Abstract

The frequency condition for the mixed modes of nonradial stellar oscillations is generally examined by a simple physical model based on a running-wave picture. The coupling coefficient between the gravity-wave oscillation in the core and the acoustic-wave oscillation in the envelope is expressed in terms of the reflection coefficient at the intermediate evanescent region. It is also argued that the eigenmode condition should appropriately be modified if the wave generated near the surface and transmitted to the core is (partially) lost either by damping or scattering in the core. The derived formulae should be helpful in understanding the physics of the mixed modes in general, the origin of the red giants with depressed dipolar modes, and the effect of radiative damping in the core of the red giant stars.

Case of the wave leakage through the boundaries

The partially reflecting boundary conditions lead to

\[ \cot \left( \phi_G - i \mu_G \right) \tan \left( \phi_P + i \mu_P \right) = \frac{1 - R}{1 + R}, \]

where

\[ \mu_{P,G} = \frac{1}{2} \ln r_{P,G} \]

with

\[ r_{P,G} : \text{reflection coefficients at the outer edge of the P cavity and the inner edge of the G cavity.} \]

The complex frequencies are determined under the assumptions of

\[ \varphi_P = \pi \left( \frac{\nu}{\Delta \nu} - \frac{1}{2} \right), \quad \varphi_G = \pi \left( \frac{1}{\nu \Delta \Pi} - \frac{\phi_G}{\nu \Delta \Pi} \right), \]

with, following Mosser (2012),

\[ q = 0.15, \quad \Delta \nu = 10 \text{[µHz]}, \quad \Delta \Pi = 80 \text{[sec]}, \quad \epsilon_p = 0, \quad \epsilon_G = 0, \quad r_P = 0.96. \]

Conclusion

The eigenmode condition for the mixed modes is derived based on a running-wave picture.

The general relation between the coupling coefficient and the reflection coefficient at the intermediate evanescent region is established.

The eigenmode condition is extended to the case of the wave leakage through the boundaries.

The (complex) reflection coefficient \( r_G \) at the inner boundary characterizes the core leakage, which can formally describe the radiative damping (e.g. Dupret et al. 2009) and the magnetic scattering (Fuller et al. 2015).

The modulus of \( r_G \) mainly influences the damping rate and little affects (the real part of) the frequency.