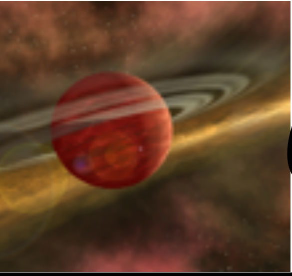
Observable sub-population
- Distribution of semi-major axis
- Distribution of masses
- Fraction of hot/cold Jupiters
- Distribution of radii

No match: improve, change parameters

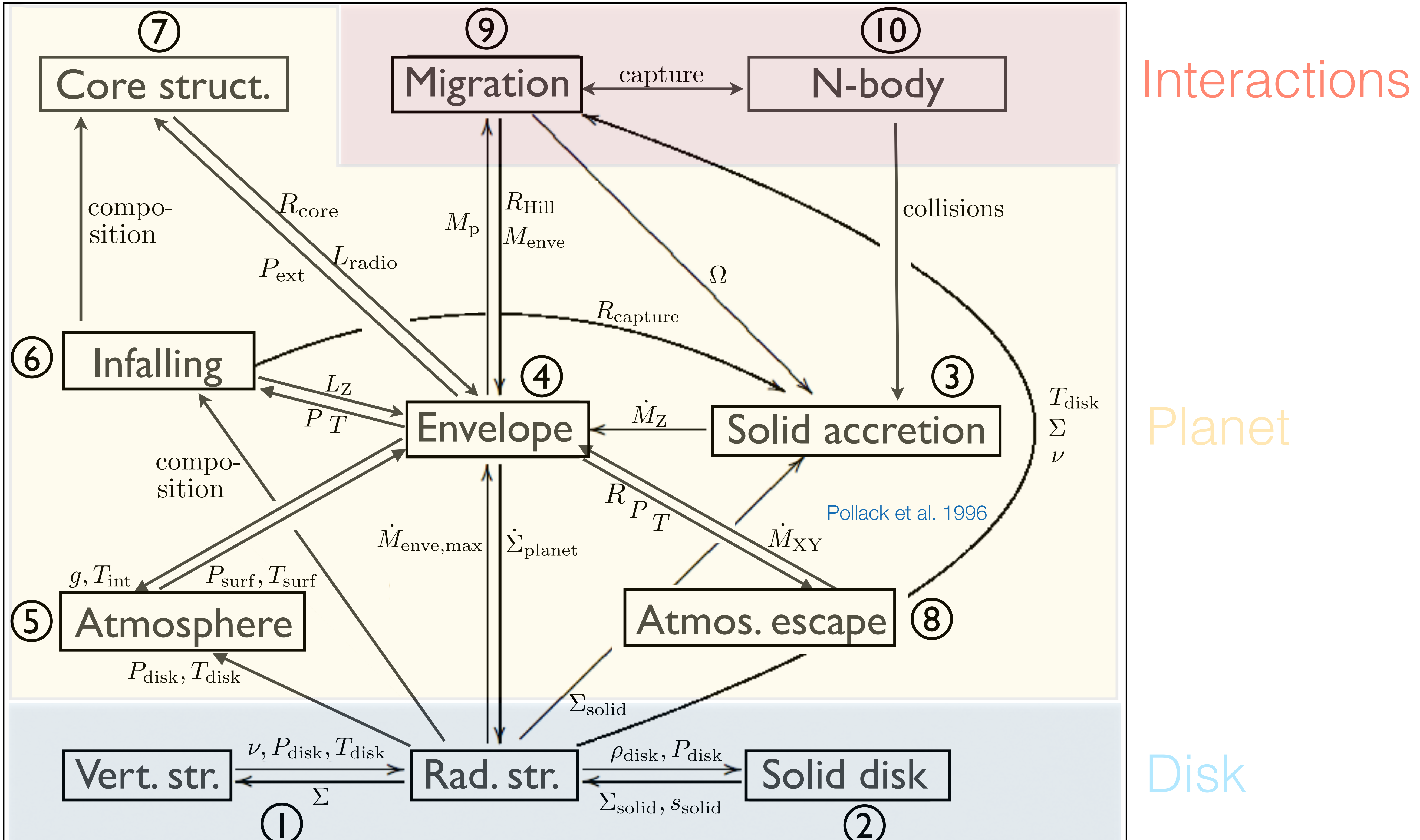
Match

Model solution found!

Predictions
(going back to the full synthetic population)

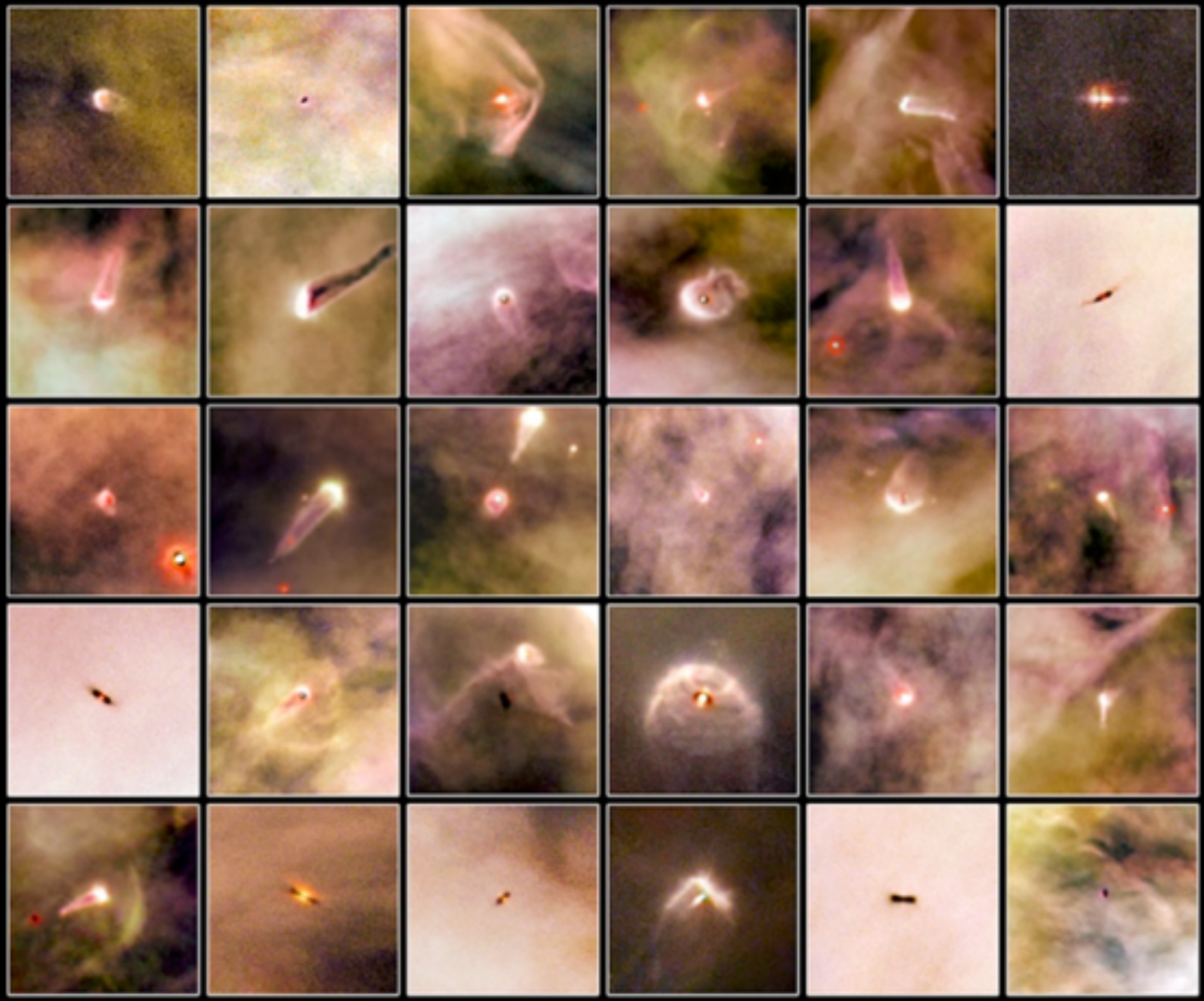


CA global formation & evolution model

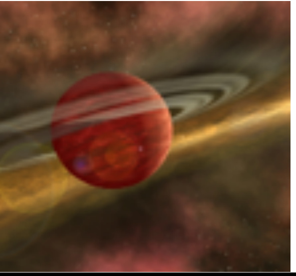


Simple standard models, but coupled: MANY links. Self-consistency.

Alibert, Mordasini, Benz 2004; Alibert et al. 2005, Mordasini et al. 2012, Alibert et al. 2013, Sheng & Mordasini 2014,…



4.
Probability distributions



Population synthesis work flow

Initial Conditions: Probability distributions & parameters

Disk gas mass
Disk dust mass
Disk lifetime

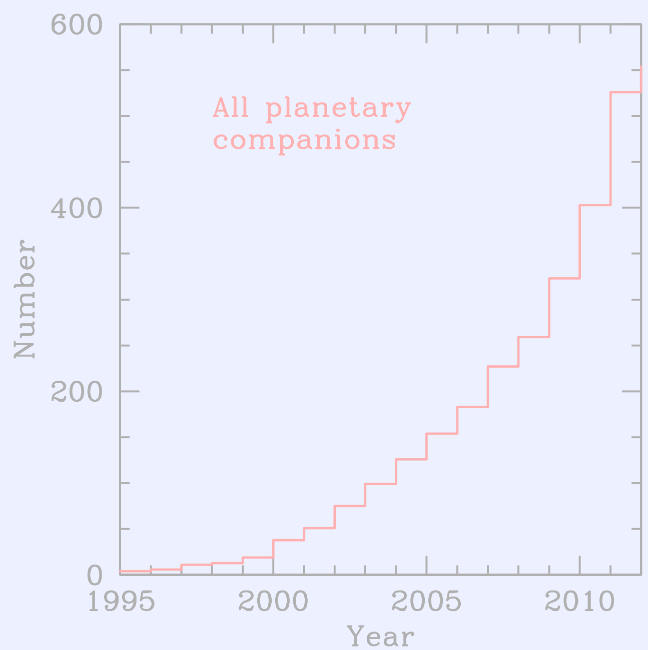
From observations

Formation model

Link disk properties ⇒ planet properties

Draw and compute synthetic planet population

Apply observational detection bias



Observed population

Comparison:

Observable sub-population
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- Fraction of hot/cold Jupiters
- Distribution of radii

No match: improve, change parameters

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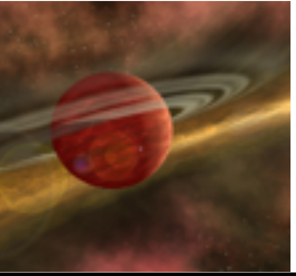
Model solution found!

Predictions

(going back to the full synthetic population)

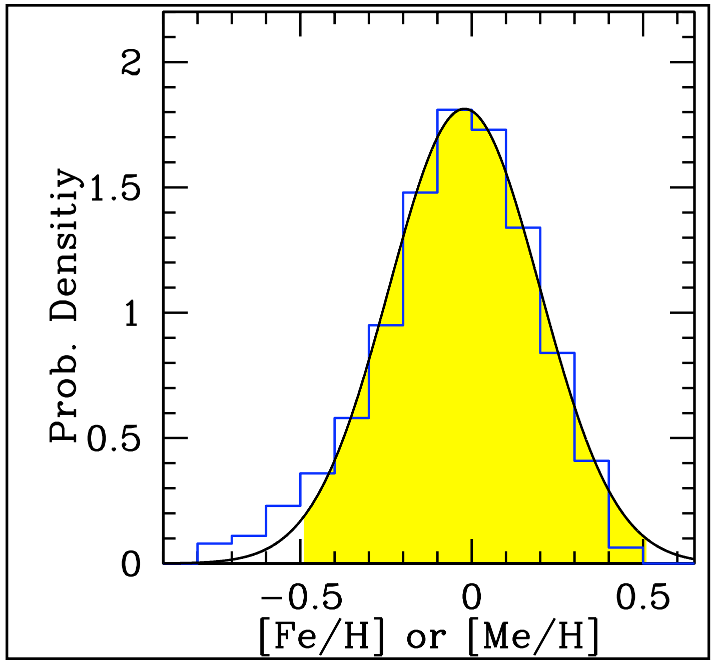


3 Monte Carlo initial conditions

1 Metallicity

assume same in star and disk

Stellar [Fe/H] from spectroscopy. Gaussian distribution for [Fe/H] with μ ~0.0, σ~ 0.2. (e.g. Santos et al. 2003)

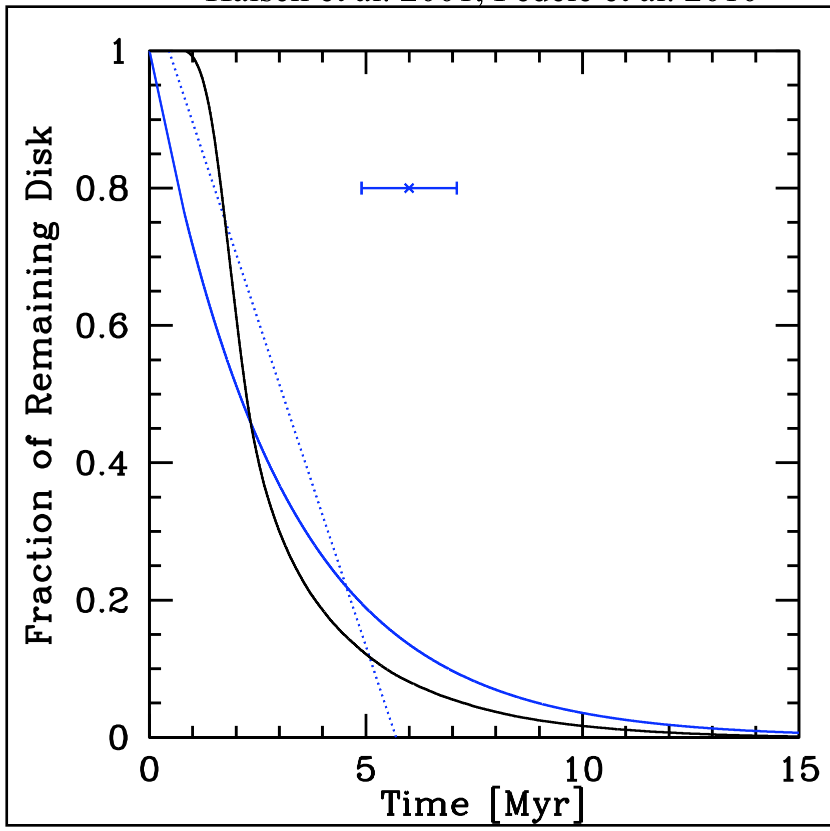


3 Disk lifetime

IR excess

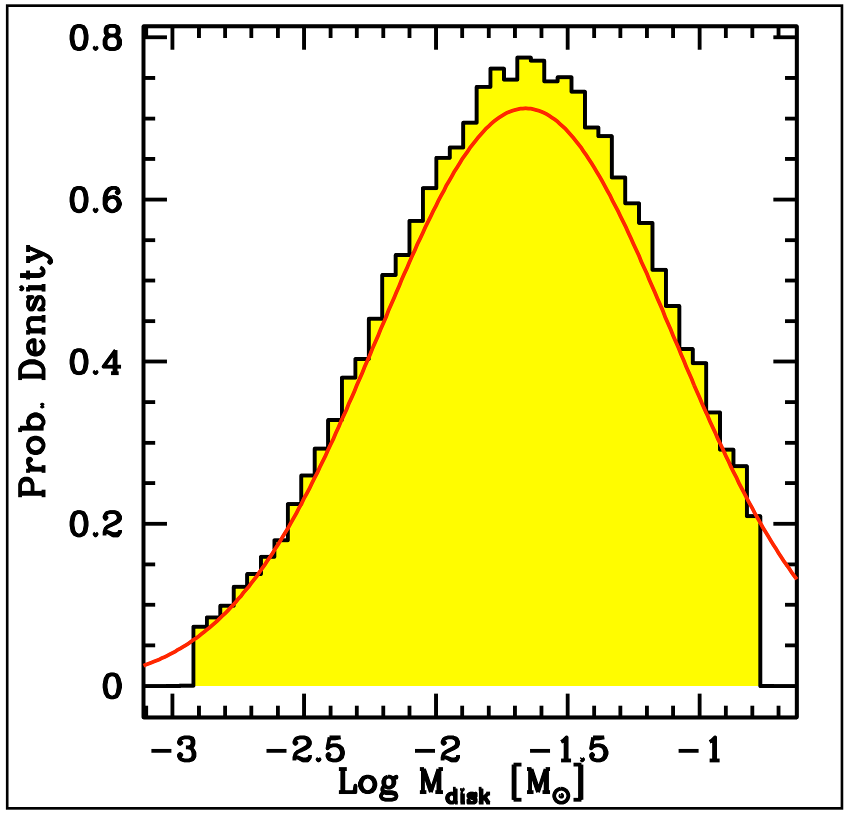
vary lifetime via photoevaporation rate

Haisch et al. 2001, Fedele et al. 2010



2 Disk (gas) masses

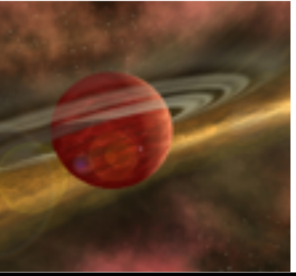
Thermal continuum emission from cold dust at mm and submm wavelengths (Ophiuchus nebula).



Draw initial conditions in Monte Carlo way to calculate synthetic population

OHP 1.93 m - 51 Peg b discovery

5. Detection biases & Statistical comparison



Population synthesis work flow

Formation model

Link disk properties ⇒ planet properties

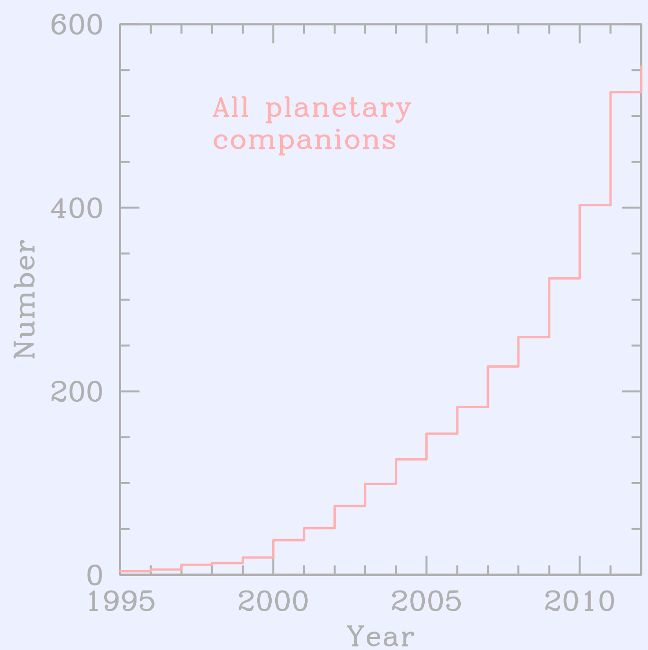
Initial Conditions: Probability distributions & parameters

Disk gas mass
Disk dust mass
Disk lifetime

From observations

Draw and compute synthetic planet population

Apply observational detection bias



Observed population

Predictions

(going back to the full synthetic population)

Comparison:

Observable sub-population
- Distribution of semi-major axis
- Distribution of masses
- Fraction of hot/cold Jupiters
- Distribution of radii

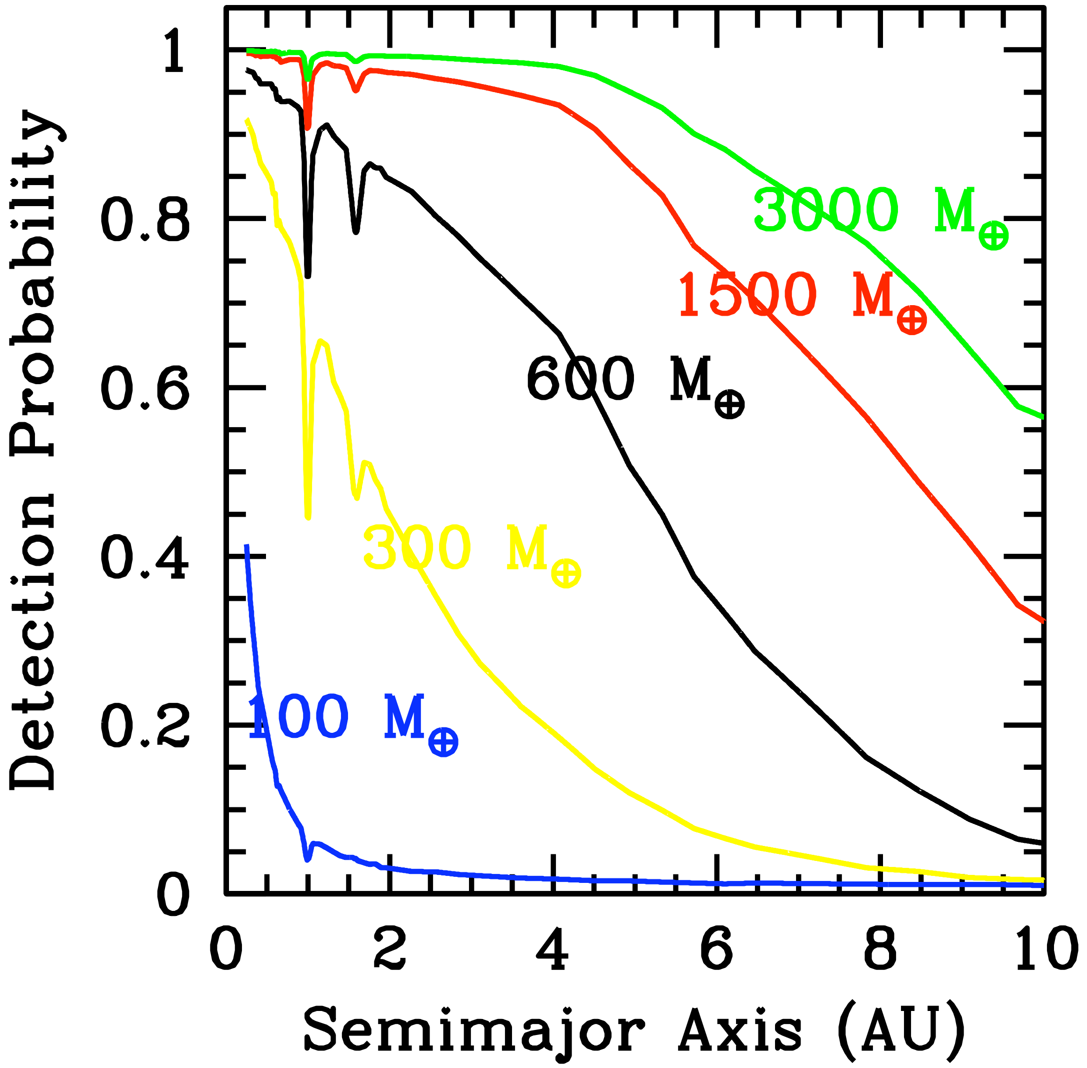
No match: improve, change parameters

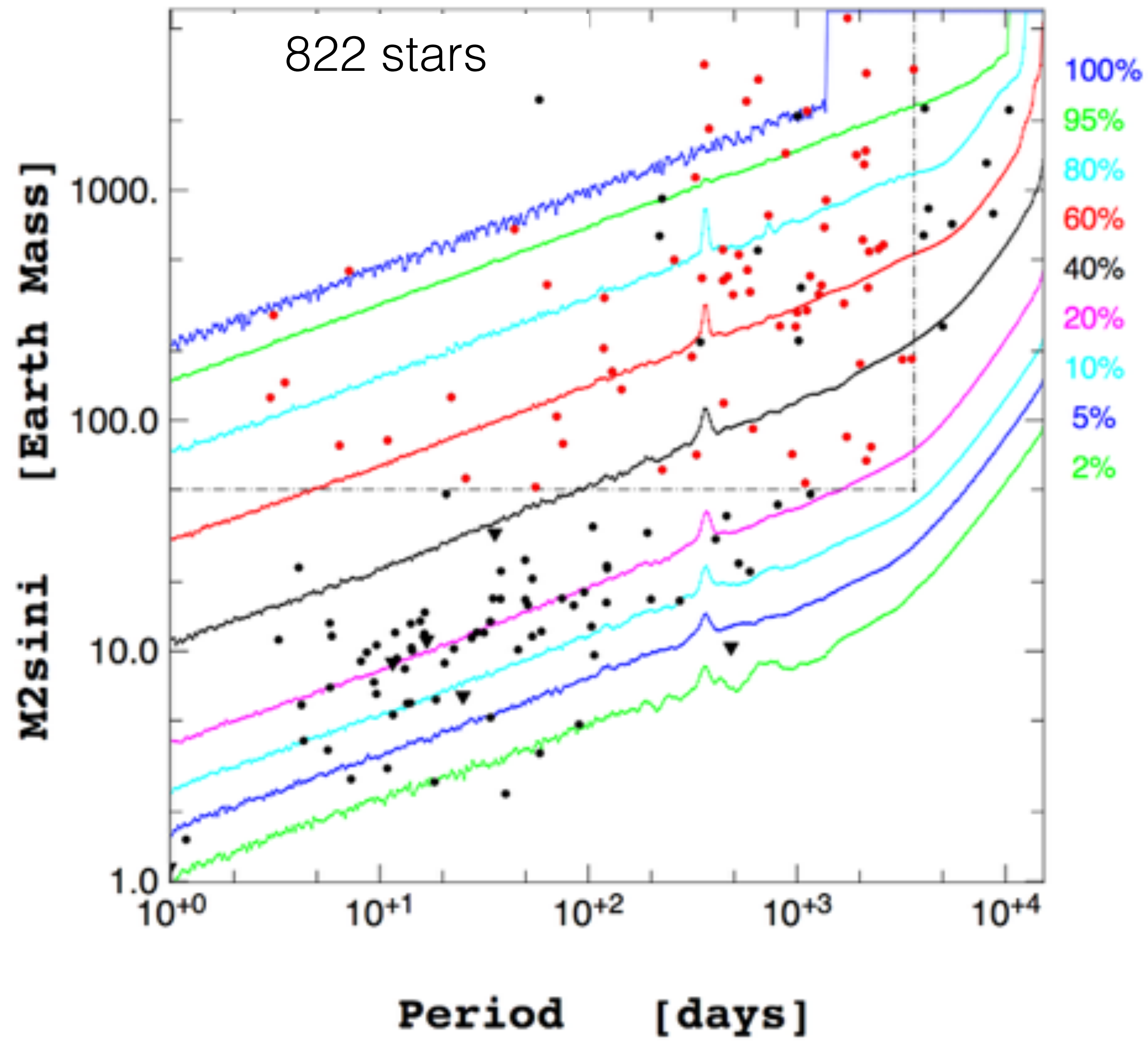
Match

Model solution found!

