Physical formulation of the eigenfrequency condition of mixed modes of stellar oscillations

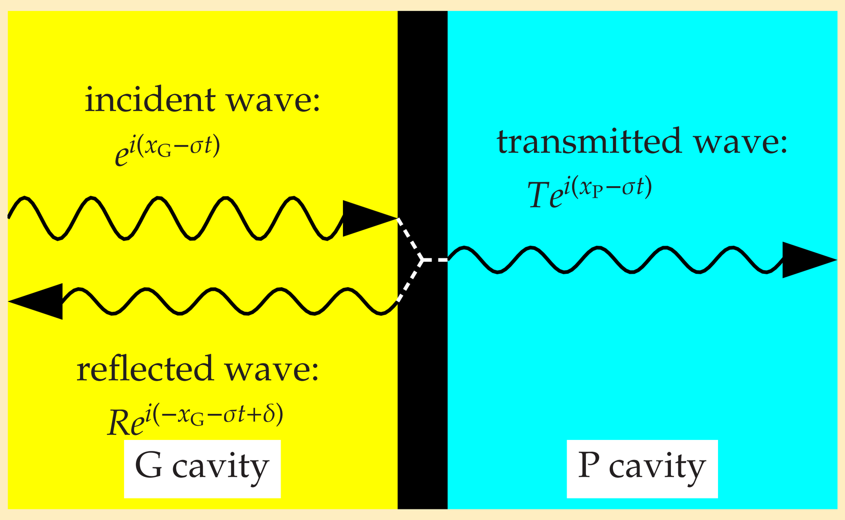
Masao Takata
Department of Astronomy, School of Science, University of Tokyo, Japan

Abstract

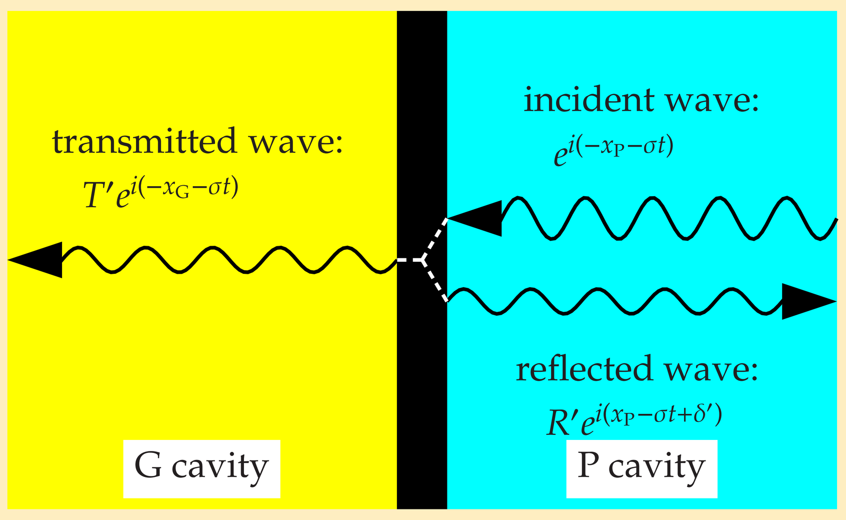
The frequency condition for the mixed modes of nonradial stellar oscillations is generally examined by a simple physical model based on a running-wave picture. The coupling coefficient between the gravity-wave oscillation in the core and the acoustic-wave oscillation in the envelope is expressed in terms of the reflection coefficient at the intermediate evanescent region. It is also argued that the eigenmode condition should appropriately be modified if the wave generated near the surface and transmitted to the core is (partially) lost either by damping or scattering in the core. The derived formulae should be helpful in understanding the physics of the mixed modes in general, the origin of the red giants with depressed dipolar modes, and the effect of radiative damping in the core of the red giant stars.

