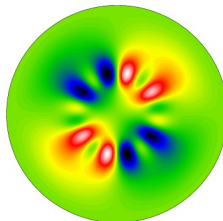


Non-adiabatic pulsations in ESTER models

D. R. Reese¹, M.-A. Dupret², and M. Rieutord³

¹LESIA, ²ULg, ³IRAP

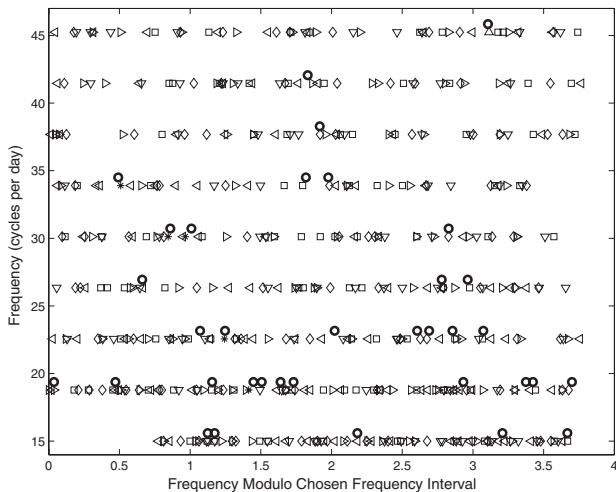
July 12, 2016



Introduction

The challenges in interpreting the pulsations of rapidly rotating stars

- theoretical challenges
 - 2D geometry – complicated formulas and numerically demanding
 - no automatic mode classification procedure
- lack of *simple* frequency patterns
 - p-modes: superposition of multiple independent patterns
 - g-modes: varying period separation + numerous inertial modes
- amplitudes are difficult to predict (classical pulsators)



(Deupree et al. 2012)

The benefits of non-adiabatic calculations

- find out which modes are excited
- consistent calculation of $\delta T_{\text{eff}}/T_{\text{eff}}$
 - amplitude ratios
 - phase shifts
 - line profile variations (LPVs)
 - mode identification

Previous 2D non-adiabatic pulsation calculations

Reference	Model	Pulsations
Lee & Baraffe (1995)	Chandrasekhar expansion	2 or 3 harmonics
Lee (2001)	Spherical	10 harmonics
Savonije (2005, 2007)	Spherical	2D calculations
Lee (2008)	Chandrasekhar expansion	4 harmonics

Comparisons with traditional approximation

- Savonije (2005, 2007): stabilising effect of Coriolis force
- Lee (2008): stabilising effect of centrifugal deformation

Previous 2D non-adiabatic pulsation calculations

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Comparisons with traditional approximation

- Savonije (2005, 2007): stabilising effect of Coriolis force
- Lee (2008): stabilising effect of centrifugal deformation
- in all cases, the effects of rotation are approximated
⇒ there is a need for full 2D calculations, with 2D models

Necessary ingredients for 2D non-adiabatic pulsations

- 2D rotating models
 - hydrostatic equilibrium: adiabatic calculations
 - energy conservation equation: non-adiabatic calculations
- a 2D pulsation code which includes non-adiabatic effects

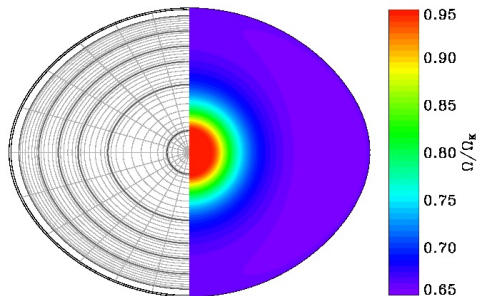
Necessary ingredients for 2D non-adiabatic pulsations

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 - energy conservation equation: non-adiabatic calculations
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ESTER (models) + TOP (pulsations)

The ESTER code (a very brief overview)

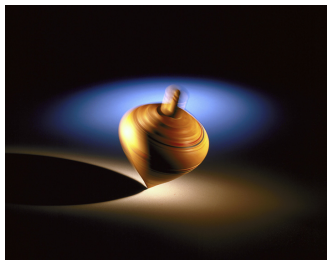
- ESTER = Evolution STEllaire en Rotation
- fully includes centrifugal deformation
- satisfies energy conservation equation:
 - baroclinic (isobars $\neq$ isochores $\neq$ isotherms)
 - self-consistent 2D rotation profile



(Rieutord & Espinosa Lara, 2009)