Radial velocity detection bias

Get sub-population of *observable* synthetic planets





Elodie ~10 m/s Instrumental precision HARPS ~1 m/s

Includes effects of

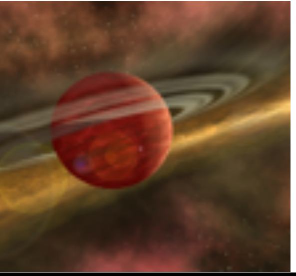
- Orbital eccentricity
- Stellar metallicity, rotation rate, and jitter
- Actual measurement schedule

Naef et al. 2004

Mayor et al. 2011

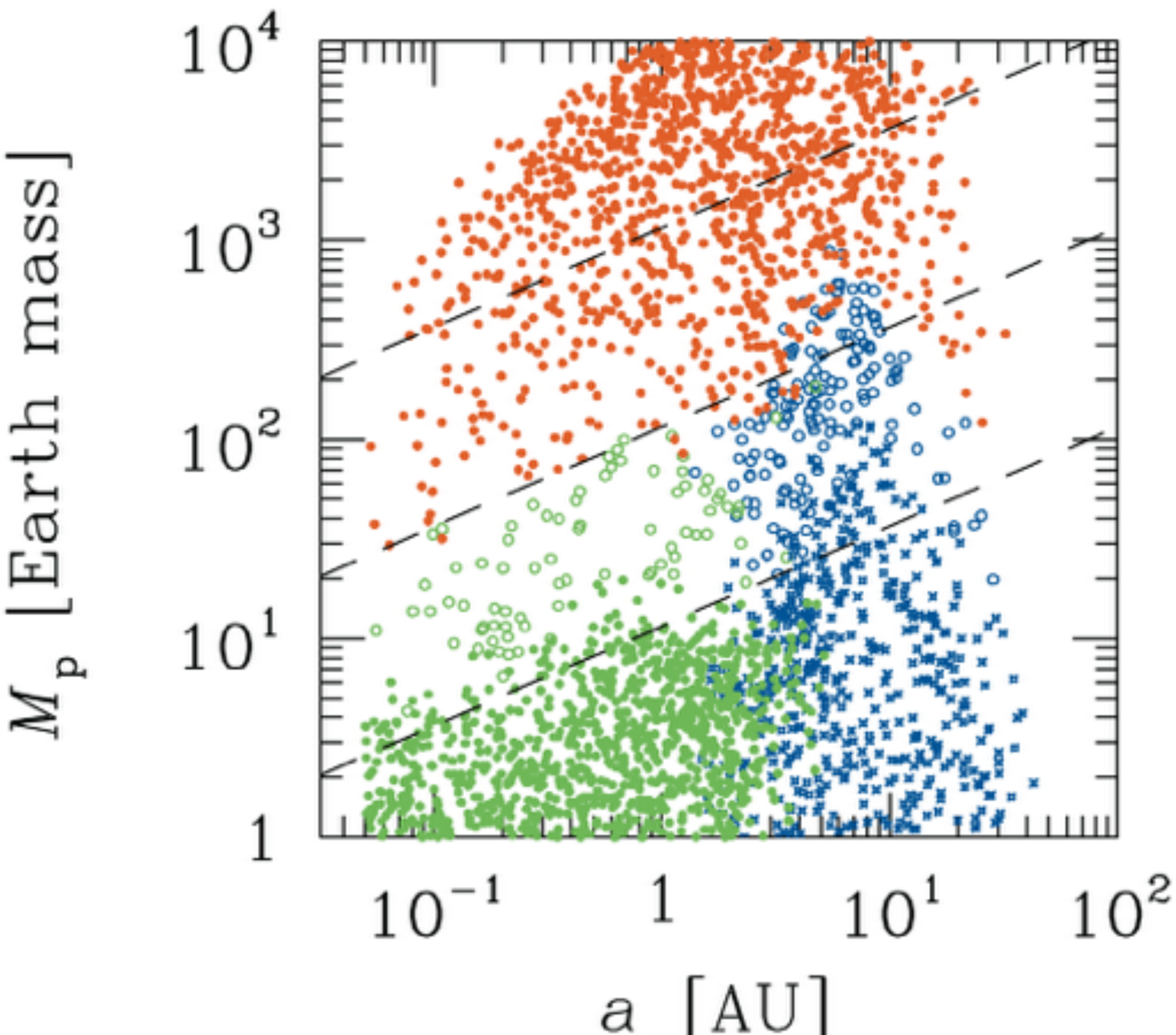
Planetary population synthesis
PART II

Results and perspectives

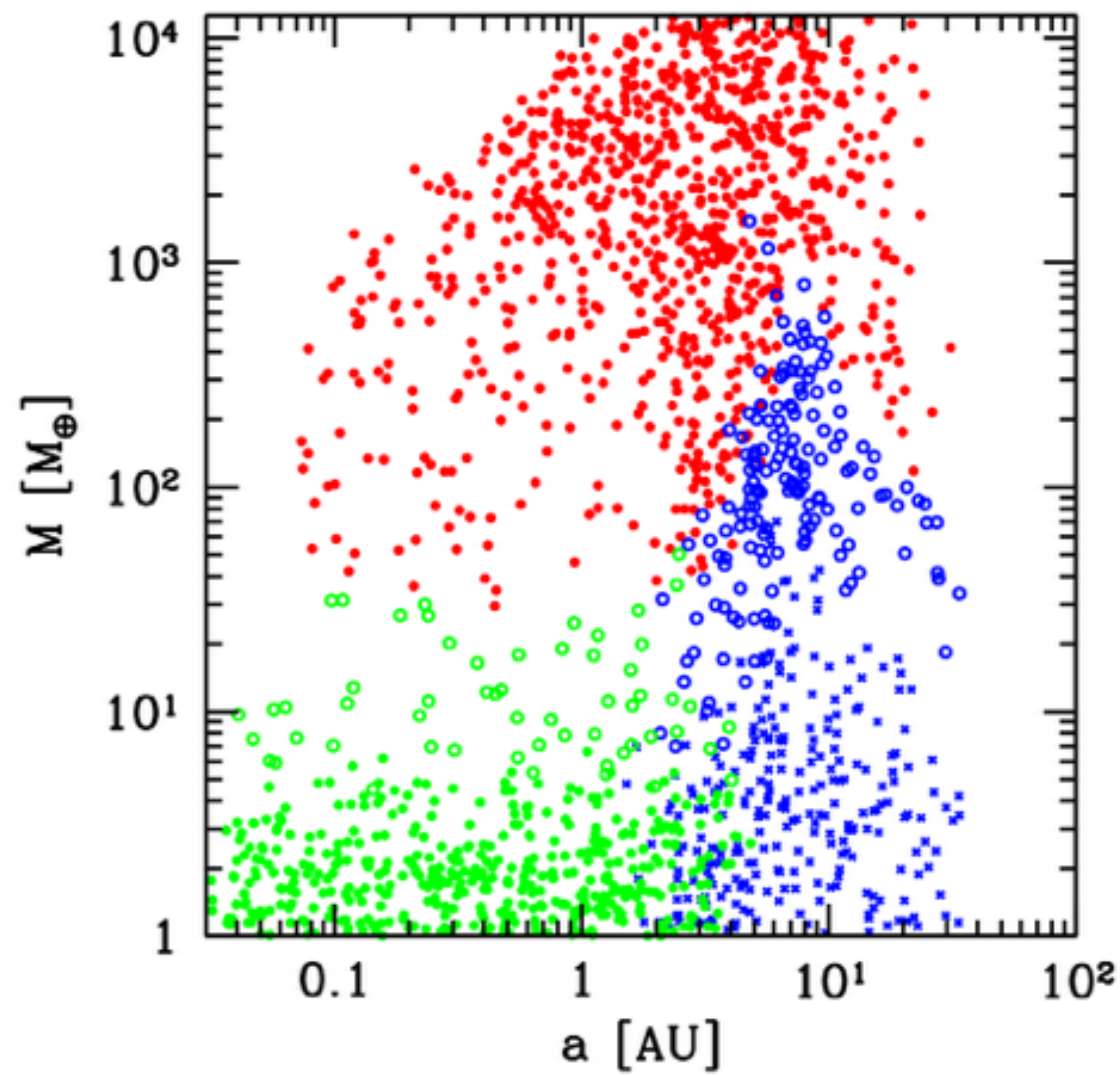


Toy population synthesis model

Freely available toy population synthesis model
http://nexsci.caltech.edu/workshop/2015/#handson
Open source, fast running time, well documented

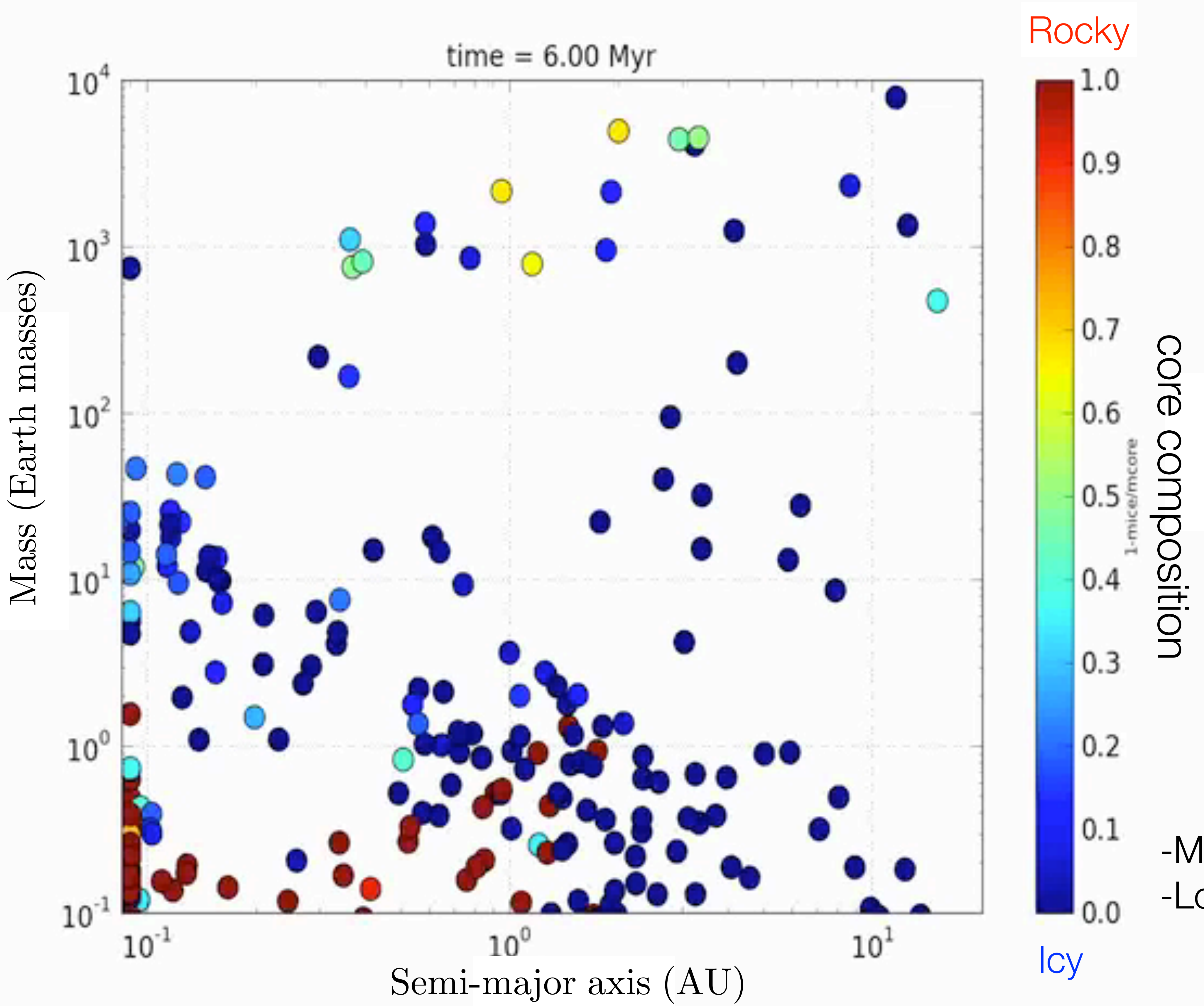


Ida & Lin 2004



This model

Formation tracks: Bern model



- 10 Embryos/disk
- $M_{ini} = 0.01\ M_\oplus$
- Full N-body
- $M_{star} = 1\ M_\odot$
- Migration (Type I and II)
- $\alpha = 2 \times 10^{-3}$

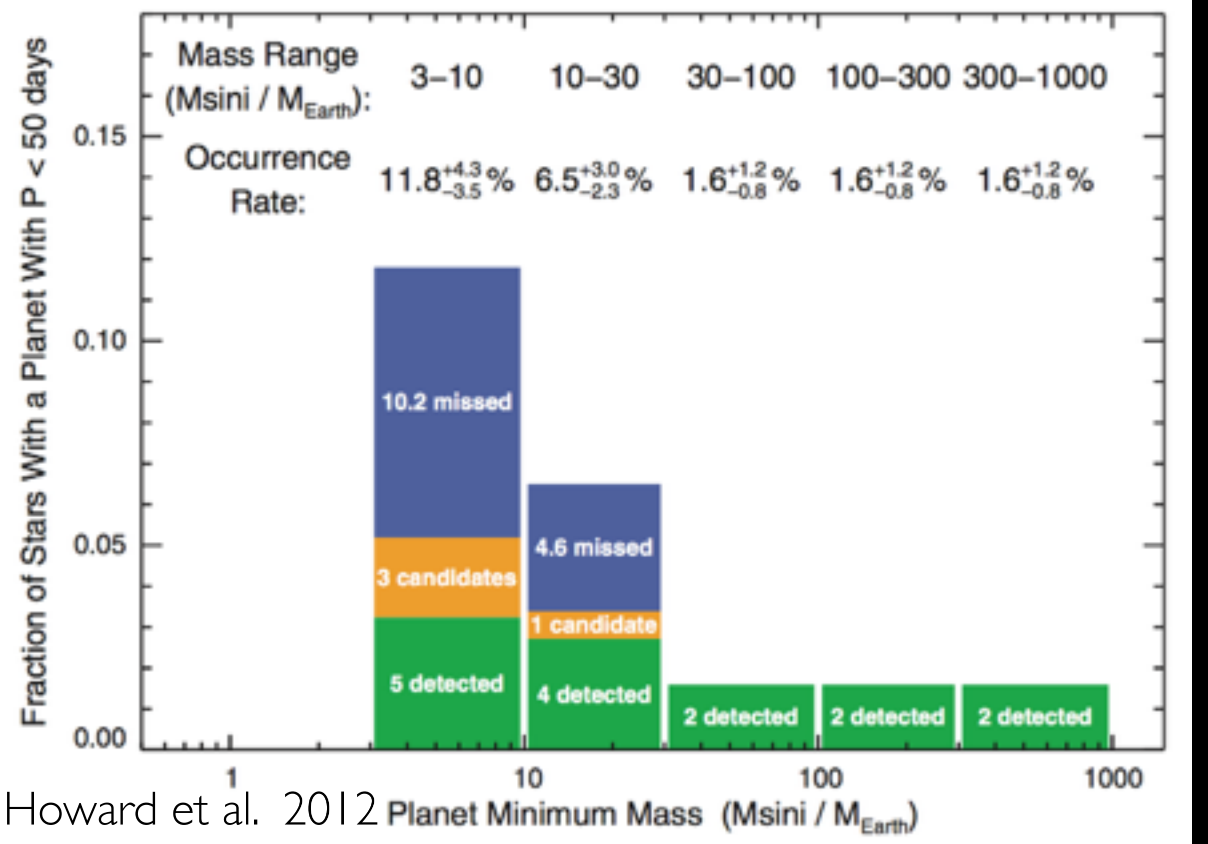
-Mostly compact systems
-Low eccentricities

Alibert et al. 2013

DACE

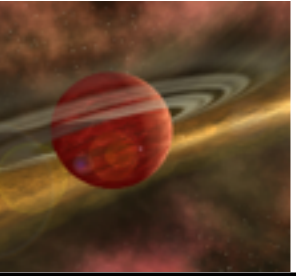
Online demonstration

https://dace.unige.ch/



Howard et al. 2012

3.
Comparisons with RV



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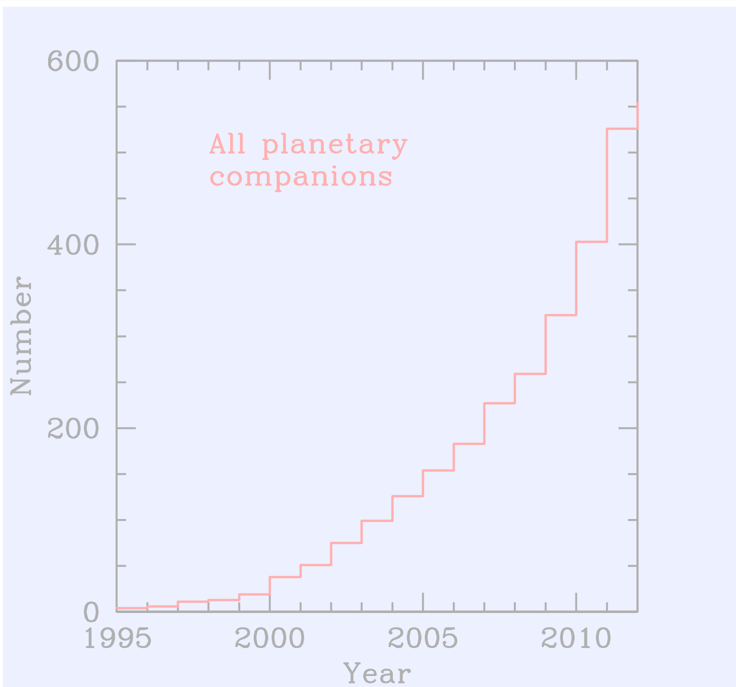

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- Fraction of hot/cold Jupiters
- Distribution of radii

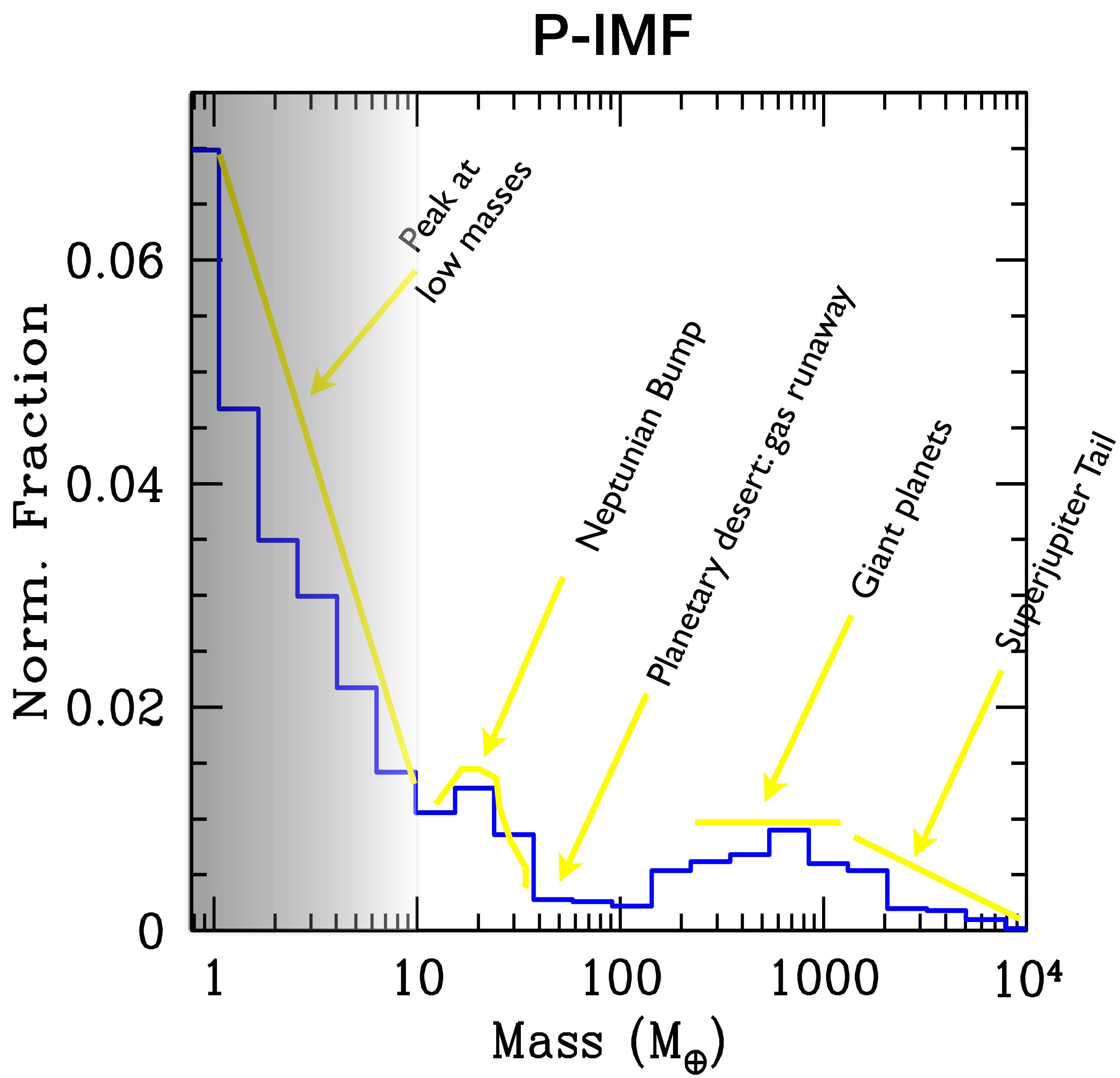
No match: improve, change parameters

Model solution found!