Reciprocal properties of a wave reflection-transmission system

Base problem



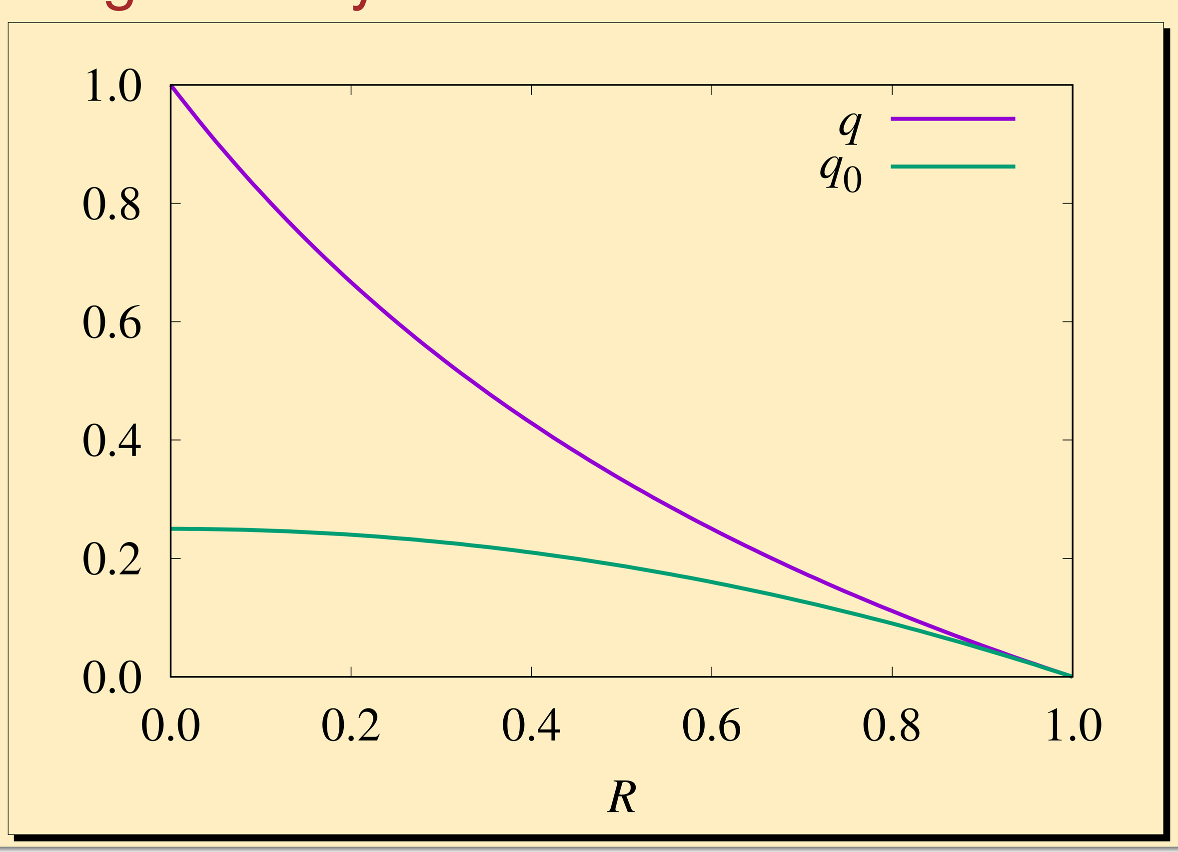
Adjoint problem



- It is possible to show
$$R' = R\,, \quad T' = T\,, \quad \delta + \delta' = \pi\,,$$
based on the following fundamental physical properties of the solutions:
  - time-reversal and time-shift symmetry
  - superposition principle
  - energy conservation ($R^2 + T^2 = 1$)
- No mirror symmetry about the middle wall is assumed.