The TOP pulsation code

- TOP = Two-dimensional Oscillation Program
- fully includes centrifugal deformation
- can handle baroclinic models
- includes non-adiabatic effects



<http://johnmannphoto.com/blog/?p=103>

Pulsation equations

Continuity equation (conservation of mass)

$$0 = \frac{\delta\rho}{\rho_o} + \vec{\nabla} \cdot \vec{\xi}$$

Poisson's equation

$$0 = \Delta\Psi - 4\pi G \left(\rho_o \frac{\delta\rho}{\rho_o} - \vec{\xi} \cdot \vec{\nabla} \rho_o \right)$$

$\delta\rho$ = Lagrangian density perturbation

ρ_o = equilibrium density profile

$\vec{\xi}$ = Lagrangian displacement

Ψ = Eulerian perturbation to the gravitational potential

Pulsation equations

Euler's equations (conservation of momentum)

$$\begin{aligned}
 0 = & [\omega + m\Omega]^2 \vec{\xi} - 2i\vec{\Omega} \times [\omega + m\Omega] \vec{\xi} - \vec{\Omega} \times (\vec{\Omega} \times \vec{\xi}) \\
 & - \vec{\xi} \cdot \vec{\nabla} (\varpi \Omega^2 \vec{e}_\varpi) - \frac{P_o}{\rho_o} \vec{\nabla} \left(\frac{\delta P}{P_o} \right) + \frac{\vec{\nabla} P_o}{\rho_o} \left(\frac{\delta \rho}{\rho_o} - \frac{\delta P}{P_o} \right) - \vec{\nabla} \psi \\
 & + \vec{\nabla} \left(\frac{\vec{\xi} \cdot \vec{\nabla} P_o}{\rho_o} \right) + \frac{(\vec{\xi} \cdot \vec{\nabla} P_o) \vec{\nabla} \rho_o - (\vec{\xi} \cdot \vec{\nabla} \rho_o) \vec{\nabla} P_o}{\rho_o^2}
 \end{aligned}$$

ω = pulsation frequency

m = azimuthal order

Ω = rotation profile

ϖ = distance to the rotation axis

δP = Lagrangian pressure perturbation

Pulsation equations

Energy conservation equation

- unperturbed form:

$$\rho_o T_o \frac{dS_o}{dt} = \epsilon_o \rho_o - \vec{\nabla} \cdot \vec{F}_o$$

- perturbed form:

$$\begin{aligned} i[\omega + m\Omega] \rho_o T_o \delta S &= \epsilon_o \rho_o \left(\frac{\delta \epsilon}{\epsilon_o} + \frac{\delta \rho}{\rho_o} \right) - \vec{\nabla} \cdot \delta \vec{F} \\ &+ \vec{\xi} \cdot \vec{\nabla} (\vec{\nabla} \cdot \vec{F}_o) - \vec{\nabla} \cdot [(\vec{\xi} \cdot \vec{\nabla}) \vec{F}_o] \end{aligned}$$

$\delta \vec{F}$ = Lagrangian perturbation to the energy flux

δS = Lagrangian entropy perturbation

$\delta \epsilon$ = Lagrangian perturbation to the energy production

Pulsation equations

Energy flux

- total energy flux

$$\vec{F}_o = \vec{F}_o^R + \vec{F}_o^C$$

- unperturbed form of radiative energy flux:

$$\vec{F}_o^R = -\frac{4acT_o^3}{3\kappa_o\rho_o}\vec{\nabla}T_o = -\chi_o\vec{\nabla}T_o$$

- perturbed form of radiative energy flux:

$$\begin{aligned}\delta\vec{F}^R &= \left[(1 + \chi_T) \frac{\delta T}{T_o} + \chi_\rho \frac{\delta\rho}{\rho_o} \right] \vec{F}_o^R \\ &- \chi_o \left[T_o \vec{\nabla} \left(\frac{\delta T}{T_o} \right) + \vec{\xi} \cdot \vec{\nabla} \left(\vec{\nabla} T_o \right) - \vec{\nabla} \left(\vec{\xi} \cdot \vec{\nabla} T_o \right) \right]\end{aligned}$$