Predictions

(going back to the full synthetic population)

Planetary initial mass function



10 embryos/disk (full N-body), start mass: 0.01 M_{Earth}
$M_{star}=1M_\odot$, full non-isothermal type I, alpha= 2 x10^{-3}

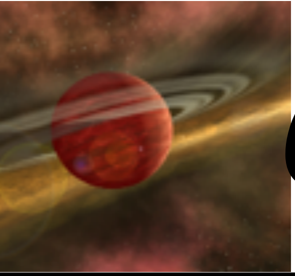
Type	Mass (M	% (of M>1 M
(Super)-Earth	< 7	61
Neptunian	7-30	17
Intermediate	30-100	3
Jovian	100-1000	13
Super-Jupiter	> 1000	5

Planets with M < 30 M_{Earth} : over 75% of all planets

Giant planets = tip of the iceberg

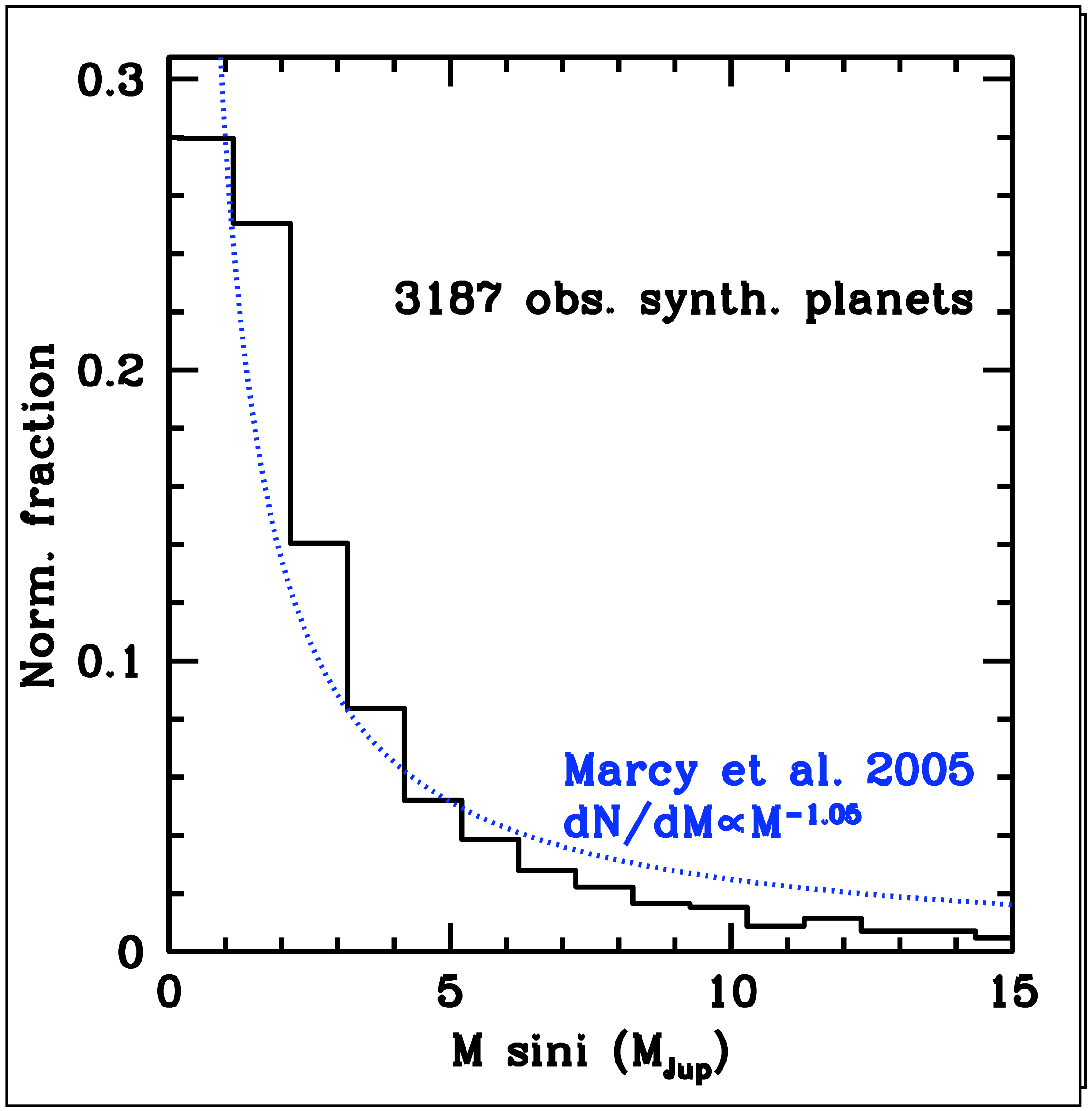
- *Complex structure, dominated by low mass planets*
- *Consistent w. non-detection of Jupiters around ~90% stars.*

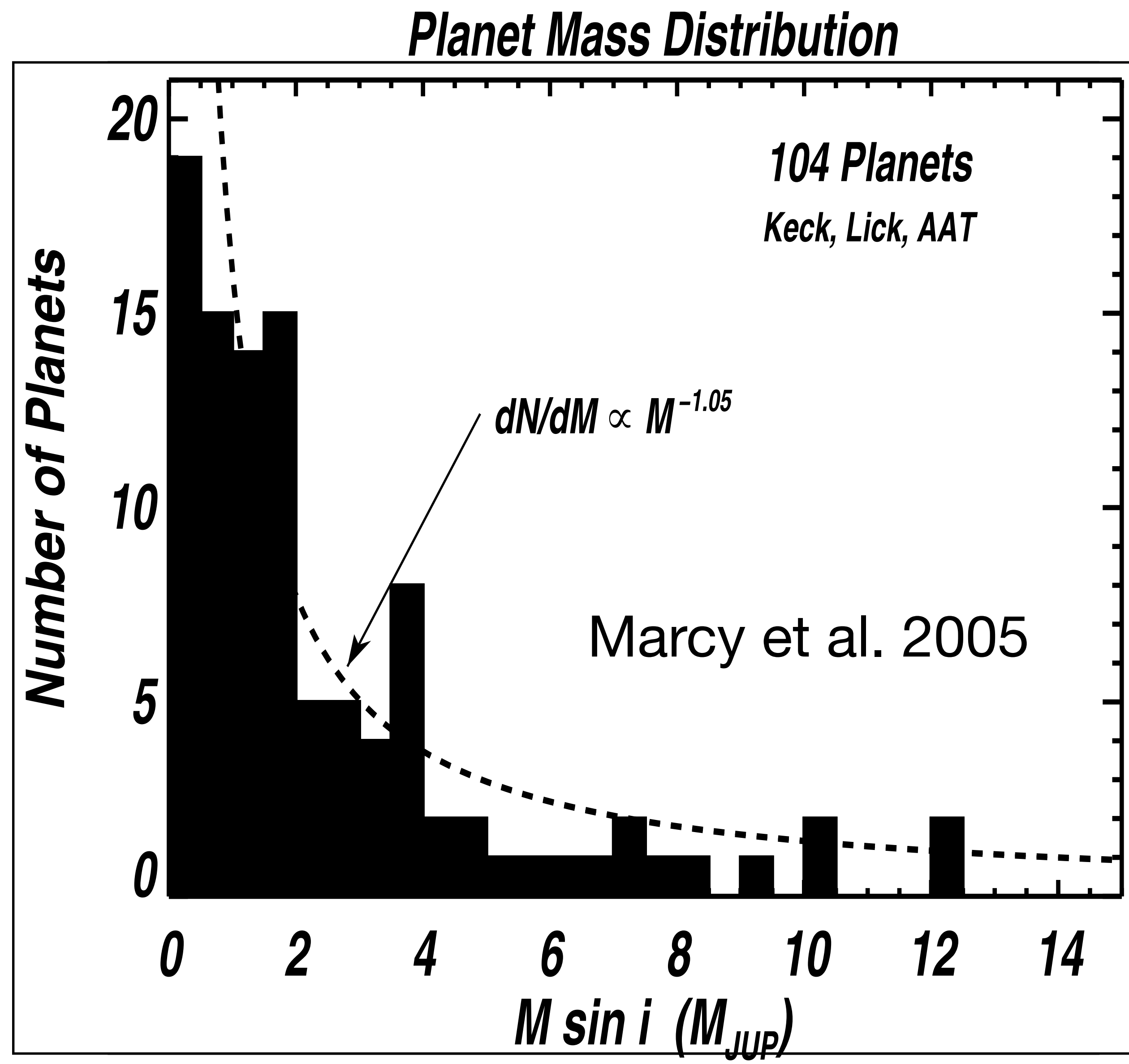
Benz et al. 2014



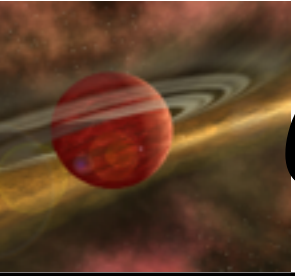
Comparison with observations: high M

Blue lines: Observational comparison sample at 10 m/s
Black lines: Detectable synthetic sub-population at 10 m/s



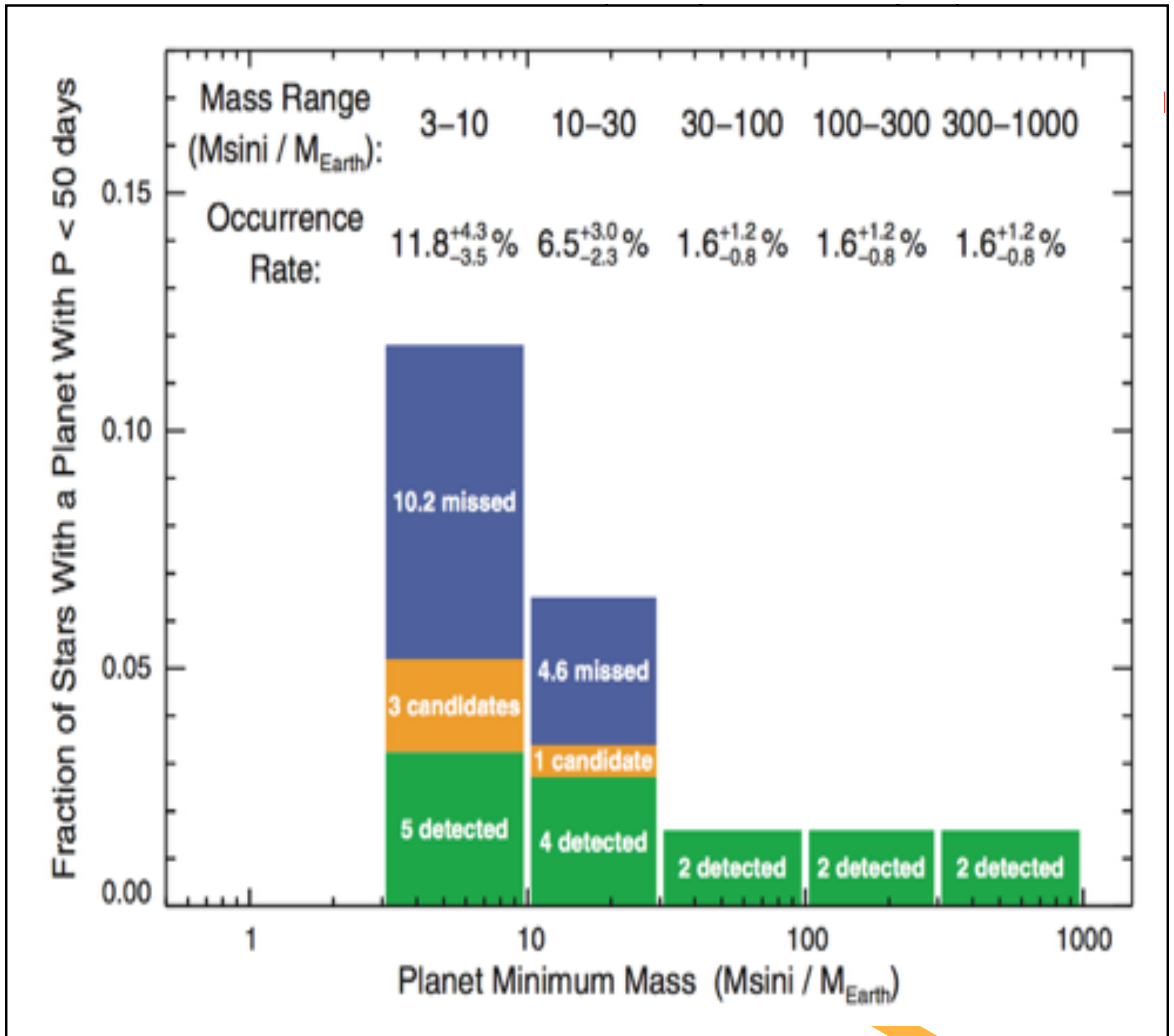


Conclusion: core accretion ~reproduces giant planet mass function

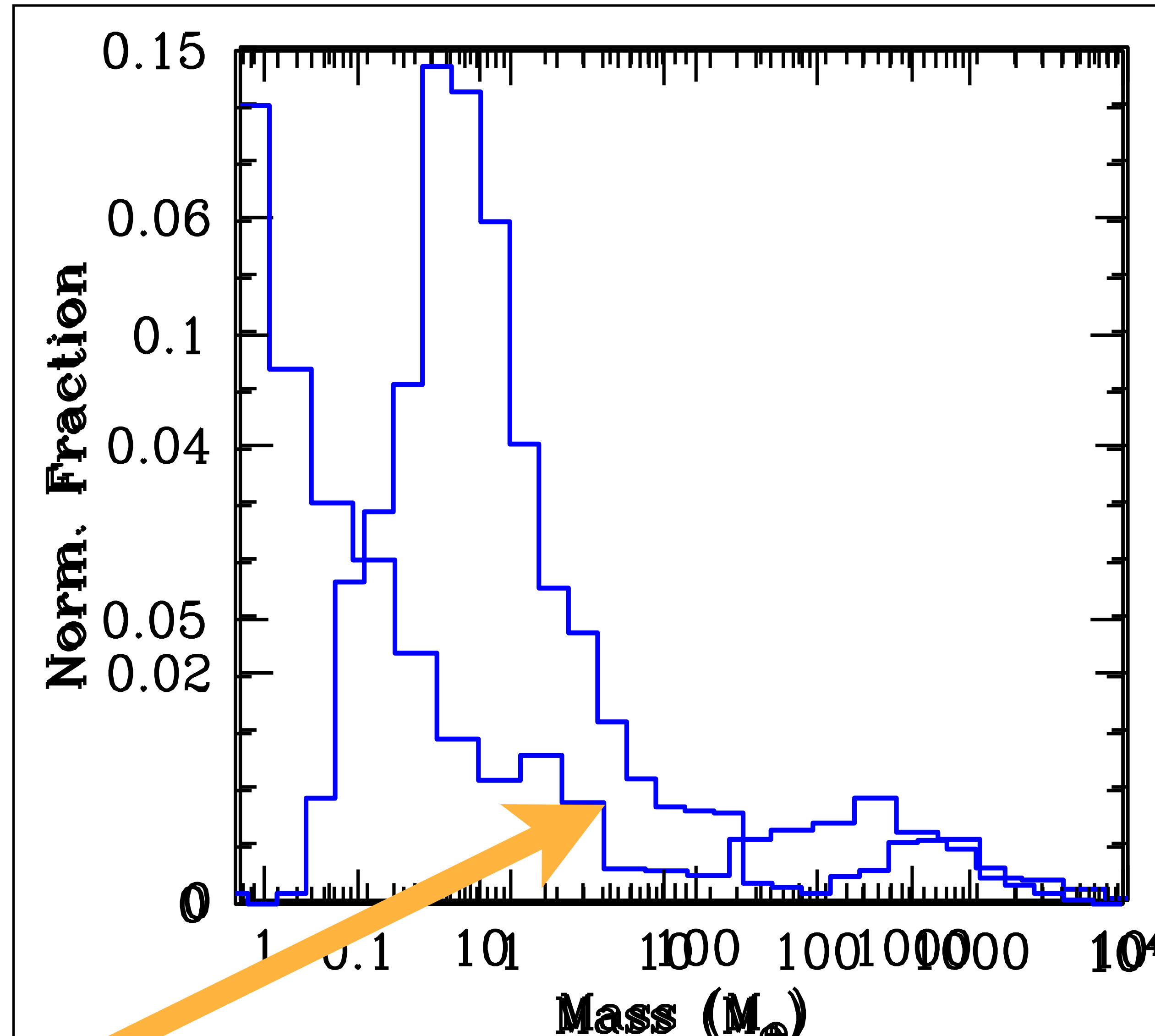


Comparison with observations: low M

Observations



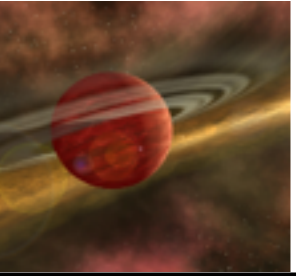
Synthetic



Mayor et al. 2011
Howard et al. 2010

Benz et al. 2014

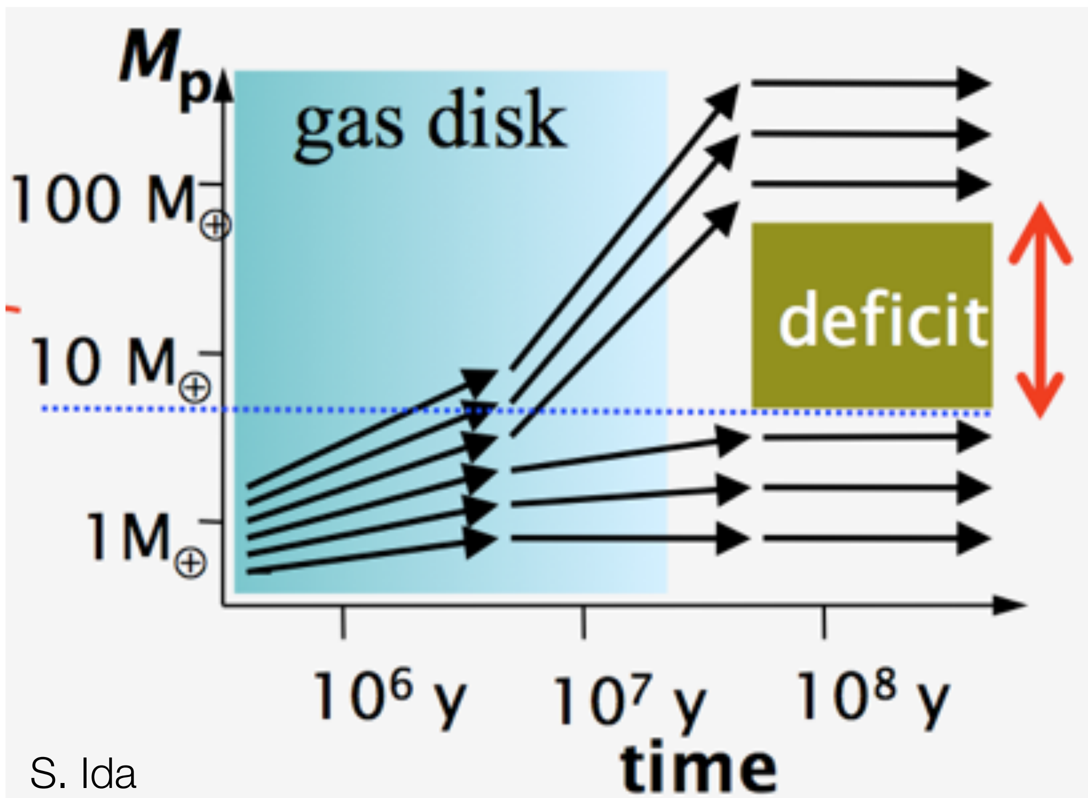
Conclusions:
-core accretion reproduces break in mass function
-Start of rapid gas accretion ~30 M_{earth}
-many low-mass planets

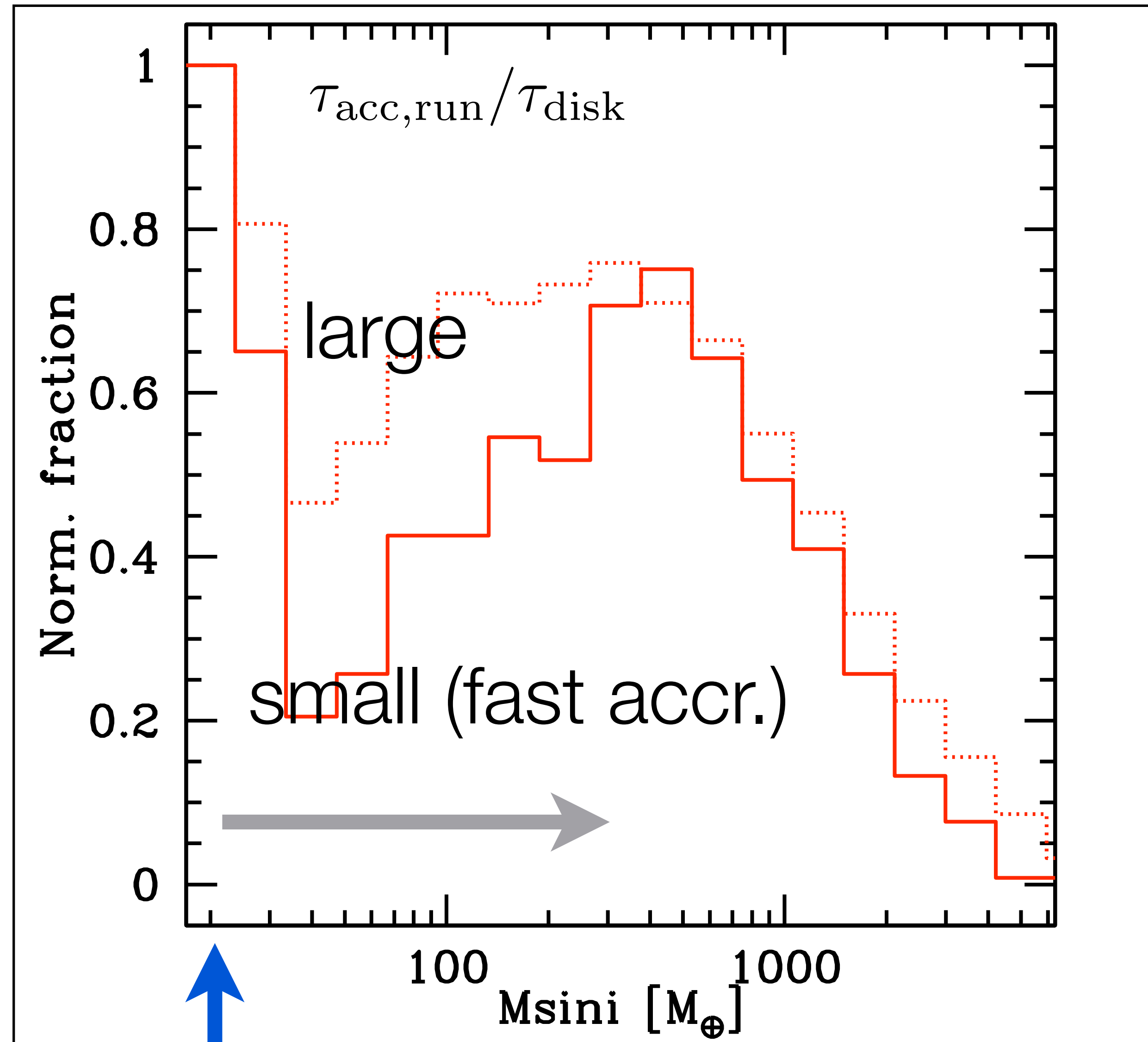


Constraints in the P-IMF: transition

M_{crit}: depends on luminosity, opacity and gas composition ~5-15 M_E

Once M_{crit} is reached, rapid gas accretion begins.



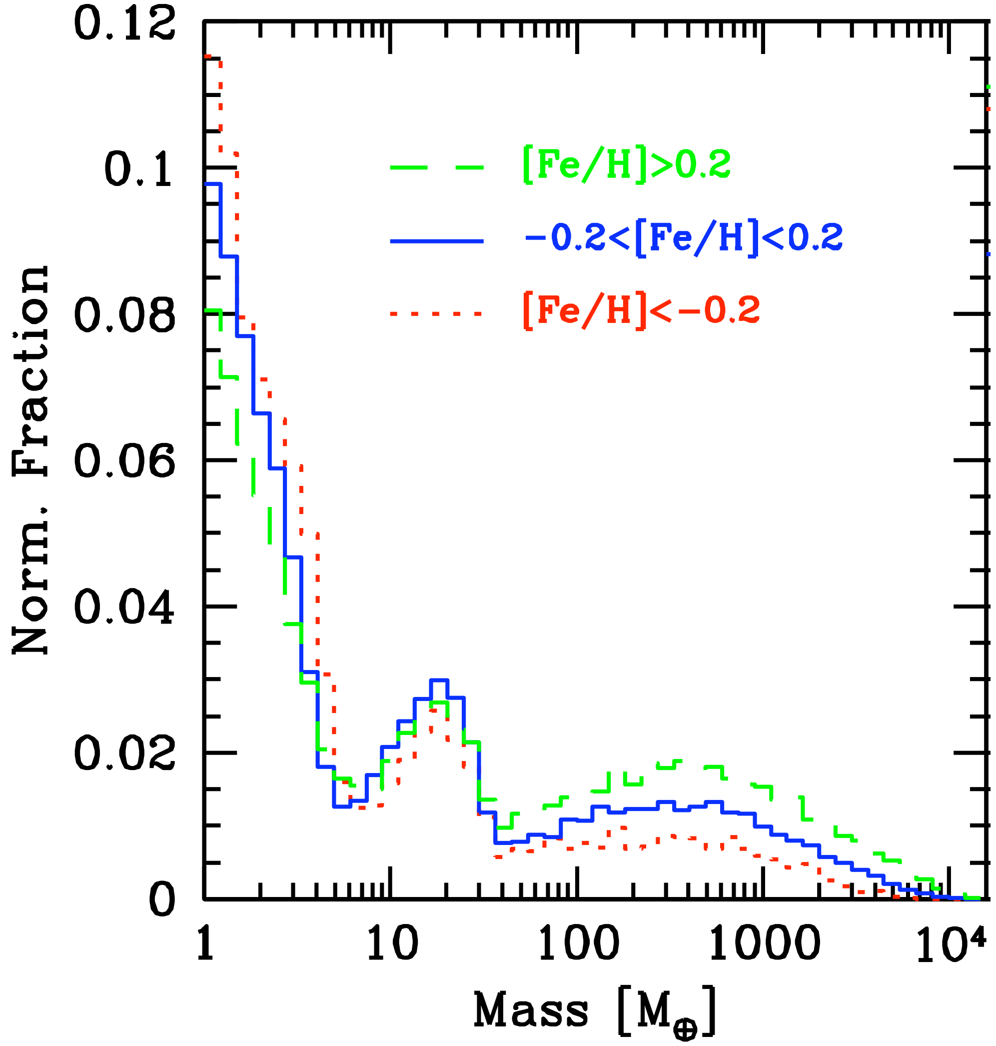


If $\tau_{acc,run}/\tau_{disk}$ << 1, a "planetary desert" can form.