Eigenmode condition for the mixed modes

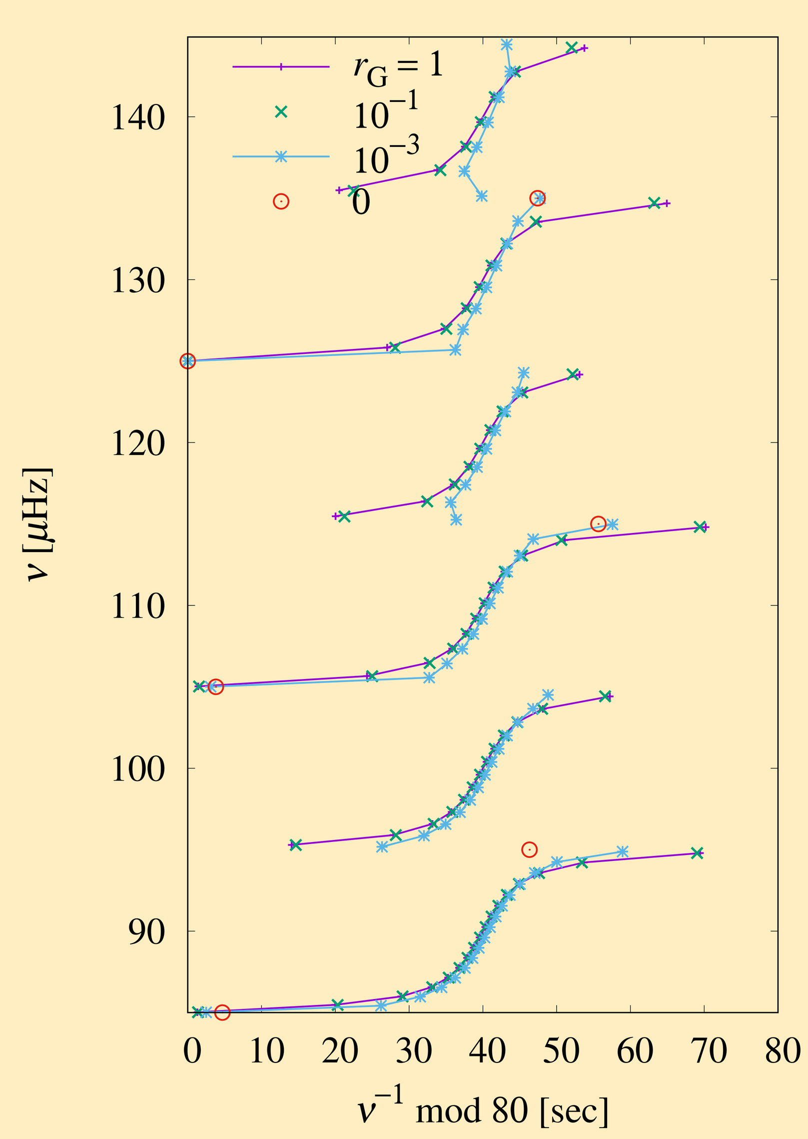
- An eigenmode solution can be constructed by the superposition of the solutions of the base and adjoint problems, with the perfectly reflecting boundary conditions at the inner edge of the G cavity and the outer edge of the P cavity.
- The eigenmode condition is given by
$$\cot\phi_G \tan\phi_P = q = \frac{1-R}{1+R}\,,$$
in which
$$\phi_G = X_G - \frac{\theta_G}{2} - \frac{\delta}{2} + \frac{\pi}{2}\,, \quad \phi_P = X_P + \frac{\theta_P}{2} - \frac{\delta}{2} + \frac{\pi}{2}$$
with
$X_{P,G}$: total phases in the P and G cavities
$\theta_{P,G}$: phase lags introduced at the reflection at the outer edge of the P cavity and the inner edge of the G cavity
- The conventional asymptotic analysis [e.g. Shibahashi (1979) and Tassoul (1980)] essentially finds
$$q \approx q_0 = \frac{T^2}{4} \quad \text{for } R \approx 1\,,$$
which is significantly different for $R \ll 1$.