Conclusion: gas accretion rate in disk-limited phase is rather low

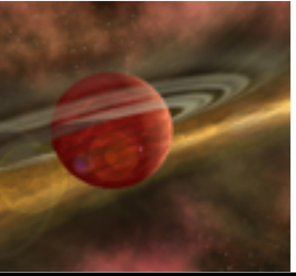
P-IMF: impact of disk properties

Metallicity



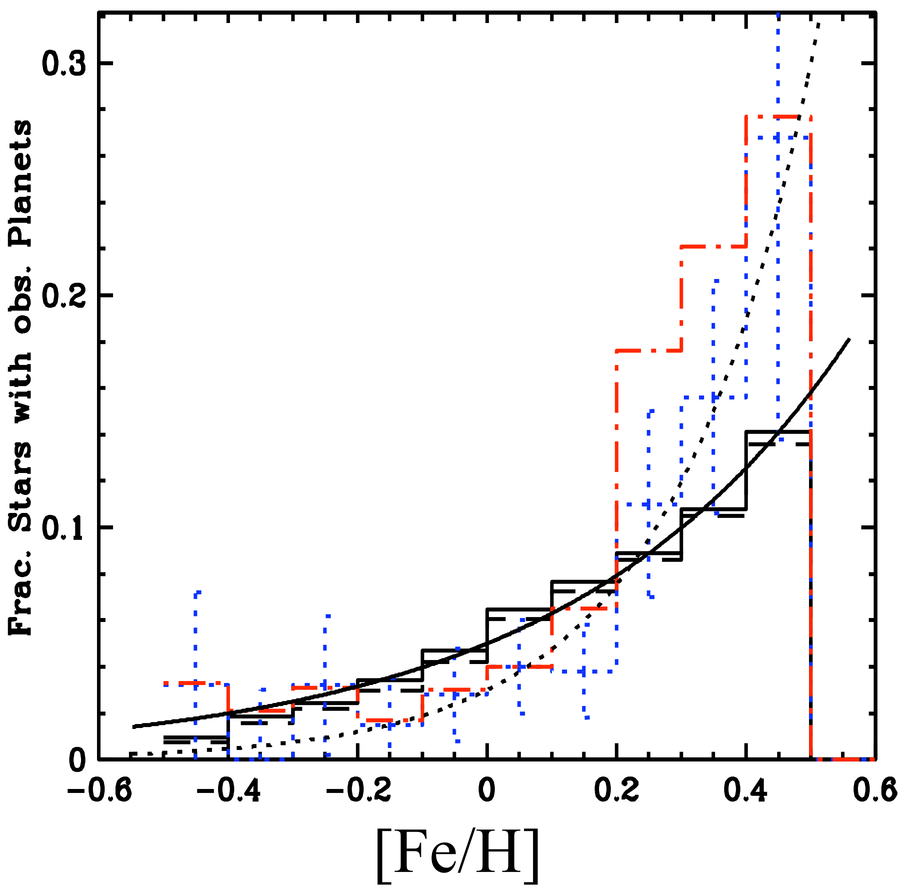
- higher number of giants
- but not more massive
- Threshold mass (M_{crit})

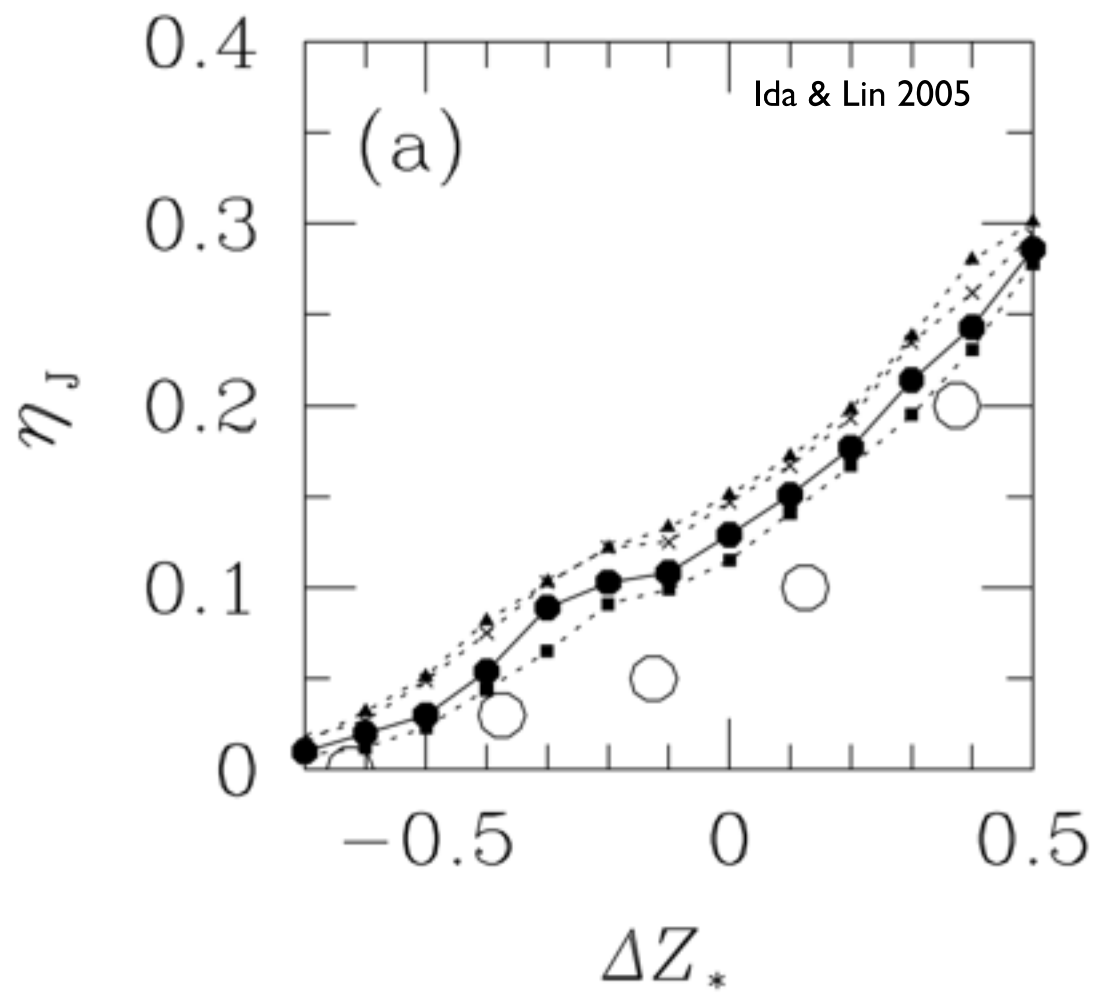
Mordasini et al. 2012



Giant planet frequency

Metallicity

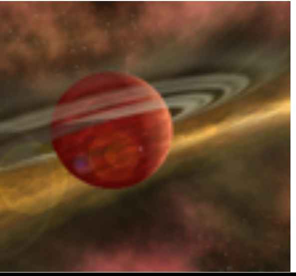




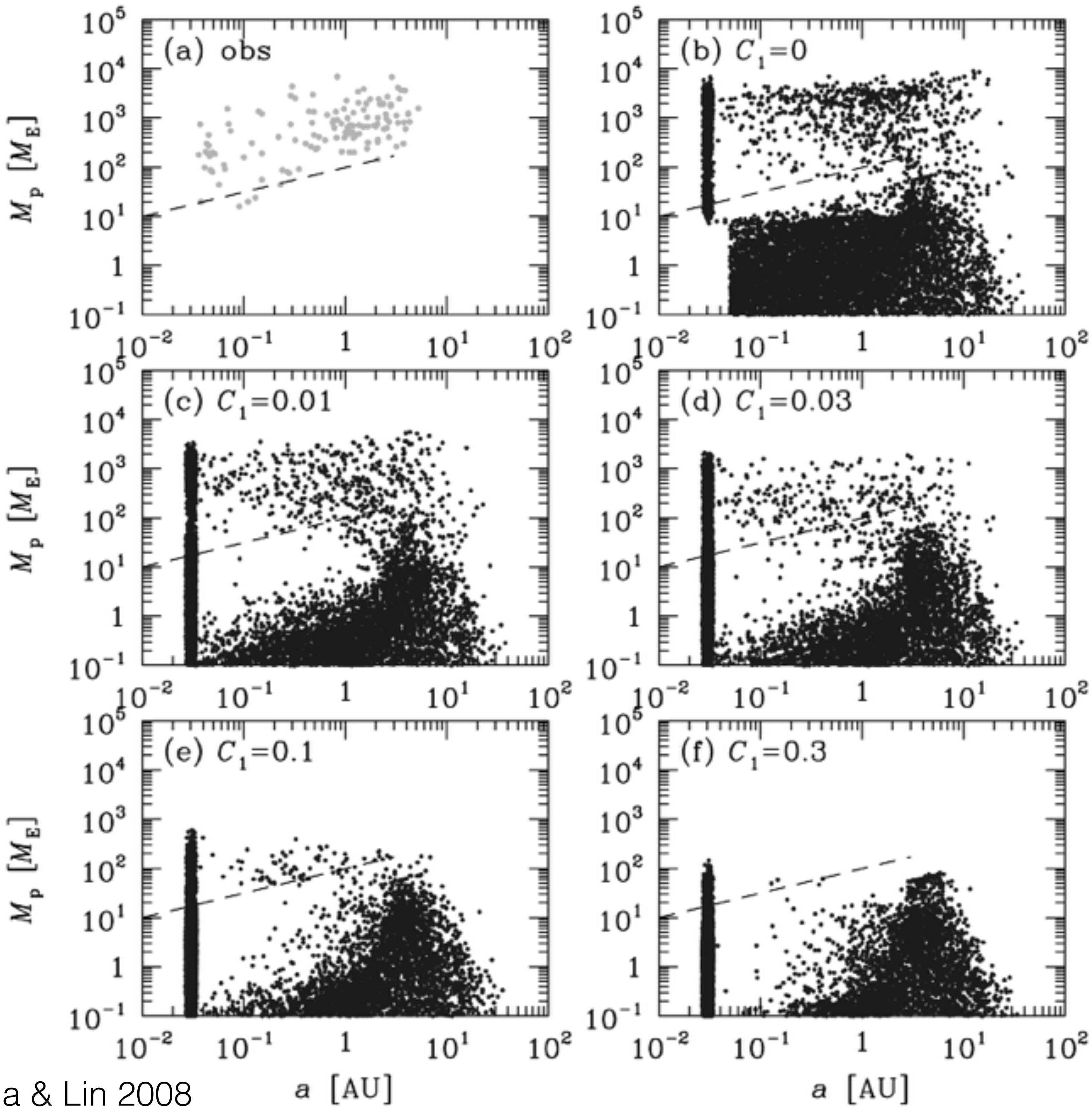
- Trend as observation, but weaker dependency
- Argument in favor of core accretion

Blue: Observation (Fischer & Valenti 2005)
Red: Observation (Udry & Santos 2007)
Black: Observable synthetic planets

Conclusions:core accretion ~reproduces the metallicity effect



Type I migration rate



Ida & Lin 2008

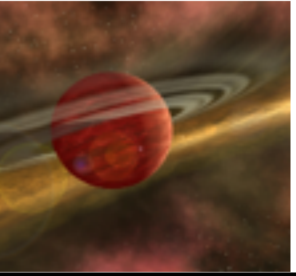
Full isothermal type I migration: cannot form Jupiters any more

Triggered many dedicated studies on type I.

New non-isothermal models now included in global models.

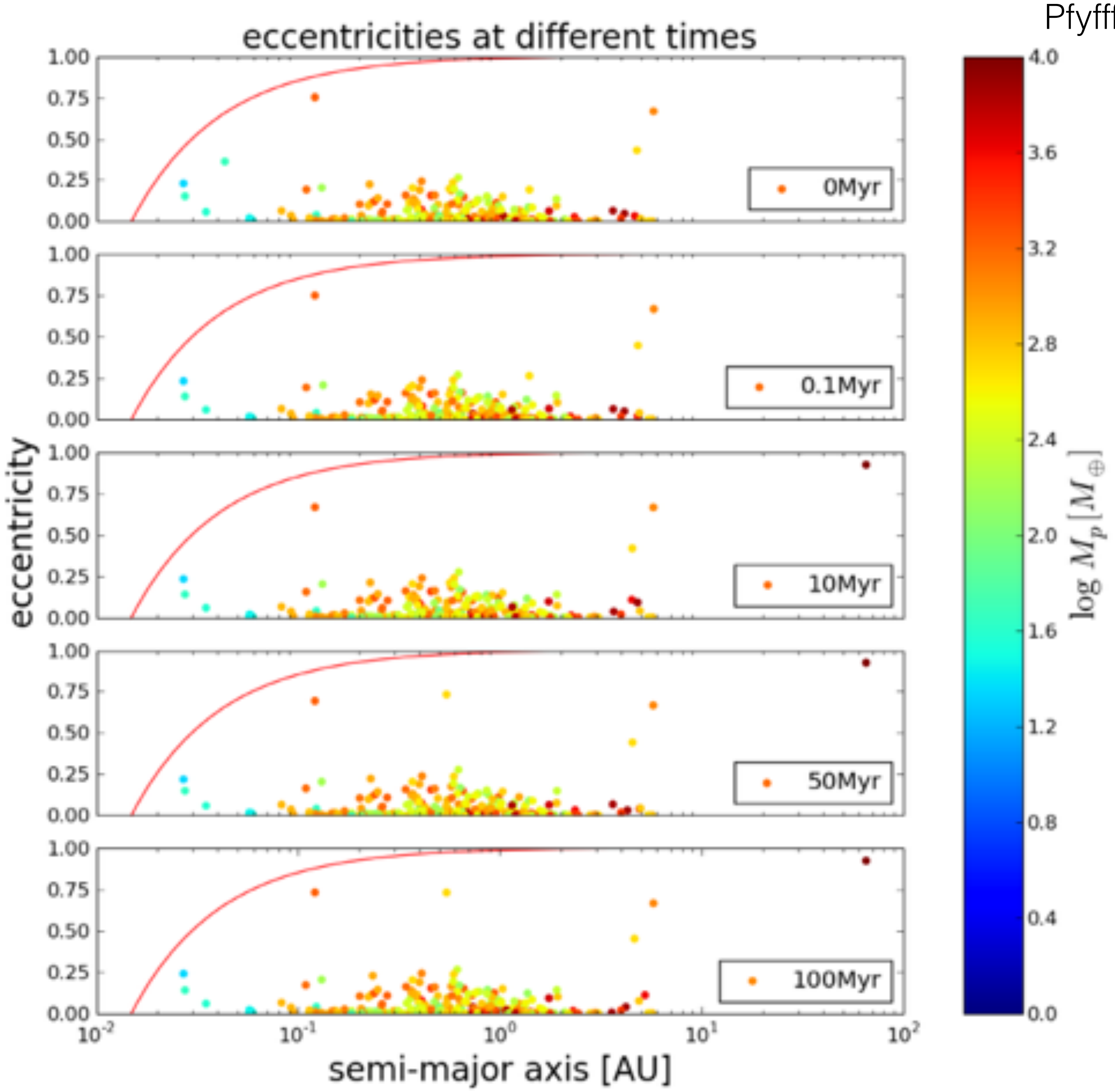
Interaction of global models and specialized studies.

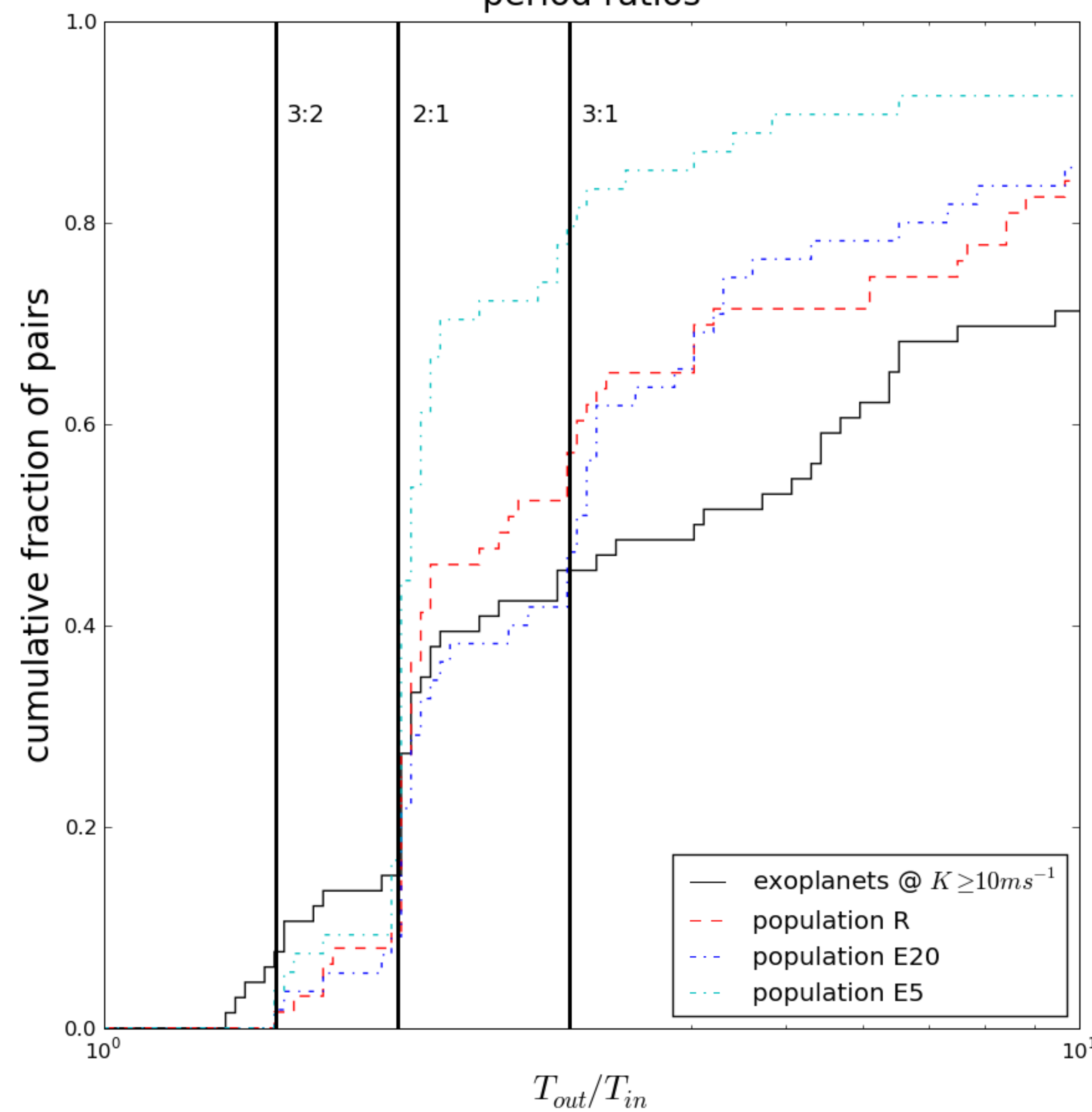
Conclusion: isothermal approximation insufficient



N-body imprints

Pfyfffer et al. 2015



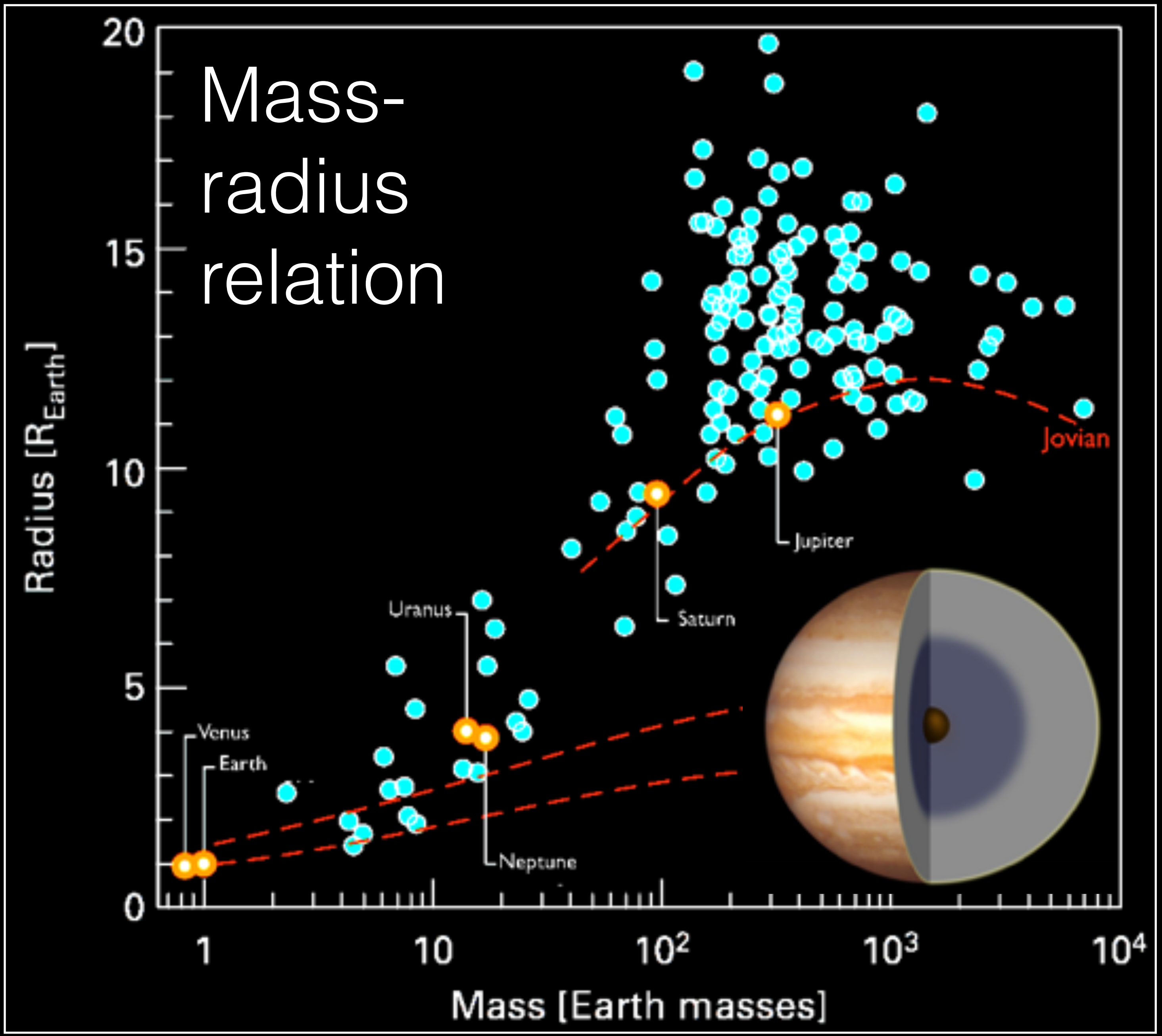


compact, low ecc. systems

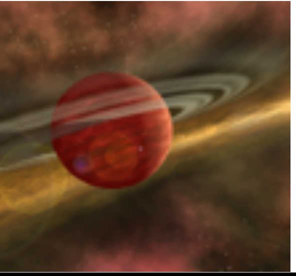
too many in MMR

Conclusions: -model cannot reproduce eccentricities
-too many MMR

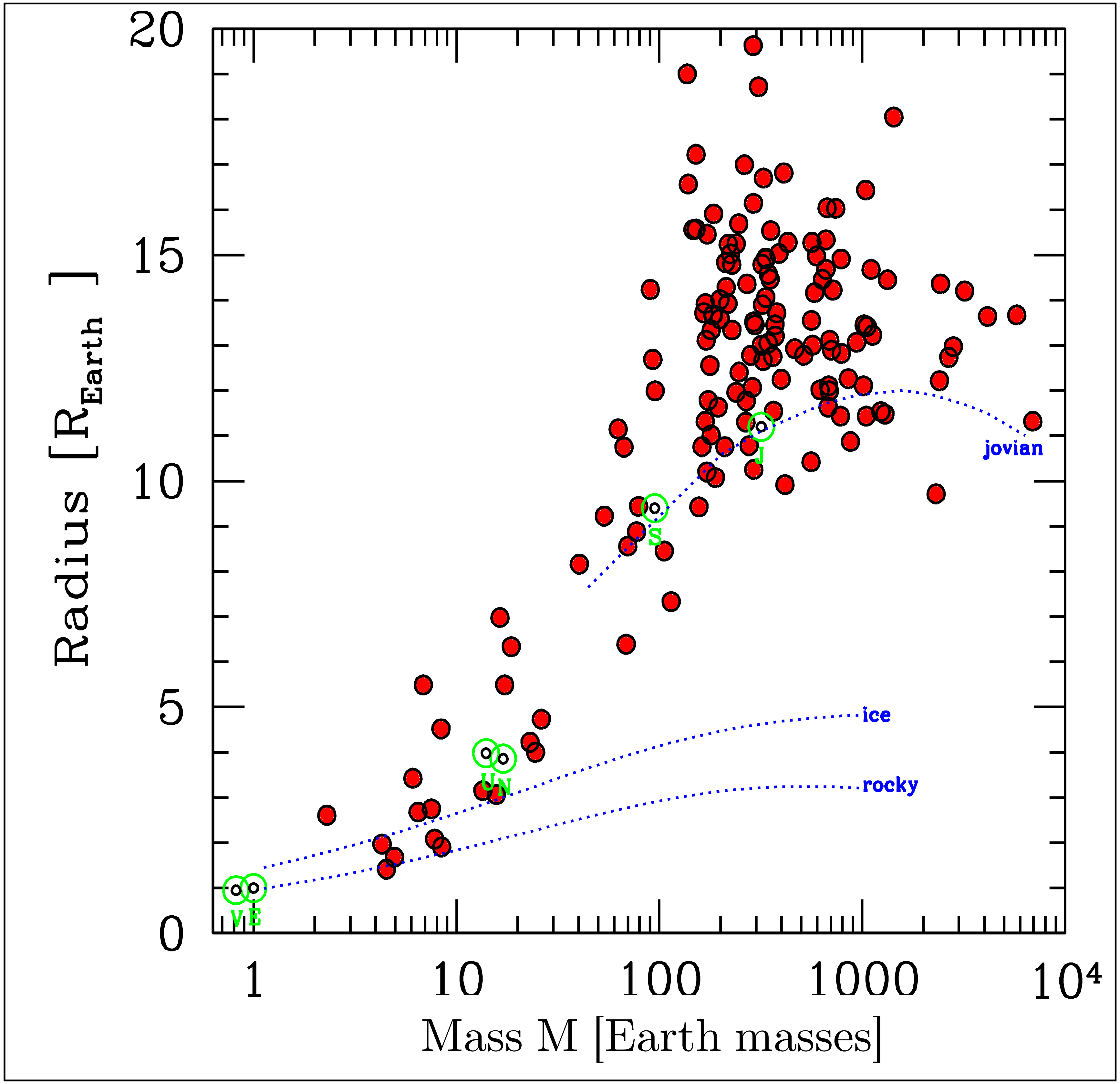
Rasio & Ford 1996, Juric & Tremaine 2008, Chatterjee et al. 2008, Malmberg & Davies (2009)



4.
Comparions with transits



Mass-radius relation



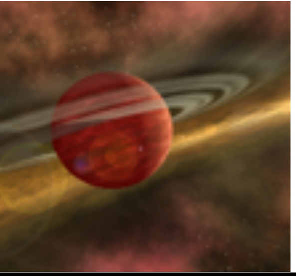
- M-R: First geophys. characterisation: rocky, icy, gaseous

- General trends
- Large diversity
- Inflated giant planets
- Empty regions

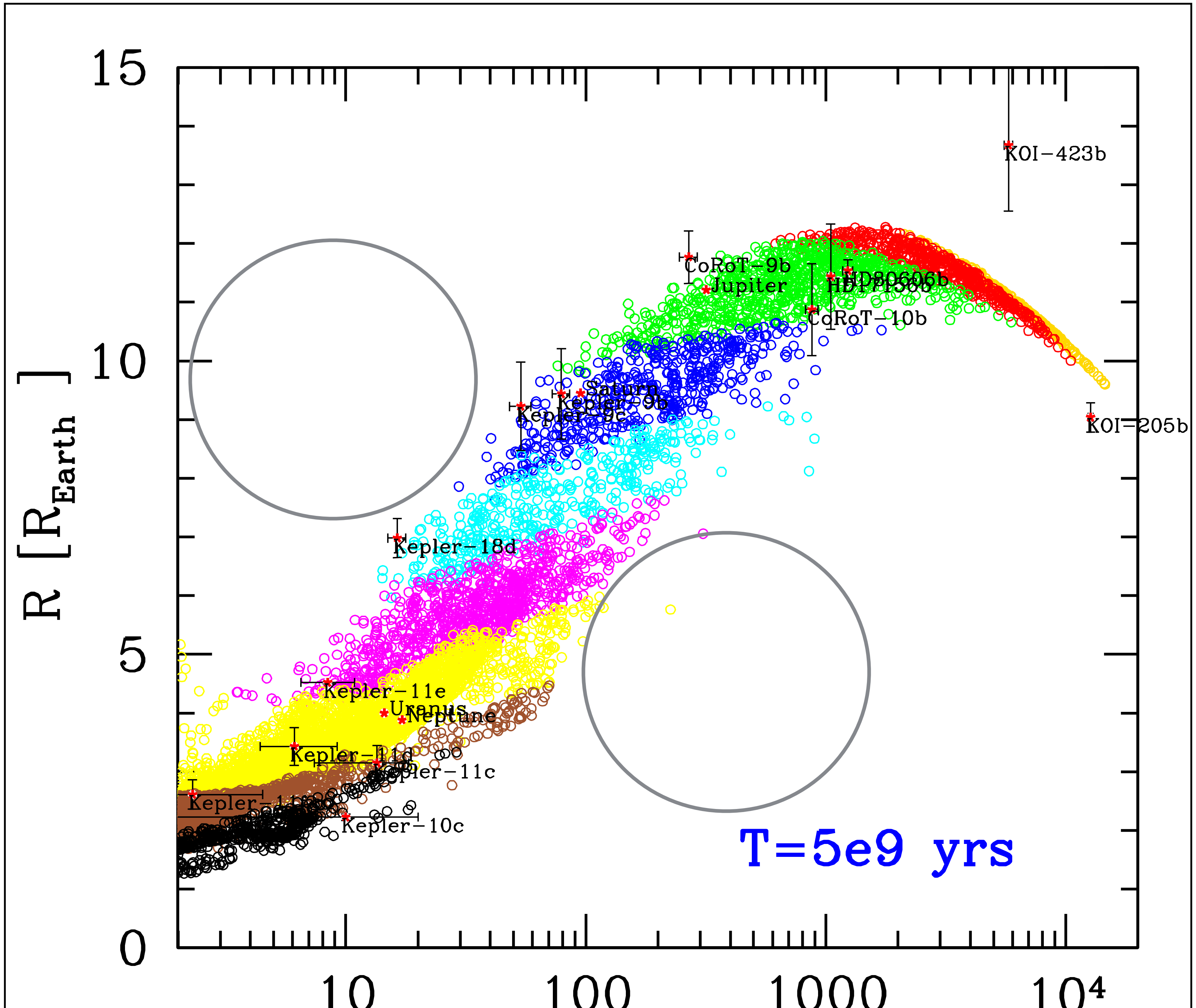
- Understandable with theoretical models?
- Constraints for formation theory beyond the a-M:

Transition solid-gas dominated planets: efficiency of H/He accretion & loss: opacity in protoplanetary atmosphere, atmospheric escape

- Must combine formation and evolution



Formation of the M-R relationship



Fraction Z of solids
(rest H/He)

Orange: $Z \leq 1\%$
Red: $1 < Z \leq 5\%$
Green: $5 < Z \leq 20\%$

Blue: $20 < Z \leq 40\%$
Cyan: $40 < Z \leq 60\%$
Magenta: $60 < Z \leq 80\%$
Yellow: $80 < Z \leq 95\%$
Brown: $95 < Z \leq 99\%$
Black: $Z > 99\%$

Rapid collapse at ~0.2 MJ when $Z \approx 0.5$ (runaway gas accretion)

After disk dispersal (T>10 Myrs), slow contraction.

Conclusion: core accretion recovers basic shape of M-R

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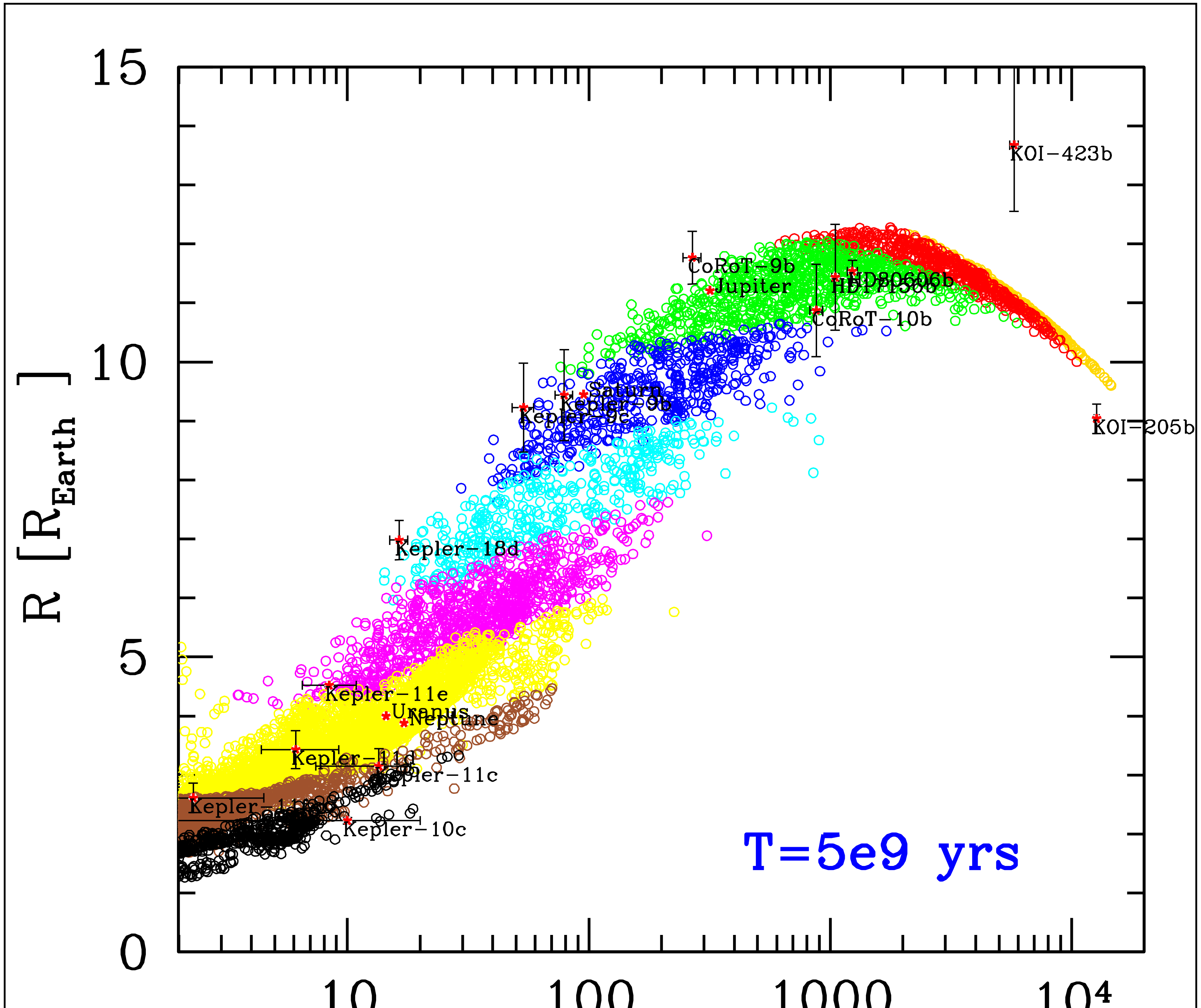

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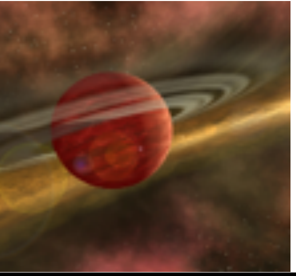

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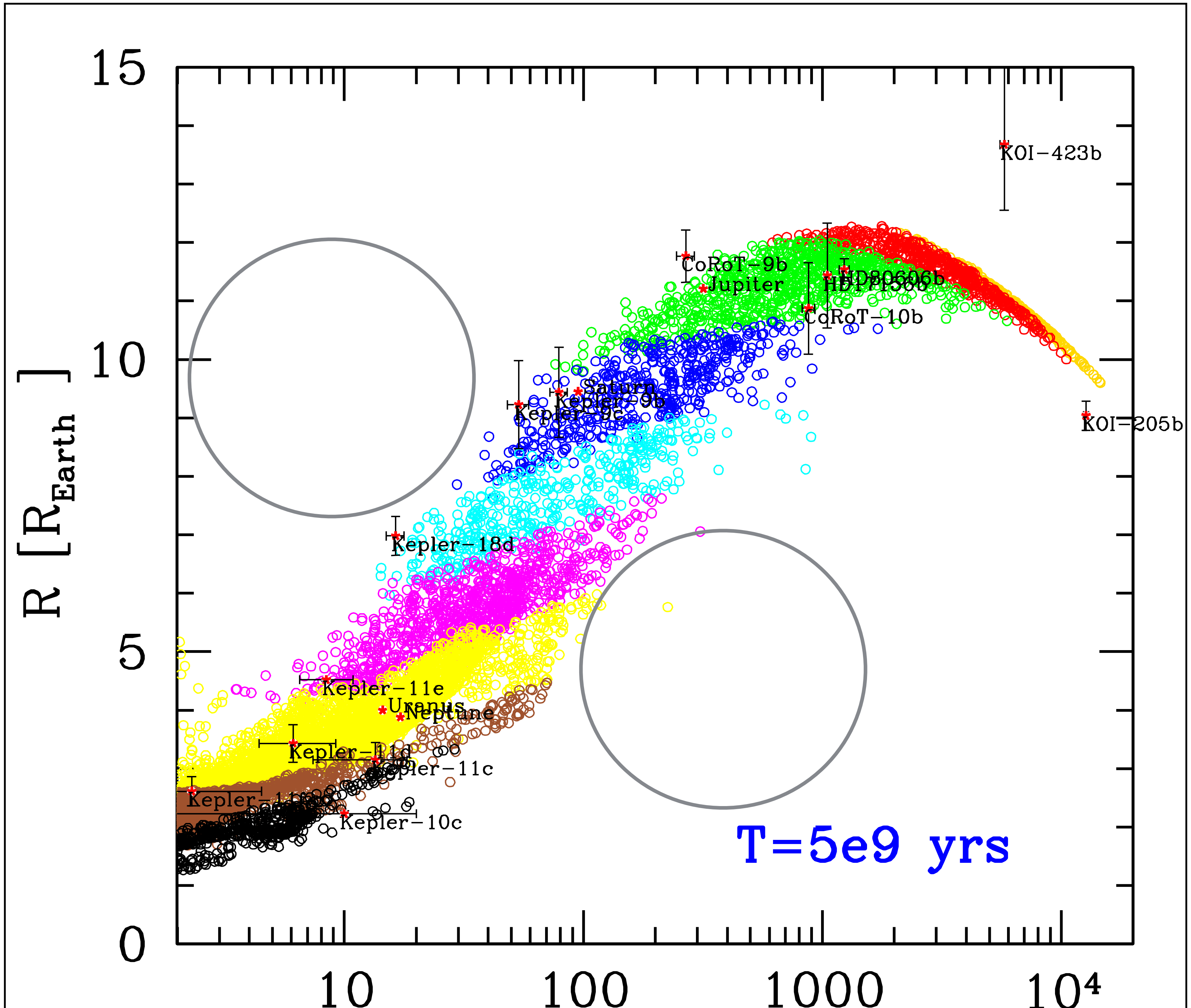

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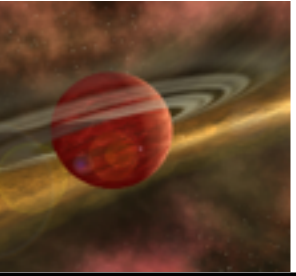
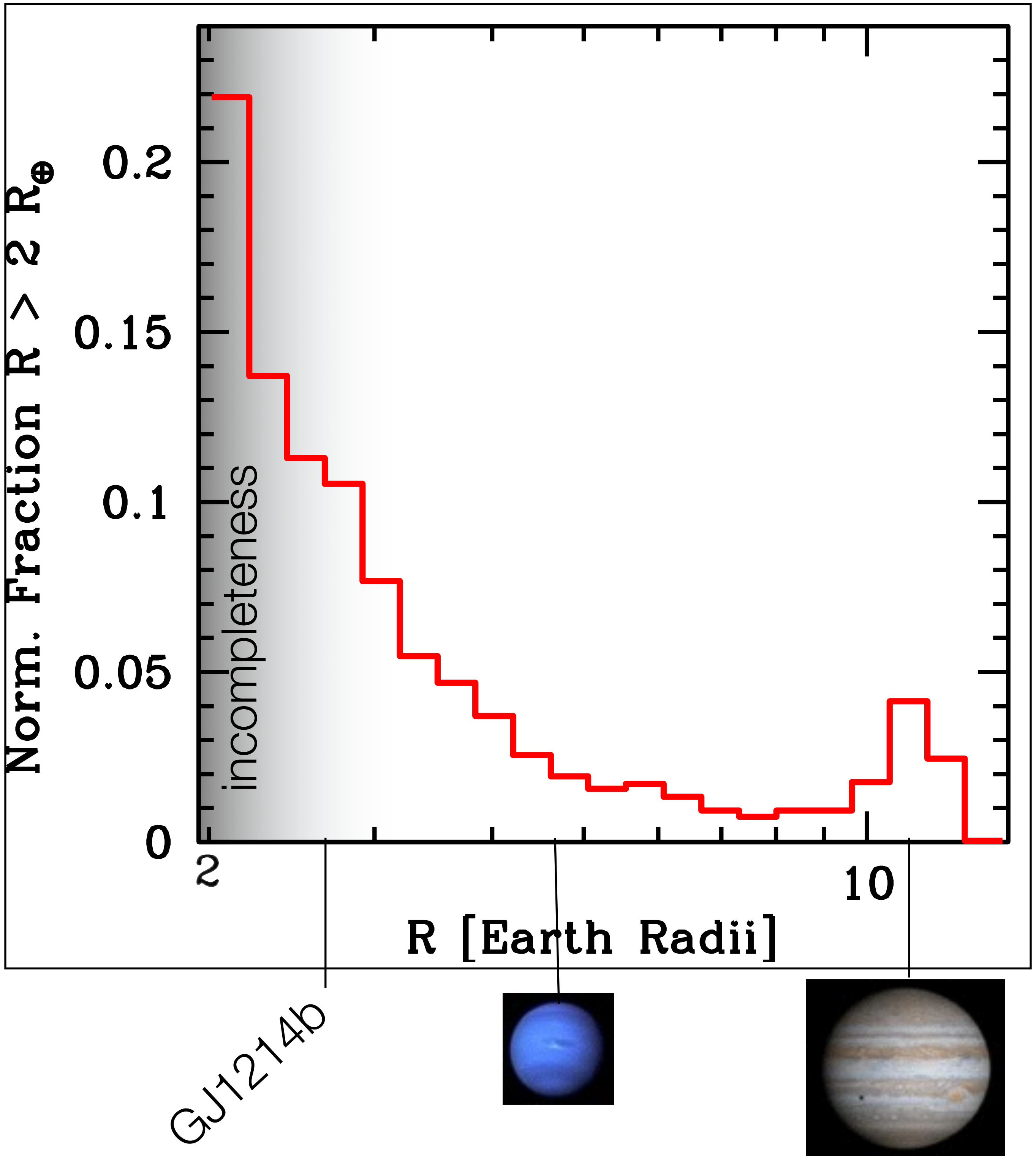


Planetary radius distribution



- Peak at lowest radii. High detection rate of Kepler.

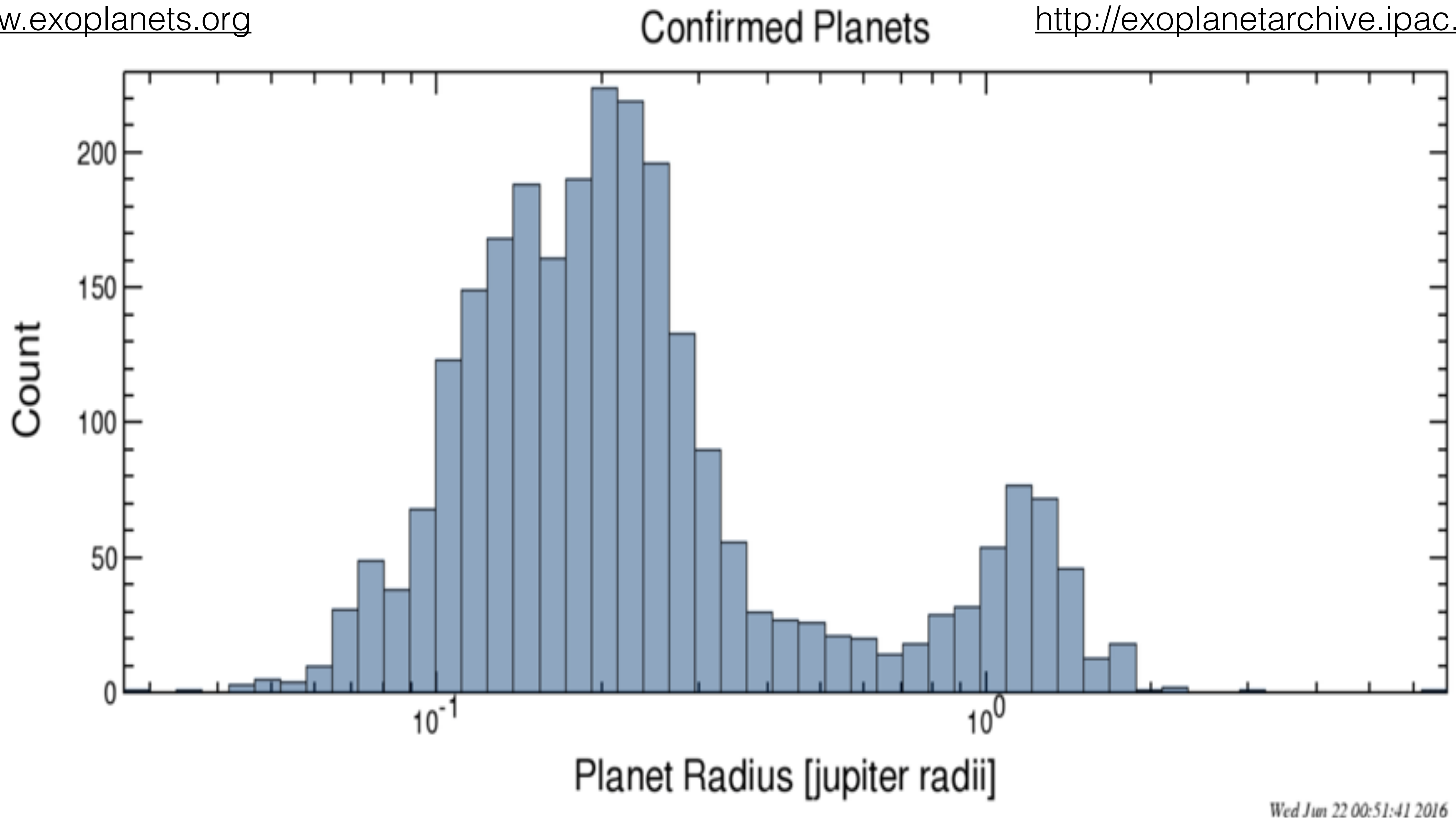
- Second peak at ~ 1 R_J ⇨ Giant planets have all approx. *the same radius independent of mass* (degenerate interiors)

M_{star}=1 M_{sun}. a>0.1AU. Non-isothermal Type I. Cold accretion. 1 embryo/disk, no special inflation mechanisms, no evap.

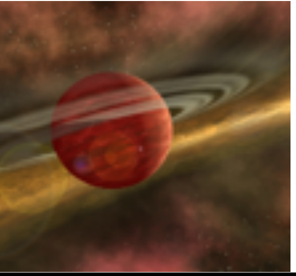
Observed radius distribution

www.exoplanets.org

http://exoplanetarchive.ipac.caltech.edu



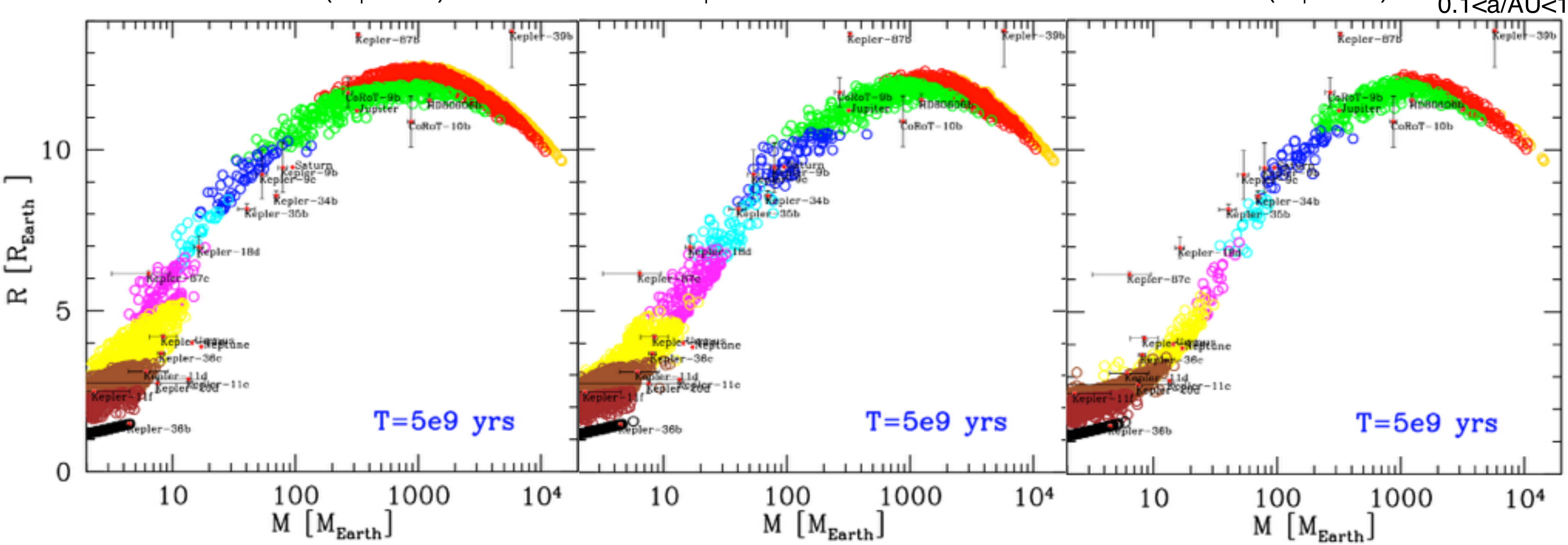
Conclusions: degeneracy (EOS) is understood & radius distribution is similar



Mass-radius relationship

Compare synthetic and observed M-R for three grain opacity reduction factors

Grain free (f_{opa}=0) | f_{opa}=0.003 | ISM (f_{opa}=1)

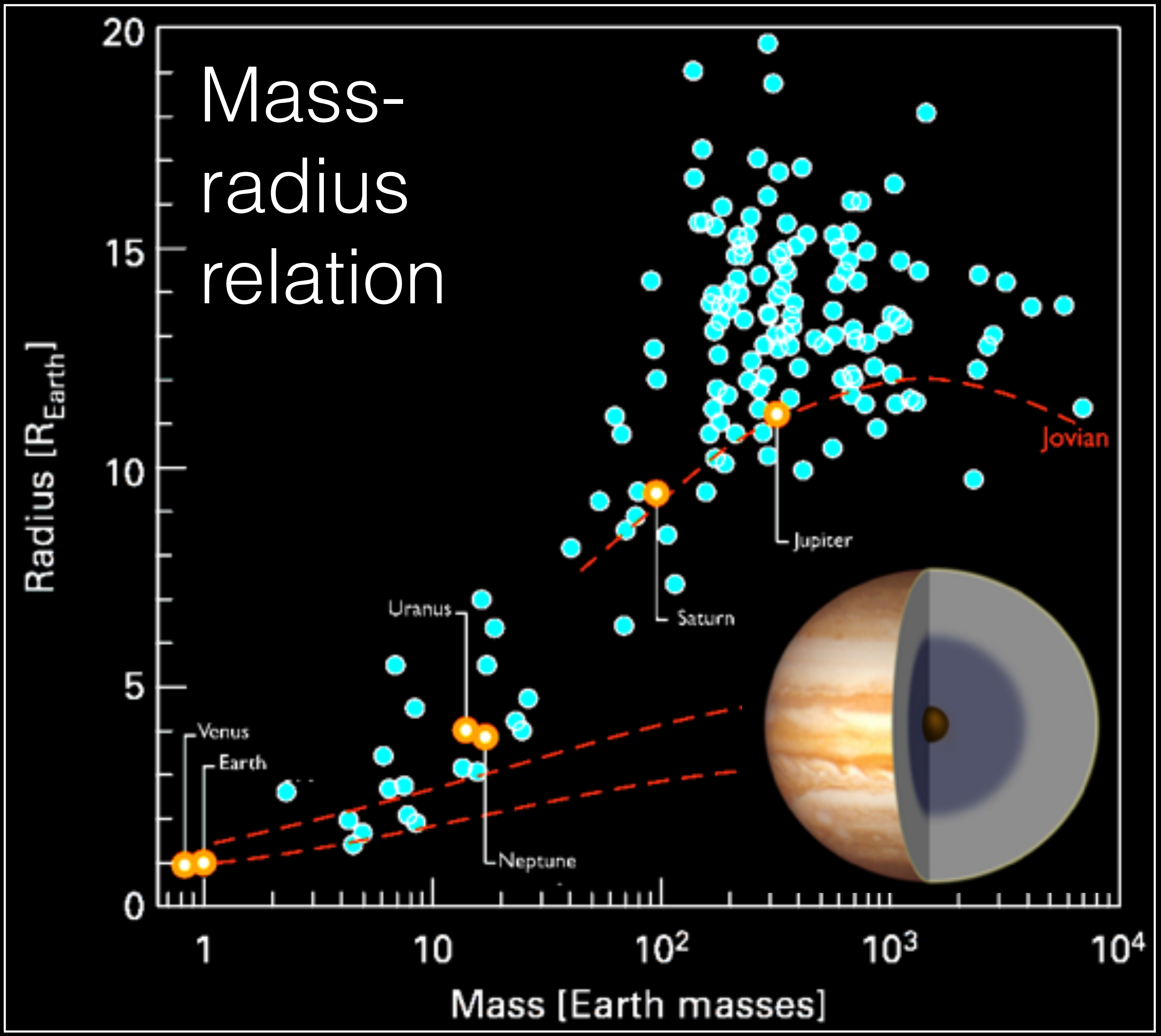


too large
too much H/He

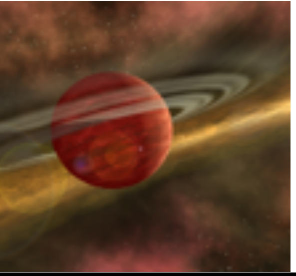
radii similar
as observed

too small
too little H/He

Conclusion: low opacities in protoplanetary atmospheres during formation



3. Perspectives



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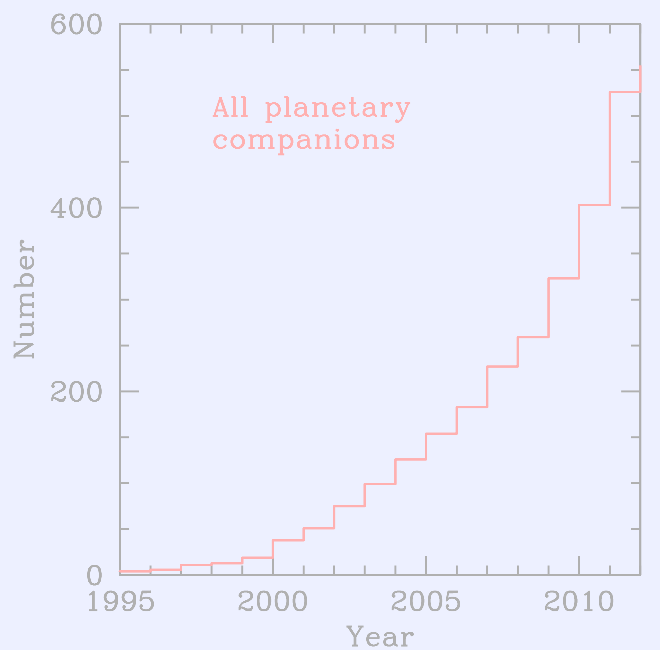
Apply observational detection bias