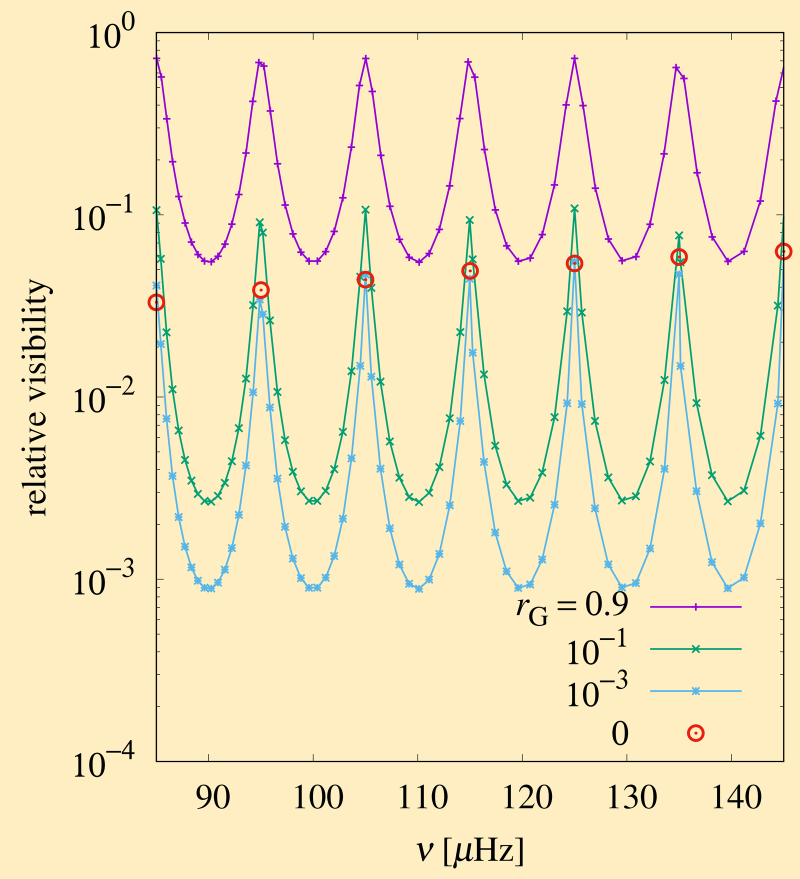
Case of the wave leakage through the boundaries

- The partially reflecting boundary conditions lead to
$$\cot\left(\phi_G - i\mu_G\right)\tan\left(\phi_P + i\mu_P\right) = \frac{1-R}{1+R}\,,$$
where
$$\mu_{P,G} = -\frac{1}{2}\ln r_{P,G}$$
with
$r_{P,G}$: reflection coefficients at the outer edge of the P cavity and the inner edge of the G cavity.
- The complex frequencies are determined under the assumptions of
$$\phi_P = \pi\left(\frac{\nu}{\Delta\nu} - \epsilon_p - \frac{1}{2}\right)\,, \quad \phi_G = \pi\left(\frac{1}{\nu\Delta\Pi} - \epsilon_g\right)\,,$$
with, following Mosser (2012),
$$q = 0.15\,, \quad \Delta\nu = 10\,[\mu\text{Hz}]\,, \quad \Delta\Pi = 80\,[\text{sec}]\,,$$
$$\epsilon_p = 0\,, \quad \epsilon_g = 0\,, \quad r_P = 0.96\,.$$

Period échelle diagram



Relative damping rate



$$v = \frac{\Im\nu\,(r_G = 1)}{\Im\nu\,(r_G)}$$

Conclusion

- The eigenmode condition for the mixed modes is derived based on a running-wave picture.
- The general relation between the coupling coefficient and the reflection coefficient at the intermediate evanescent region is established.
- The eigenmode condition is extended to the case of the wave leakage through the boundaries.
- The (complex) reflection coefficient r_G at the inner boundary characterizes the core leakage, which can formally describe the radiative damping (e.g. Dupret et al. 2009) and the magnetic scattering (Fuller et al. 2015).
- The modulus of r_G mainly influences the damping rate and little affects (the real part of) the frequency.