Observed population

Comparison:

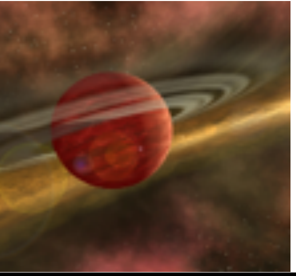
Observable sub-population
- Distribution of semi-major axis
- Distribution of masses
- Fraction of hot/cold Jupiters
- Distribution of radii

No match: improve, change parameters

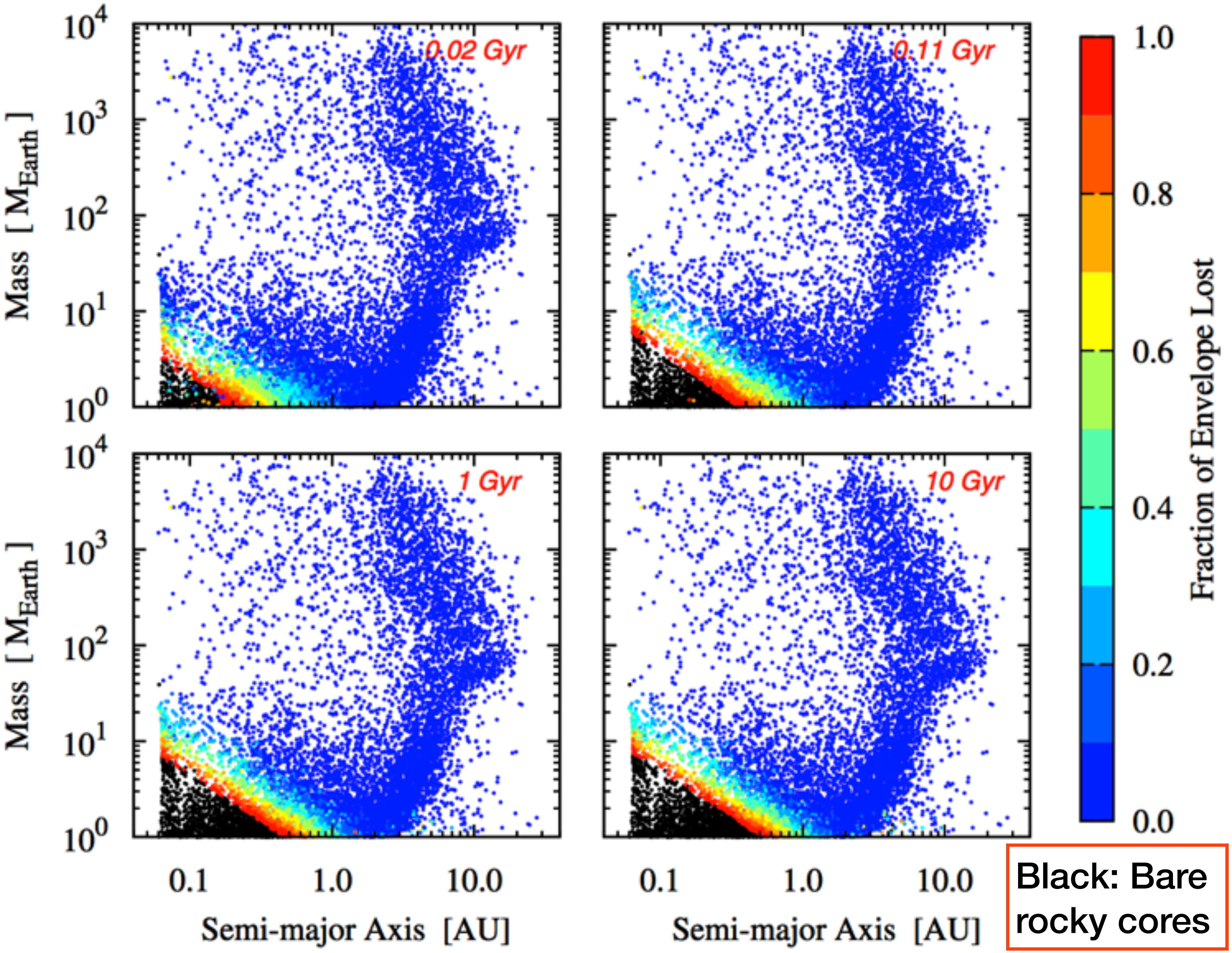
Match

Model solution found!

Predictions
(going back to the full synthetic population)



1) Adding a new dimension: time



Black: Bare rocky cores

Output of core accr. population synthesis

Thermodynamic evolution (cooling & contraction) in time w. atmos. escape

Close-in, low-mass loose the envelope.

Most of the action early on.

a-M does not change much. (a>0.06 AU)

Jin, Mordasini et al. 2013

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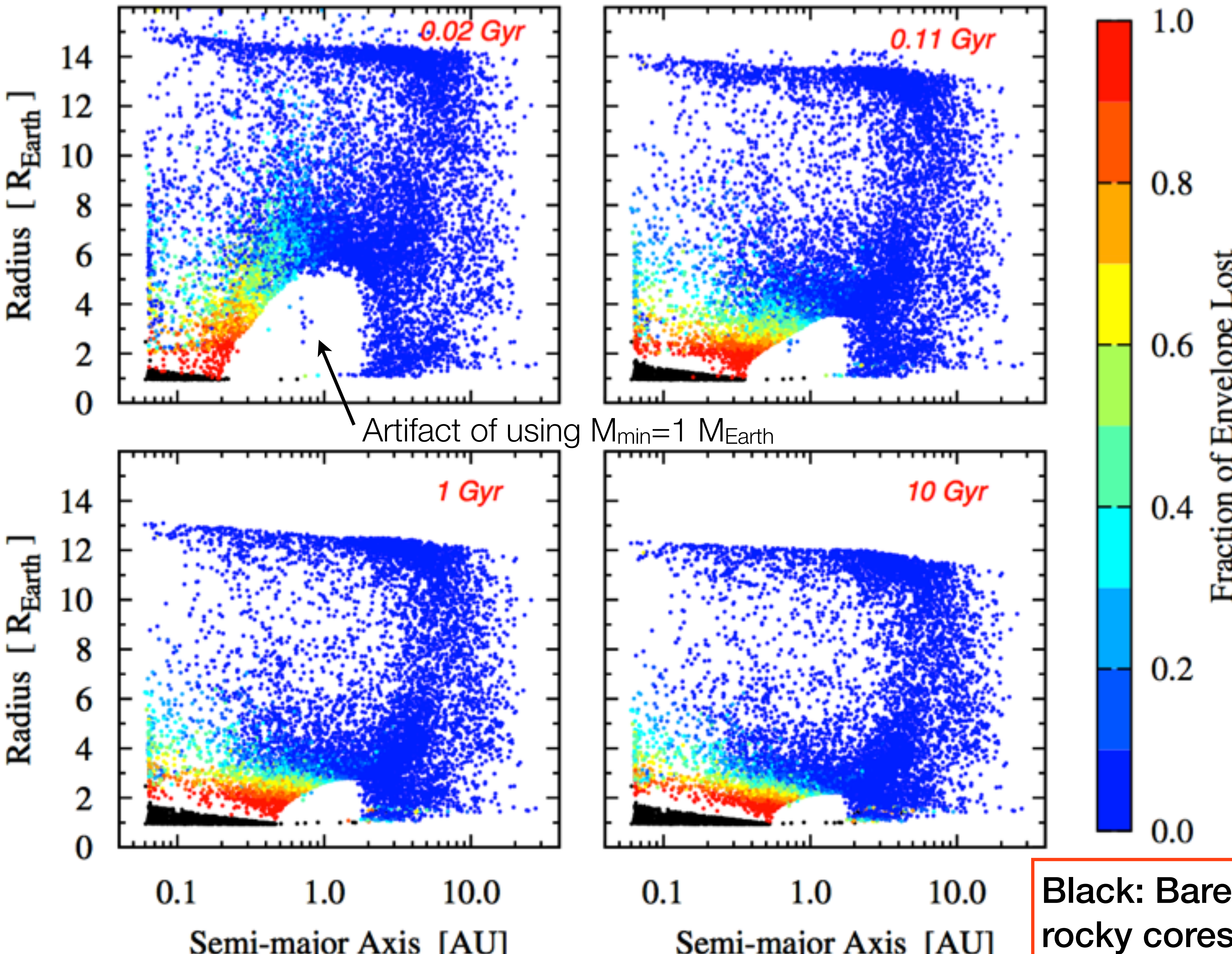
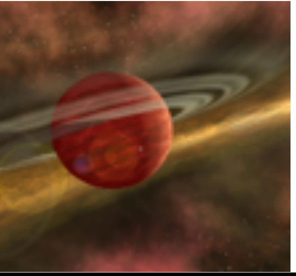


a-R does change

-Contraction
-Evaporation
-Empty valley below 1-2 R_{Earth}

Black: Bare rocky cores



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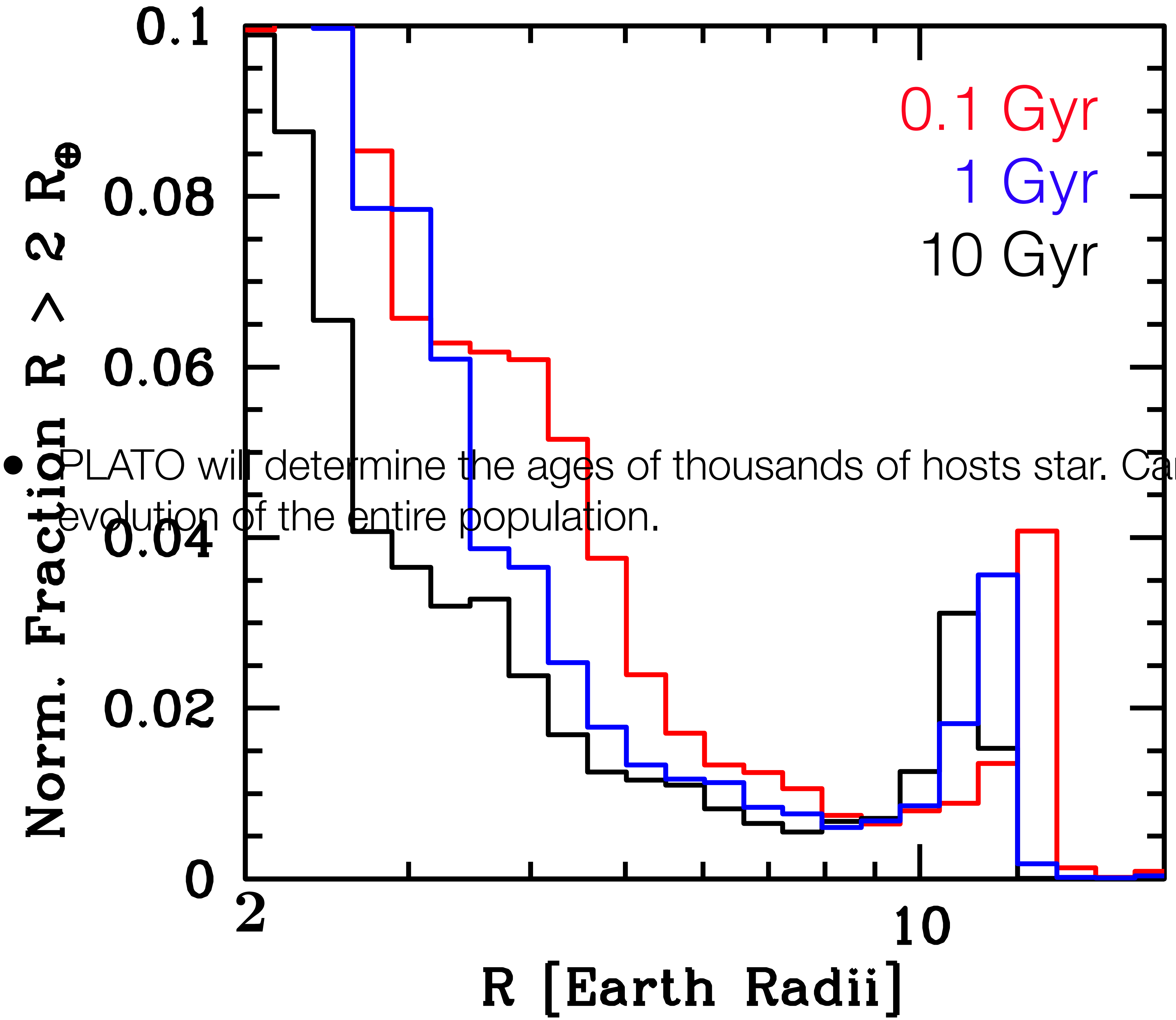
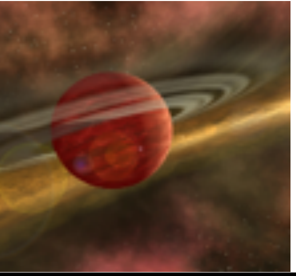


Radius distribution as function of time:

Key constraint for planet formation and evolution theory

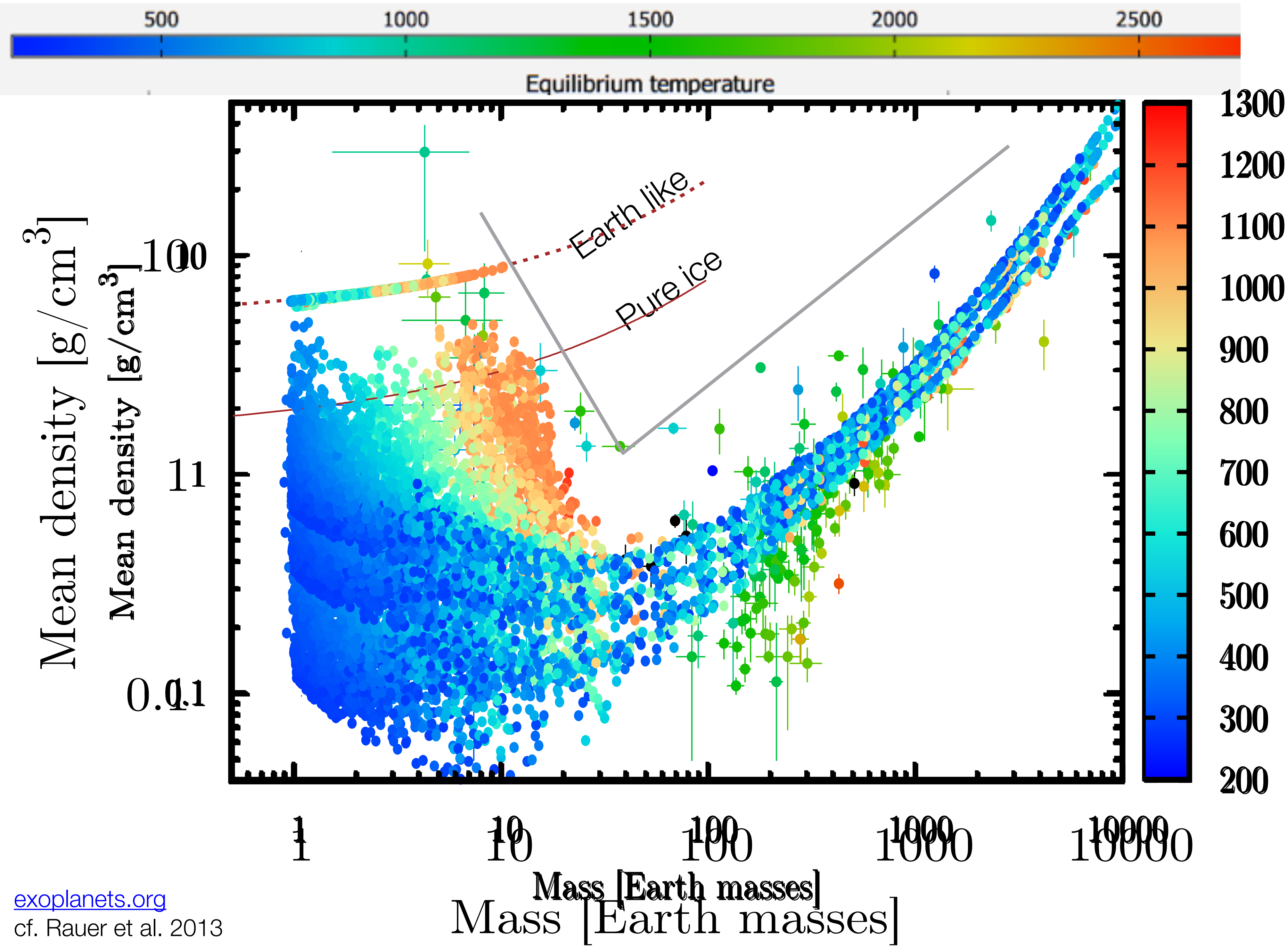
- PLATO will determine the ages of thousands of hosts star. Can see temporal evolution of the entire population.

Not (yet) directly observed



1) Adding a new dimension: time

1 M_{sun} star. No bloating mechanism.



exoplanets.org

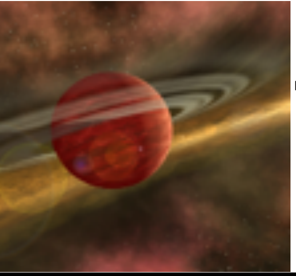
cf. Rauer et al. 2013

Search for the transition in the M-rho-t space

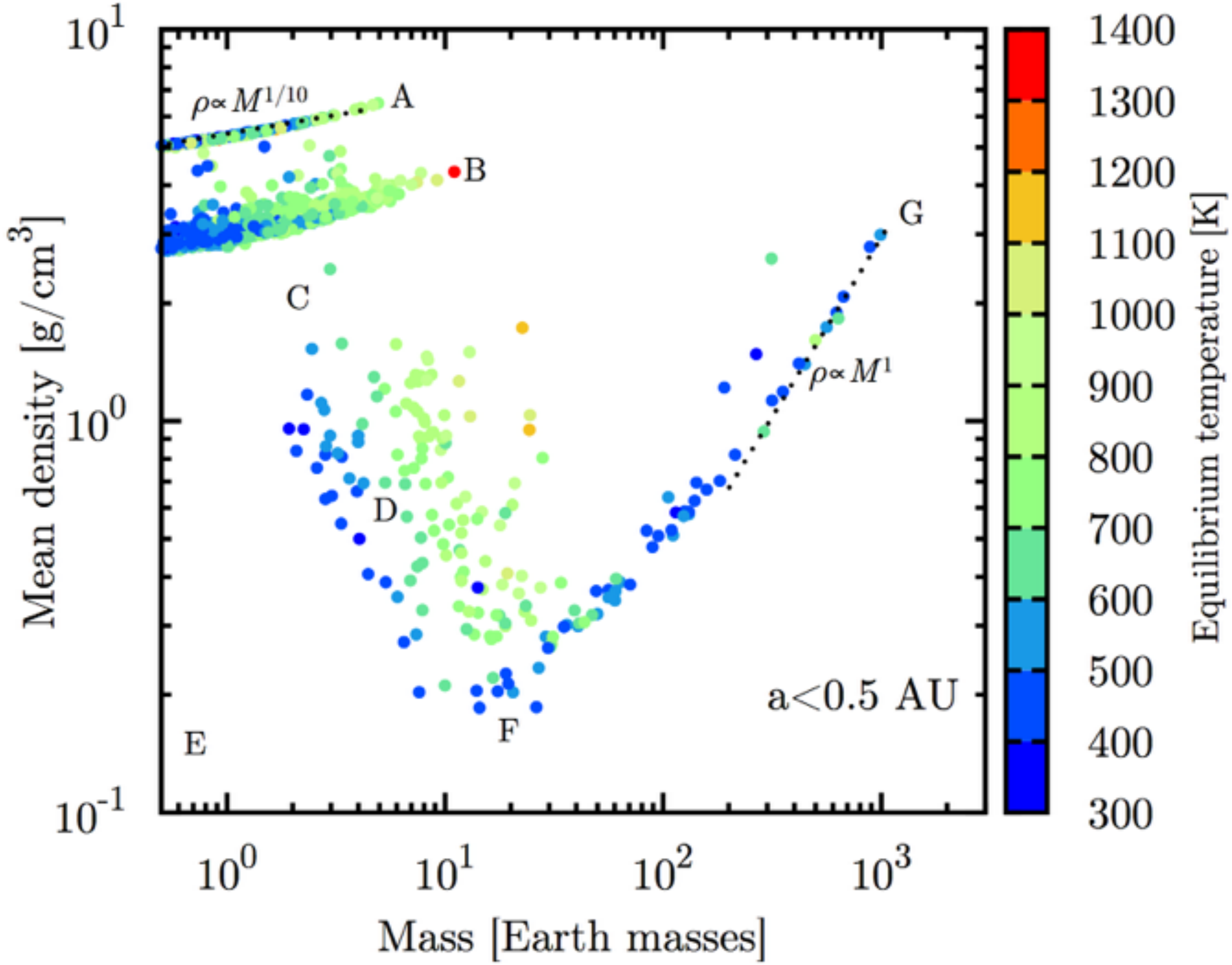
Giants: hotter less dense: bloating

Low mass: hotter denser: evaporation

- Solid planets ~don't change, those with H/He do.

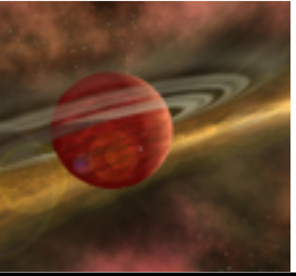


Theoretical mass - density diagram

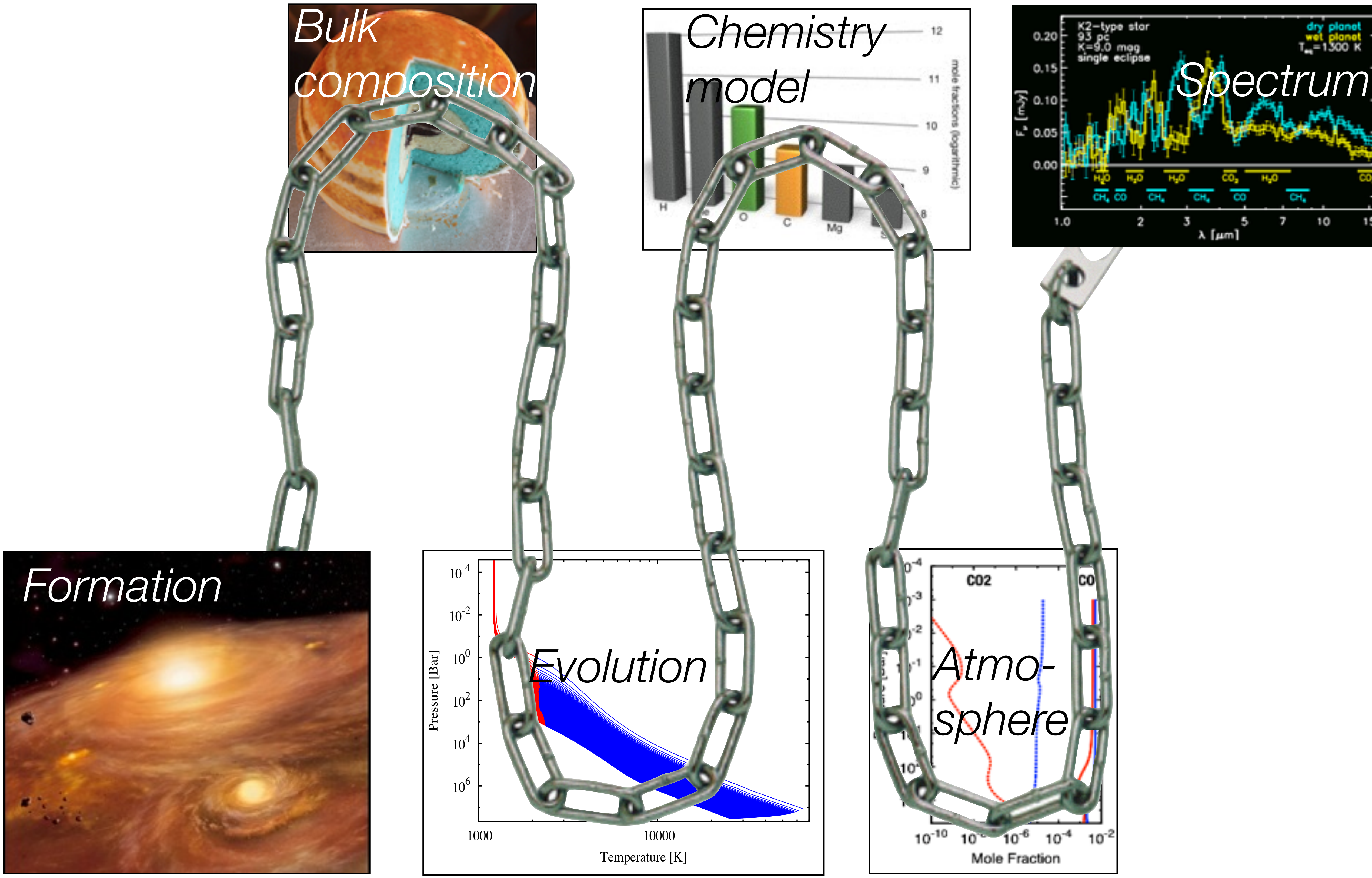


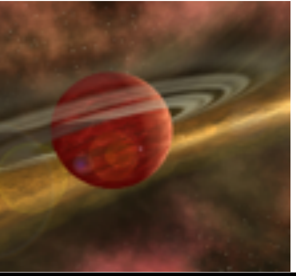
A: Bare rocky cores
B: Bare icy cores
C: Evaporation valley
D: Low-mass planets with H/He

E: Evaporation forbidden zone
F: Transition to gas dominated planets
G: Giant planets



2) Linking formation and spectra

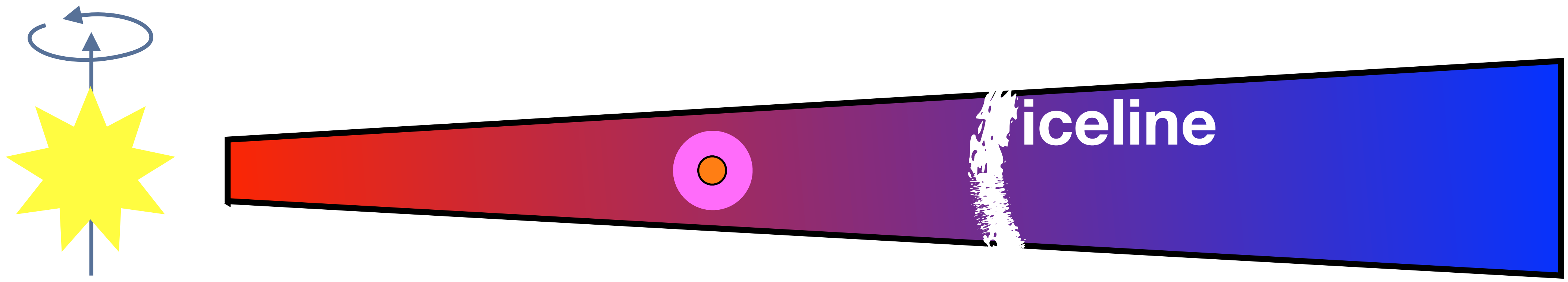




2) Linking formation and spectra

Case 1: “dry planet”

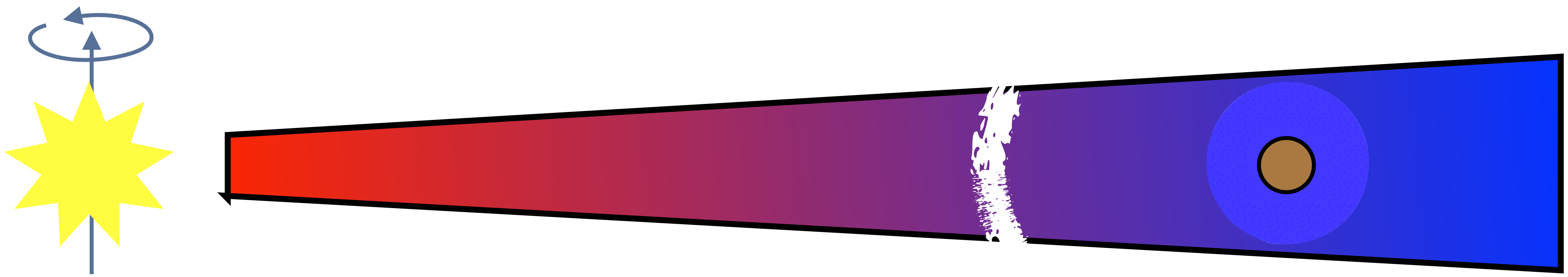
Disk migration



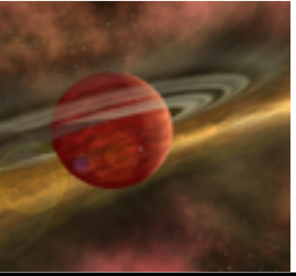
Result: aligned Hot Jupiter with chemical imprint of accretion of hot gas and rocky planetesimals

Case 2: “wet planet”

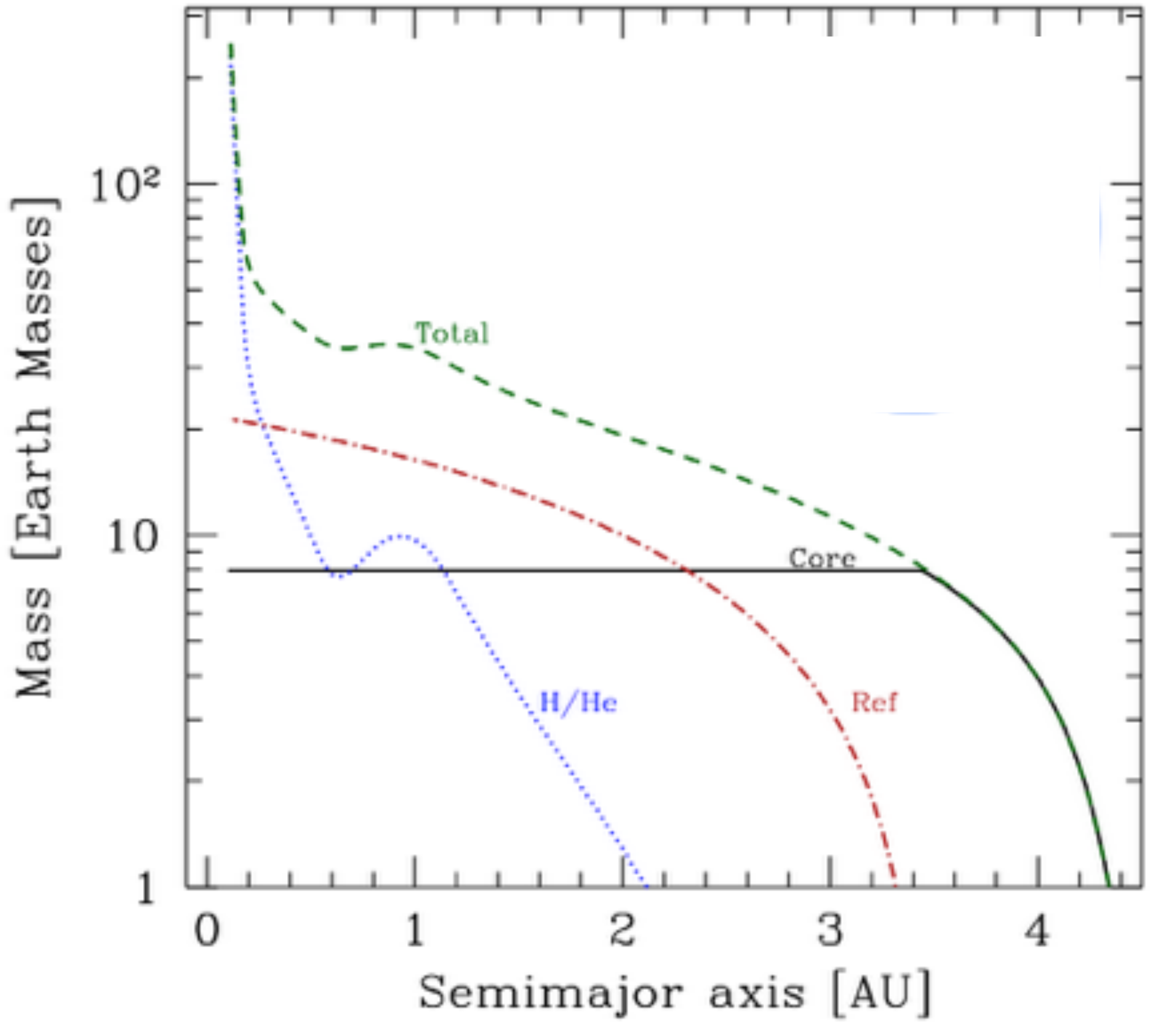
Kozai and tidal circularization

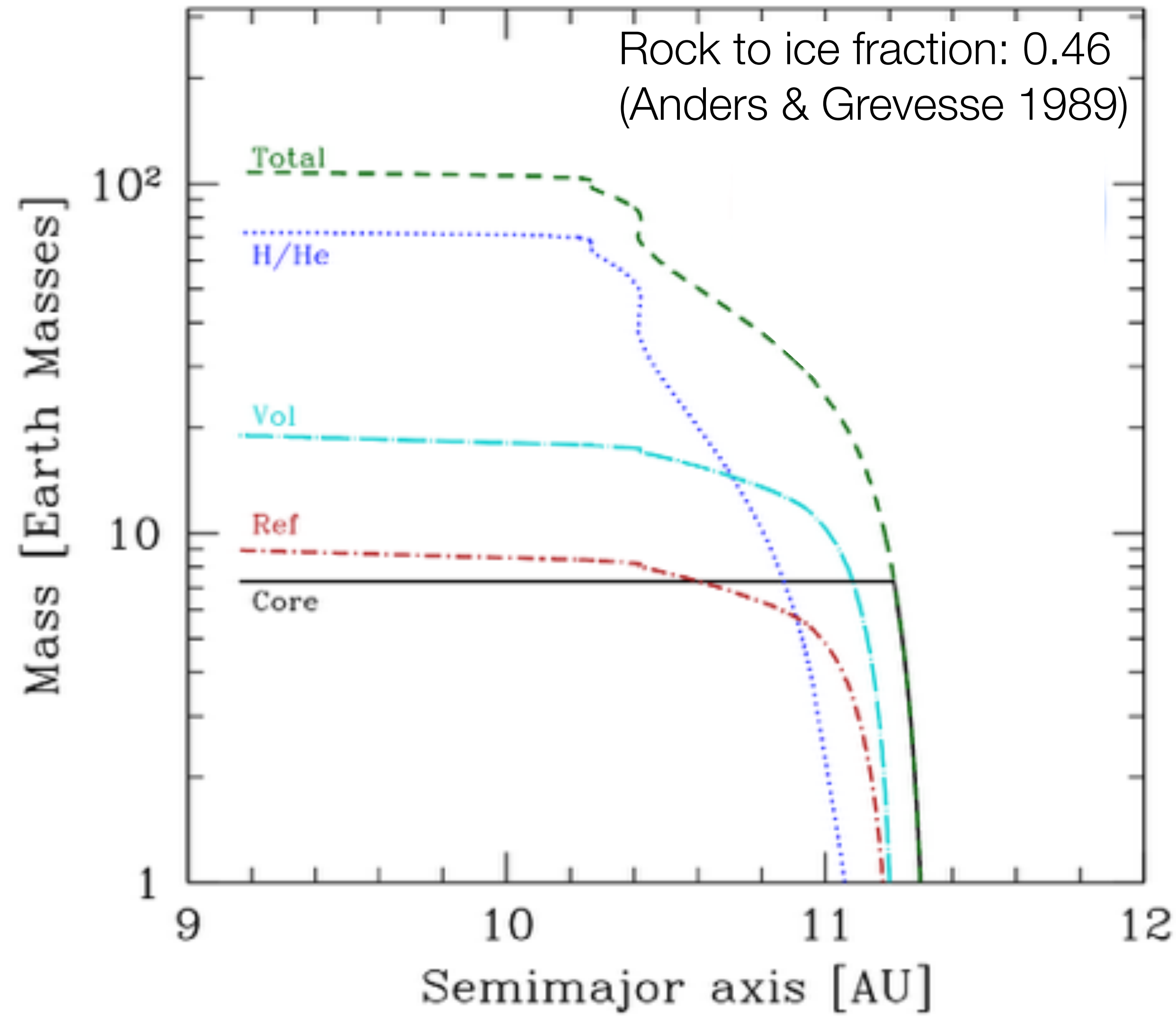


Result: potentially misaligned Hot Jupiter with chemical imprint of accretion of cold gas and icy planetesimals from beyond iceline only



Formation phase





Dry Jupiter

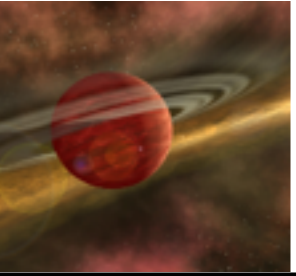
Wet Saturn

Disk migration to inner disk edge during disk lifetime.

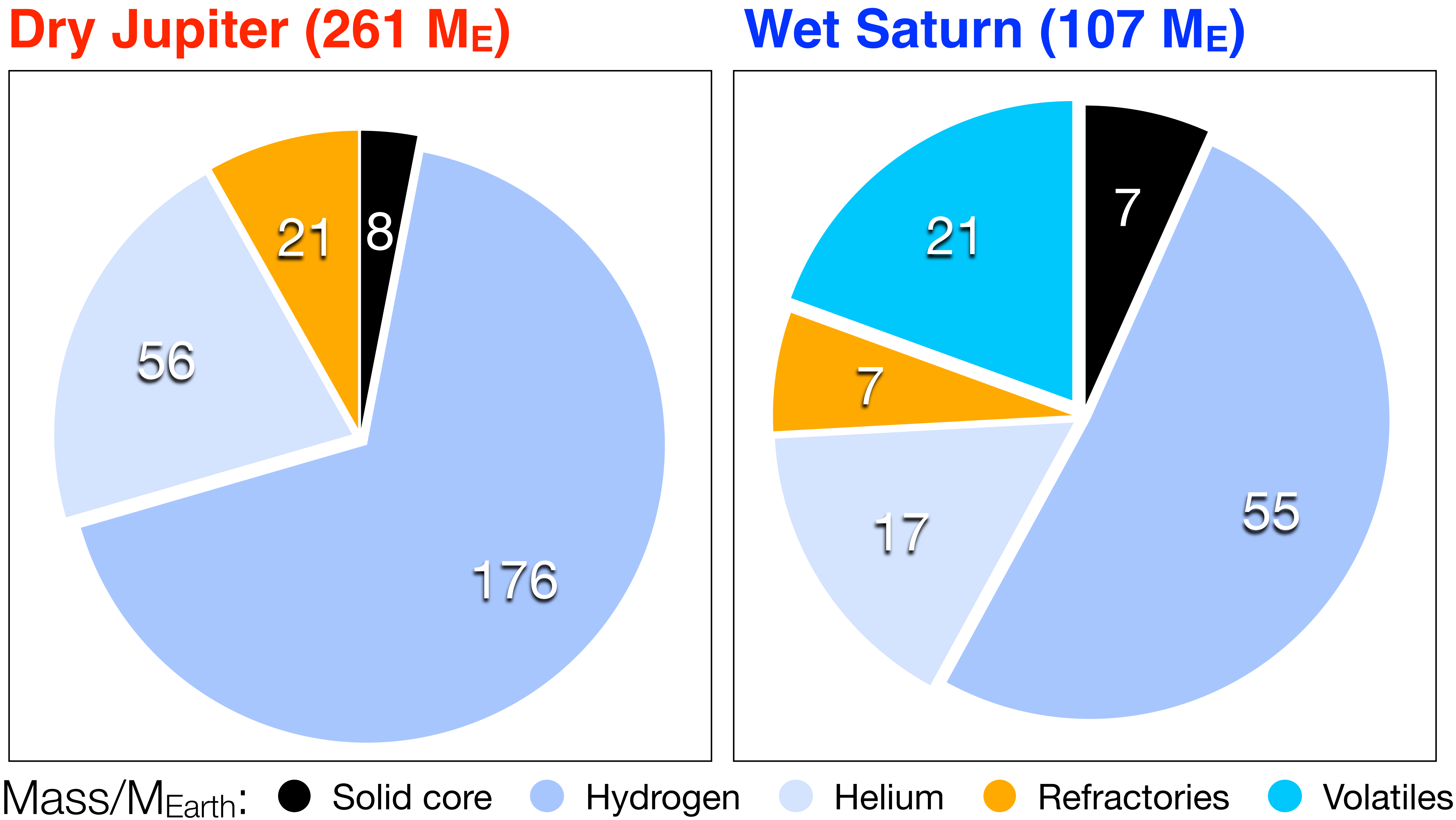
Assumption: accreted gas volatile free (might not true if disk midplane MRI dead)

Scattering/Kozai migration to 0.04 AU after disk dissipation.

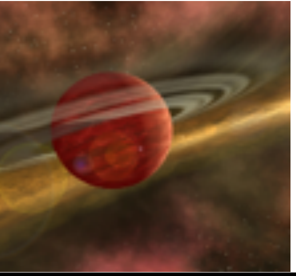
Assumption: no accretion during this process



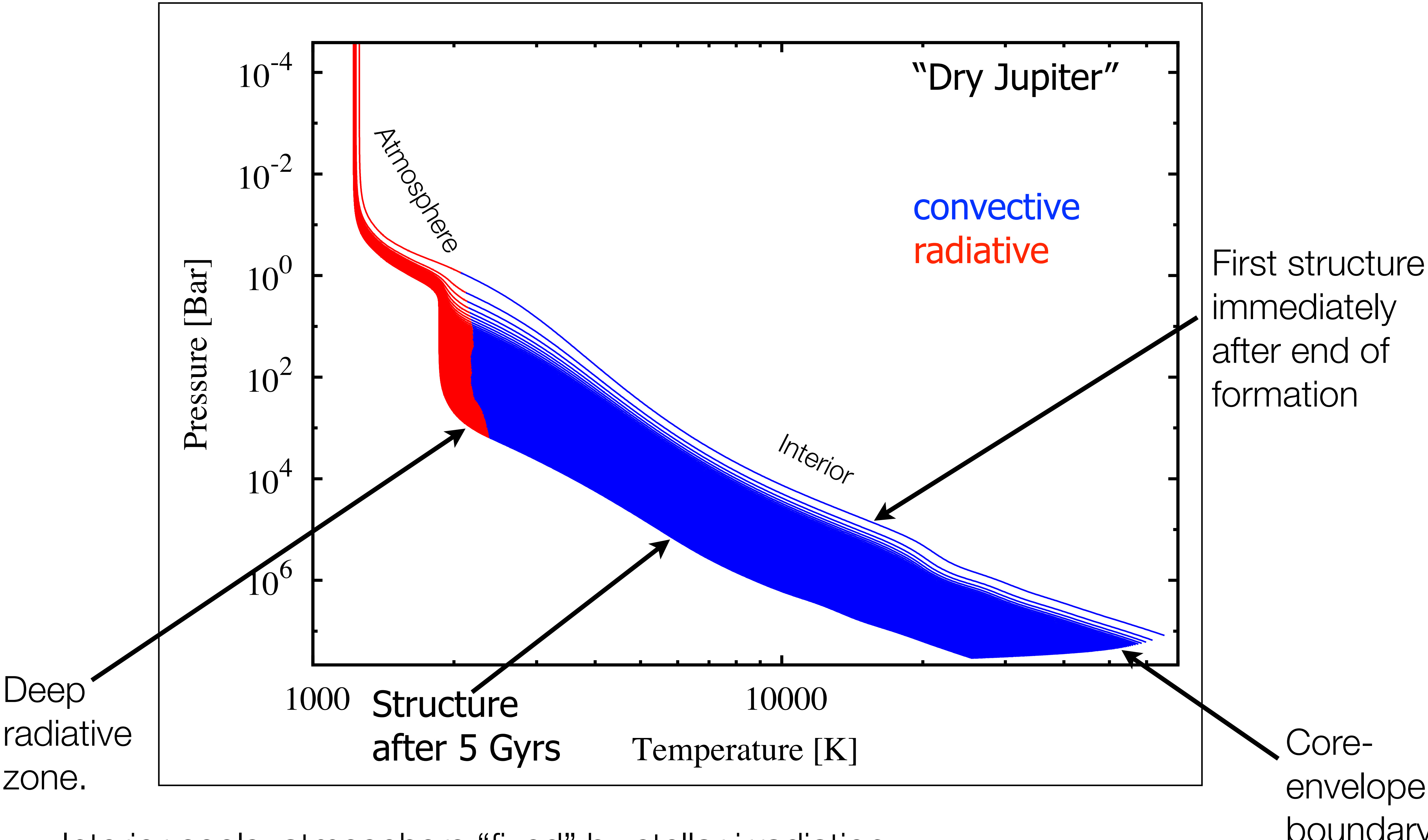
Final bulk composition



The envelope of the "wet Saturn" is more enriched since a) more solids further away from the star (larger feeding zone, ices) b) lower H/He mass c) icy planetesimal more fragile



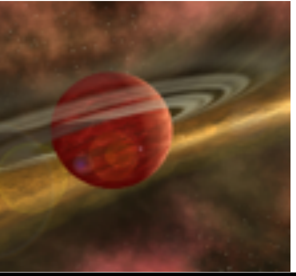
Evolution: p-T structure



-Interior cools, atmosphere "fixed" by stellar irradiation.

-Atmospheric composition may decouple from interior: but mixing strong from GCMs

cf. Guillot & Showman 2002



Chemistry model

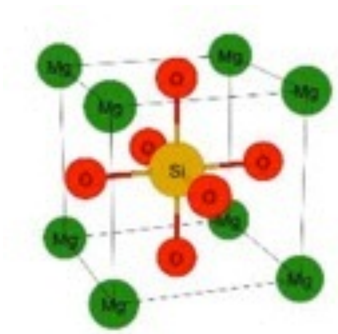
Specify what “refractory” or “ice” is in terms of atomic composition.

Refractories:

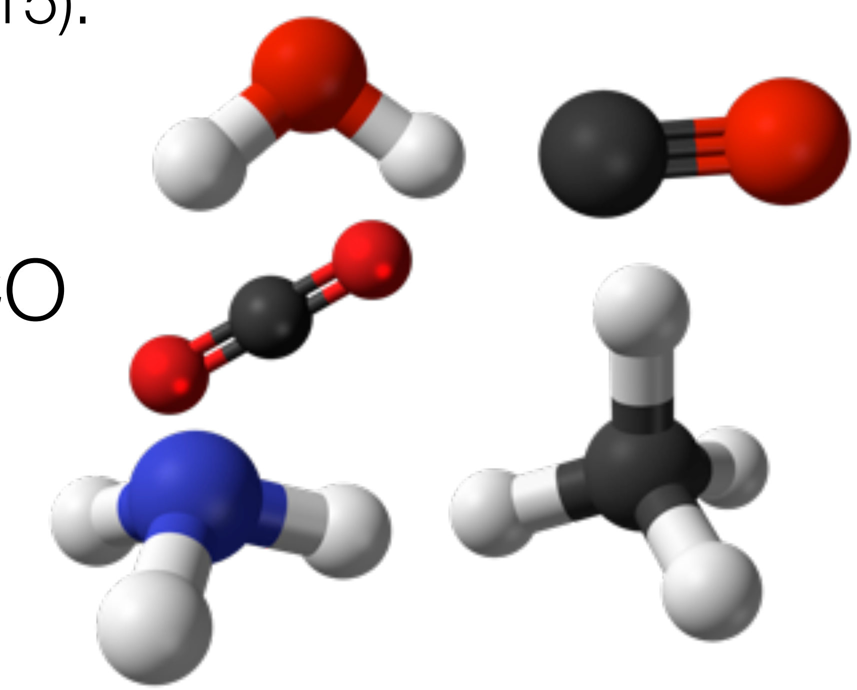
33 wt% Iron Fe

44 wt% Silicate Perovskite $MgSiO_3$

22 wt% Carbon C



From local ISM dust composition (Nuth et al. 1998). Assume no evaporation and re-condensation during solar nebula formation (Gaidos et al. 2015).



Volatiles:

61 nb% Water H_2O

12 nb% Carbon monoxide CO

19 nb% Carbon dioxide CO_2

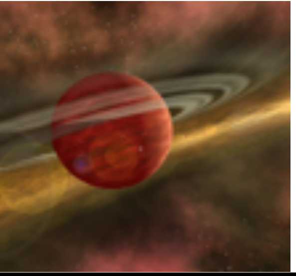
2.4 nb% Methane CH_4

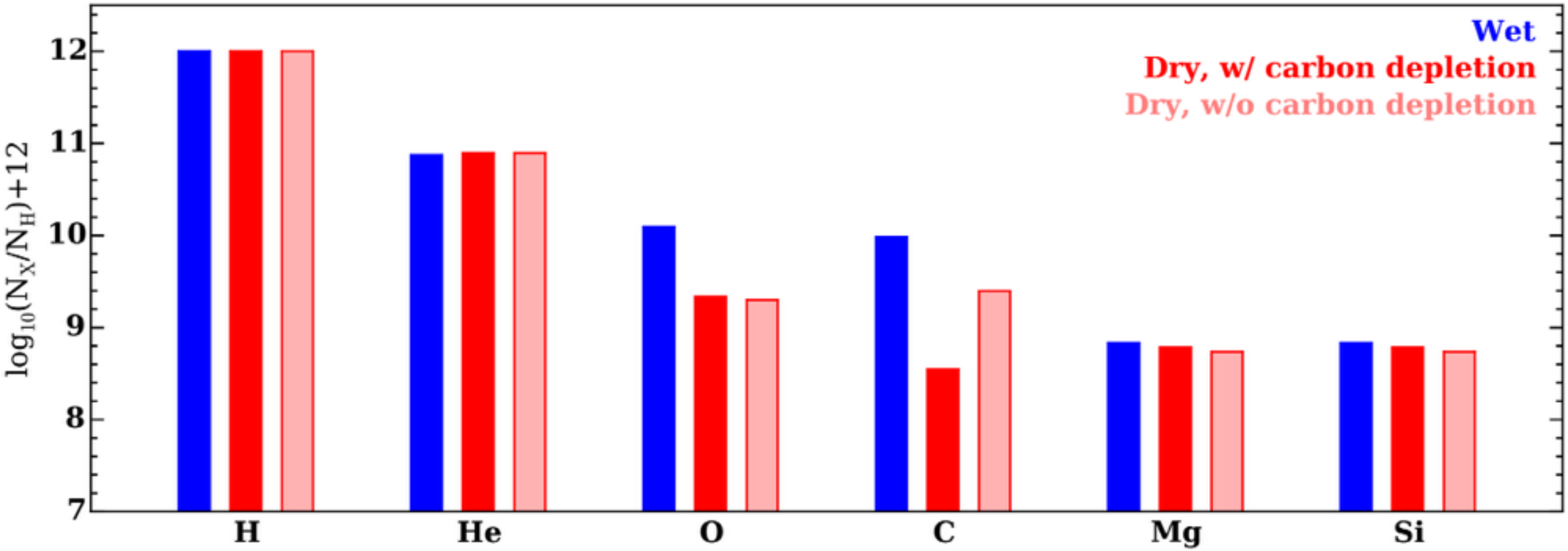
6.1 nb% Ammonia NH_3

From observed abundances in protoplanetary disks (Pontoppidan et al. 2005). Similar in comets (Bockelee-Morvan et al. 2004) and protostellar cloud cores.

Assume uniform mixing of atmosphere and envelope. No temporal evolution. Heavy atoms might settle to the deep interior (Fortney et al. 2008, Spiegel et al. 2009)

Resulting abundances

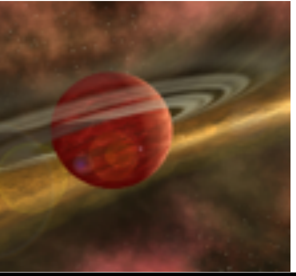




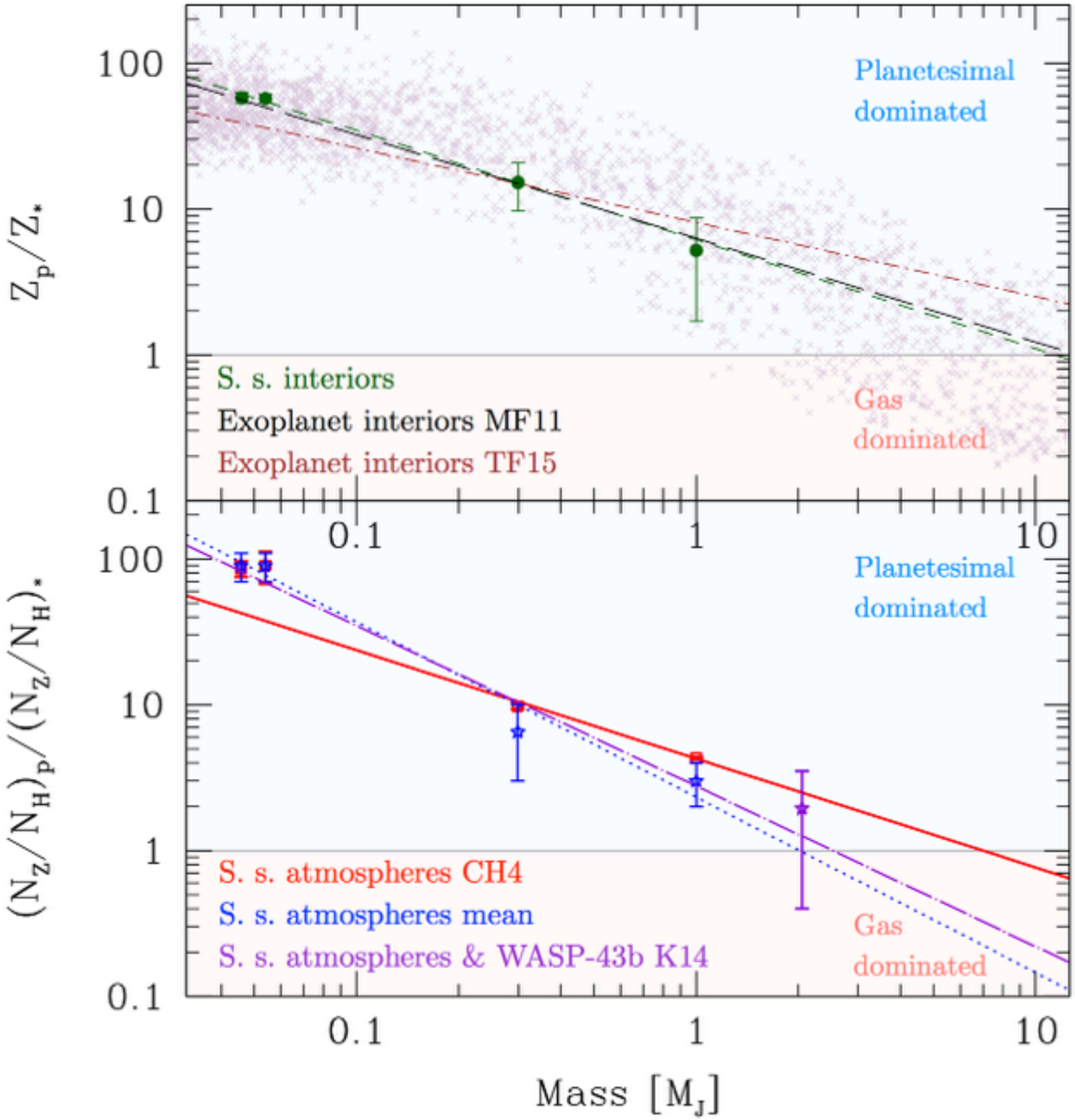
New constraints from spectra

Here: formation location, migration mode → C/O ratio

- EGPs formed outside water iceline: O-rich
- EGPs formed inside water iceline:
    - O - rich (carbon poor rocky planetesimals - likely)
    - C - rich (ISM-like carbon-rich grains - unlikely)



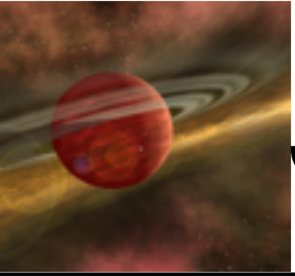
Conclusions



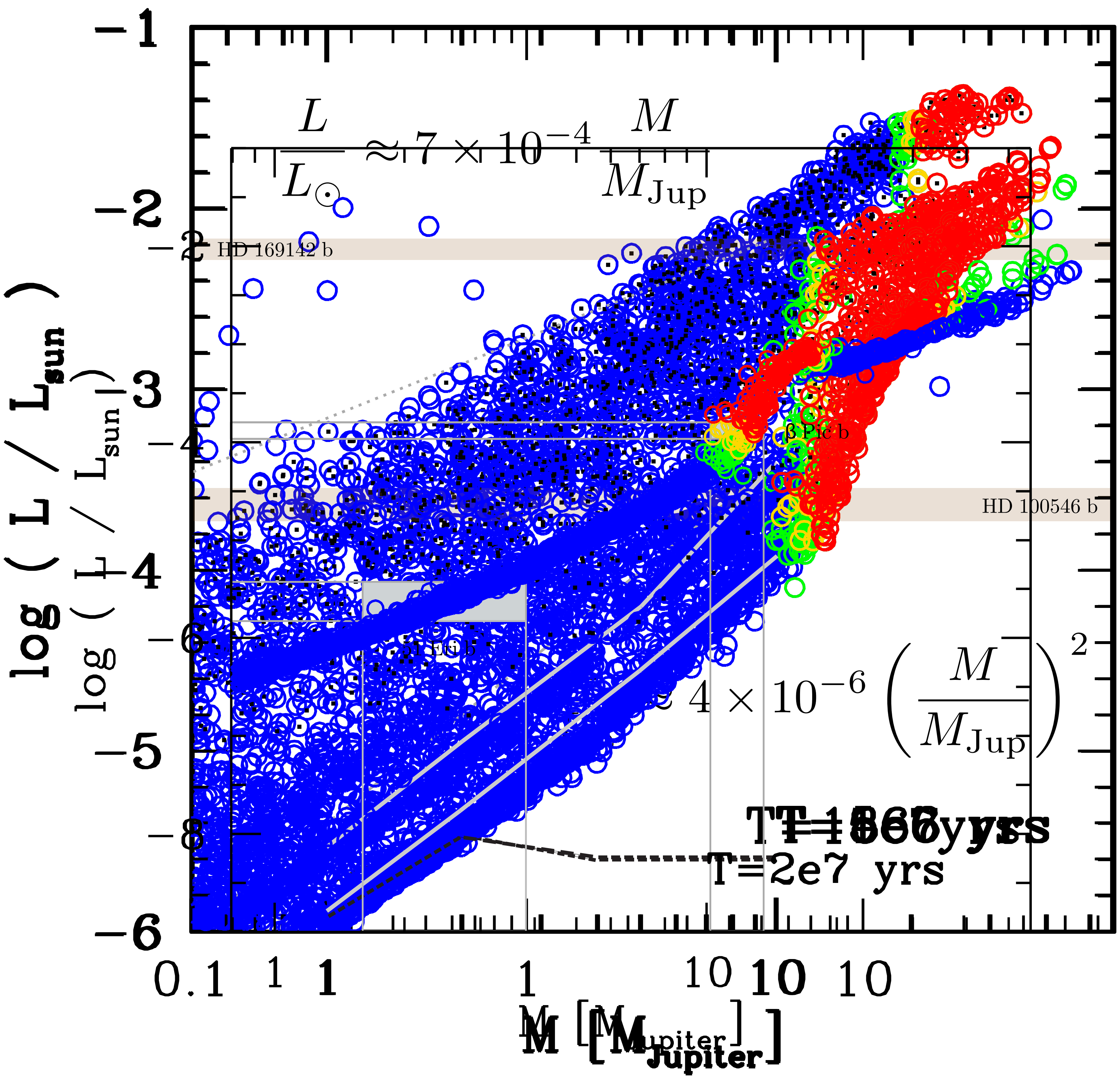
Conclusions for Hot Jupiters

1) Planetesimal enrichment is dominant ($M<2-10\ M_J$)

2) Hot Jupiters have water dominated atmospheres with $C/O<1$.



3) Observing planet formation as it happens



cold gas accretion

D-burning

$L_D \geq 5\%\ L_{int}$

$L_D \geq 25\%\ L_{int}$

$L_D \geq 50\%\ L_{int}$

Dot: $L_{acc} > L_{int}$

Accreting sequence:

$$L \approx L_{\rm acc} \propto M$$

Evolving sequence

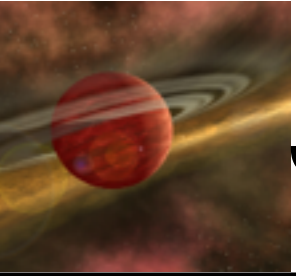
$$L \approx L_{\rm int} \propto M^2$$

Burrows & Liebert 1993, Marleau & Cumming 2014

- L almost as Hot Start.
- Intrinsic scatter in M-L
- Core mass effect: enrichment relative to star

Mordasini et al. in prep

Quanz et al. 2013, 2015, Reggiani et al. 2014



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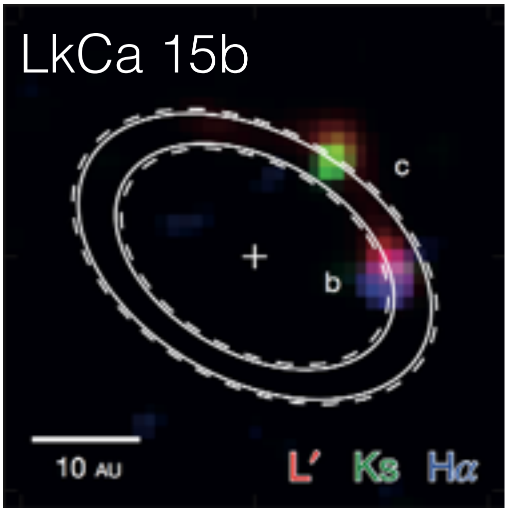
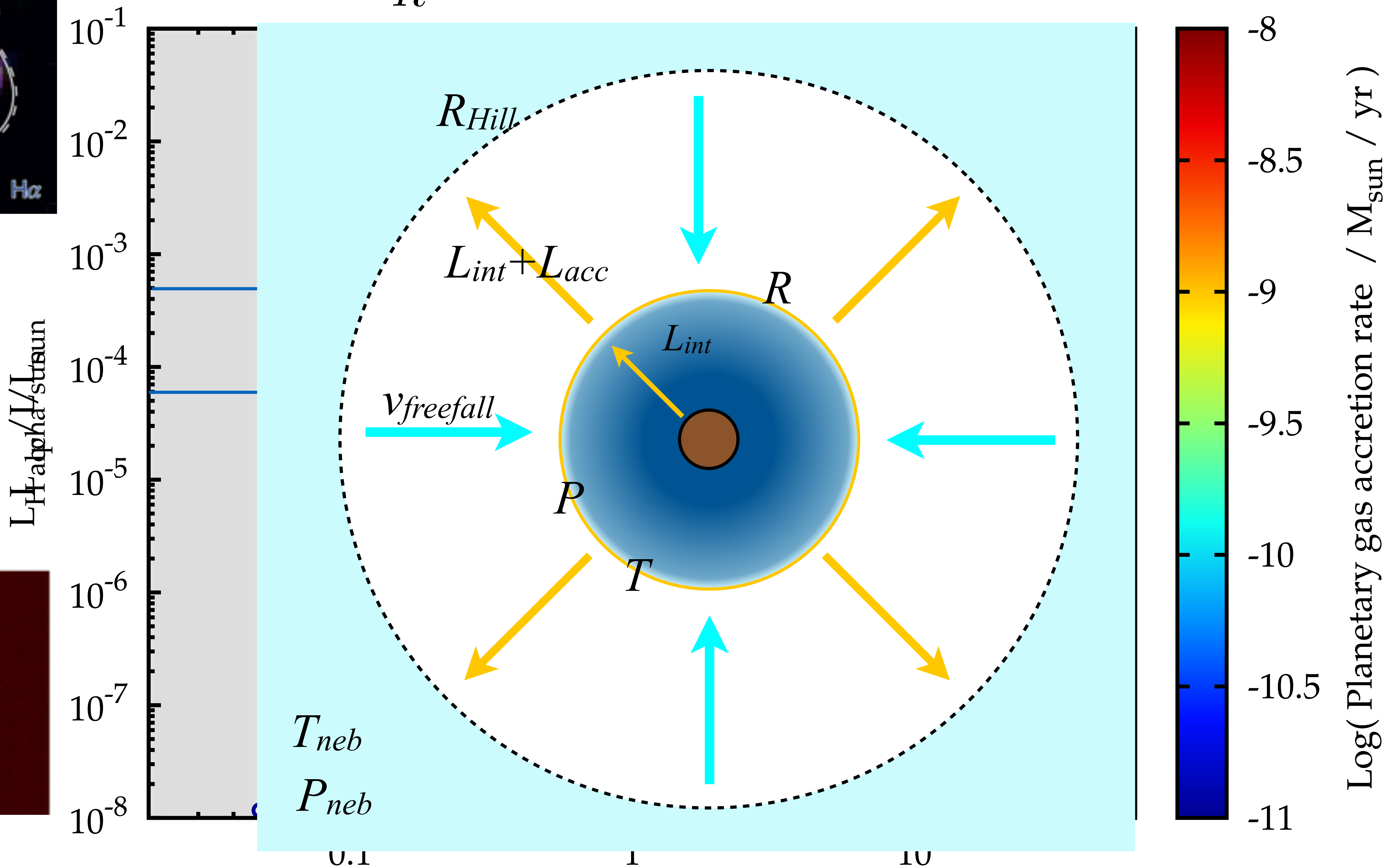
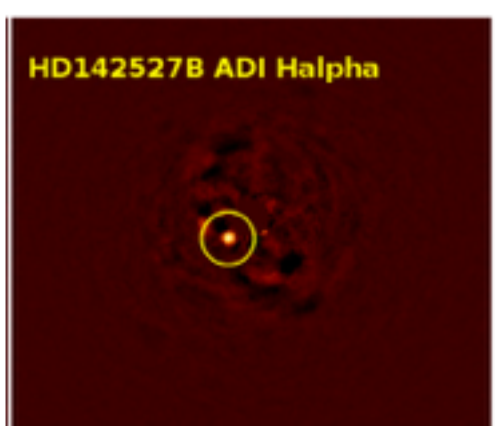


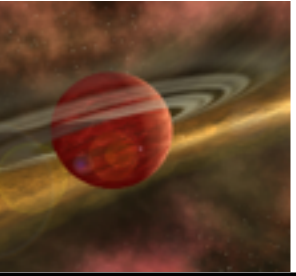
$$L_{\rm acc} = \frac{G M \dot{M}_{\rm gas}}{R} \qquad \log(L_{\rm acc}) = b + a \times \log(L_{\rm H\alpha})$$ from T Tauris





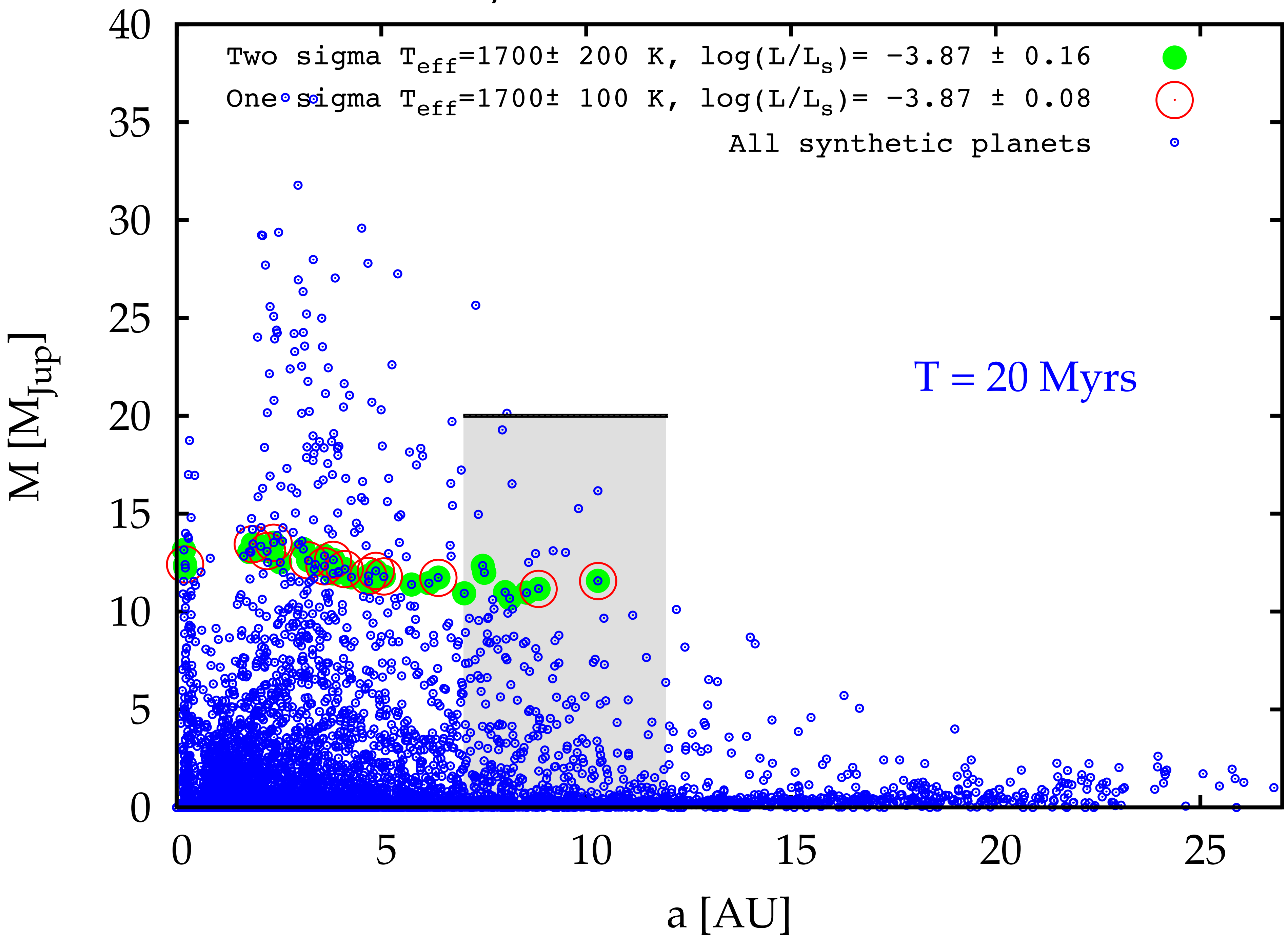
Close et al. 2014
Sallum et al. 2015

t=3 Myr, cold accretion



Application: Beta Pic b

Bonnefoy et al. 2013, Mordasini et al. 2014



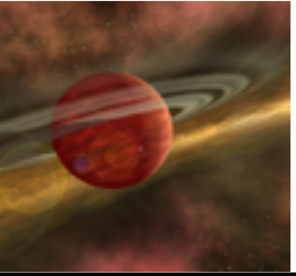
Beta Pic b can be explained with "cold" core accretion:

Core mass effect. Planets with many heavy elements.

Mass: ~11 M_J

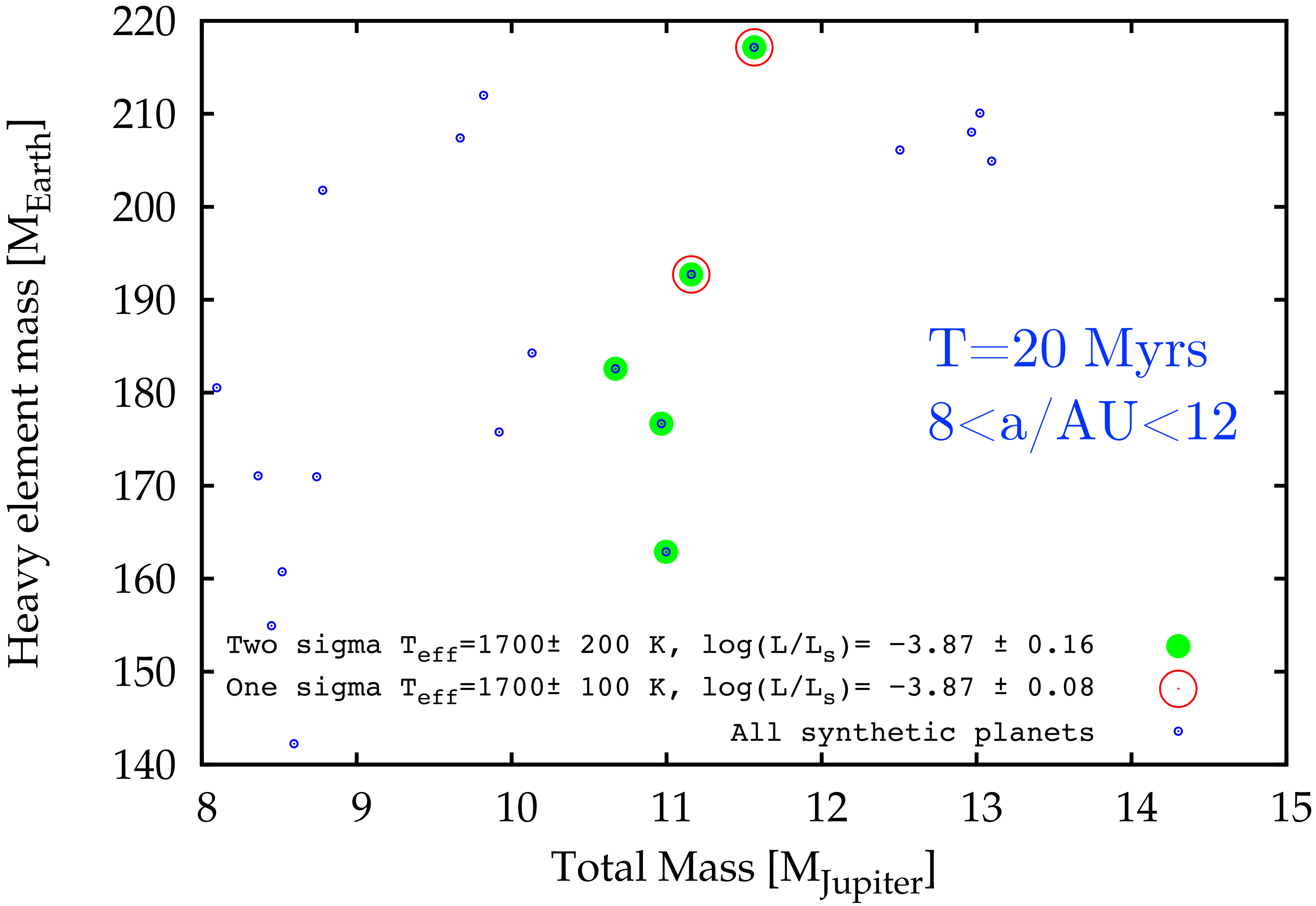
-Specialized population synthesis for Beta Pictoris.

-Combine constraints from RV and direct imaging.



Beta Pic b: enrichment ?

Beta Pic b: For cold accretion, needs large core masses for observed L and T_{eff}.



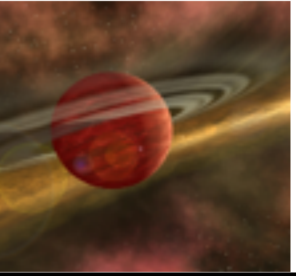
Total mass:
~10-12 M_J
Core mass:
~150-200 M_E

Corresponds to ~2 to 3 times stellar Z, i.e., [M/H]=0.3-0.5

Some metals might get mixed back into envelope and atmosphere: Enrichment (spectroscopy)

Conclusions

- Population synthesis is a tool to compare theory and observation to improve understanding of planet formation
  - use full wealth of observational constraints
  - put detailed models to the test
  - see global statistical consequences
- Observational constraints on many processes
  - solid and gas accretion rate (T_{KH})
  - grain dynamics
  - orbital migration rate
- See link between disk and planetary properties
- Predict yield of future instruments/space missions
- Continuously evolving models
  - population syntheses depend on progress of formation theory as a whole
  - a lot to do



Resources

Population synthesis review papers

-Benz et al., Protostars & Planets VI, 691, 2014

-Mordasini et al., IJA, 201, 2015

Freely available toy population synthesis model

http://nexsci.caltech.edu/workshop/2015/#handson

DACE data base

https://dace.unige.ch/evolution/index