- frozen convection approximation:

$$\delta\vec{F}^C \simeq \vec{0}$$

Pulsation equations

Equation of state, opacities, and nuclear reaction rates

$$\begin{aligned}\frac{\delta P}{P_o} &= \Gamma_1 \frac{\delta \rho}{\rho_o} + P_T \frac{\delta S}{c_v} = P_\rho \frac{\delta \rho}{\rho_o} + P_T \frac{\delta T}{T_o} \\ \frac{\delta T}{T_o} &= \frac{\delta S}{c_v} + (\Gamma_3 - 1) \frac{\delta \rho}{\rho_o} = \frac{\delta S}{c_p} + \nabla_{\text{ad}} \frac{\delta P}{P_o} \\ \frac{\delta \chi}{\chi_o} &= \chi_\rho \frac{\delta \rho}{\rho_o} + \chi_T \frac{\delta T}{T_o} \\ \frac{\delta \epsilon}{\epsilon_o} &= \epsilon_T(\omega) \frac{\delta T}{T_o} + \epsilon_\rho(\omega) \frac{\delta \rho}{\rho_o}\end{aligned}$$

- in what follows we will neglect $\delta \epsilon$

Pulsation equations

Boundary conditions

- in the centre: regularity conditions
- at infinity: gravitational potential perturbation goes to zero
- at the surface:

$$\nabla_{\text{vert.}} \left(\frac{\delta P}{P_o} \right) = 0$$

$$4 \frac{\delta T}{T_o} = \frac{\delta F^R}{F_o^R}$$

Pulsation equations

Summary

- final result: a system of 10 equations with 10 unknowns:

$$\frac{\delta P}{P_o}, \quad \vec{\xi}, \quad \frac{\delta S}{c_p}, \quad \delta \vec{F}^R, \quad \frac{\delta T}{T_o}, \quad \psi \quad (1)$$

- although some of these variables can be cancelled algebraically, they are needed to ensure good convergence

Work integral

- it is possible to derive an integral expression for the complex frequencies:

$$A\omega^2 + 2B\omega + C = 0$$

where

$$A = \int_V \rho_0 \xi^2 dV,$$

$$B = \int_V \rho_0 \left[m\Omega \xi^2 - i\vec{\Omega} \cdot \left(\vec{\xi} \times \vec{\xi}^* \right) \right] dV,$$

$$\Re(C) = \text{a complicated expression}$$

$$\Im(C) = - \int_V \Im \left\{ \frac{\delta P \delta \rho^*}{\rho_0} \right\} dV$$

- From this we deduce the excitation rate:

$$\Im(\omega) = - \frac{\Im(C)}{2(A\Re(\omega) + B)}$$

Comparison with Lee & Baraffe (1995)

Lee & Baraffe (1995)	Current work
Model based on Chandrasekhar expansion	2D baroclinic model
Eulerian perturbations	Lagrangian perturbations
$\vec{F}^C = 0$	$\delta\vec{F}^C = 0$

Numerical implementation

- explicit expression in spheroidal coordinates
- projection onto spherical harmonics
- radial discretisation using Chebyshev polynomials

N_r	N_h	Memory (in Gb)	Time (in min)	Num. proc.
400	10	3.5		
400	15	7.9		
400	20	13.4	5	4
400	29	28.0	10	8
400	40	52.7	22	8
400	50	82.3	26	16

Estimated accuracy

- the problem is stiff: reduced numerical accuracy
- estimated accuracy based on variational expression:
 - frequencies: $\sim 10^{-4}$
 - excitation/damping rates: 10^{-2} to 10^{-1}

Description

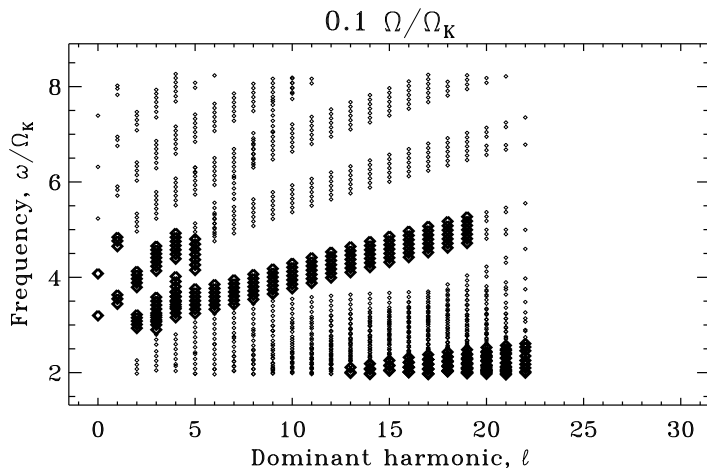
Model

- $9 M_{\odot}$ models
- $\Omega = 0.0$ to $0.8 \Omega_K$
- $z = 0.025$
- OPAL opacities

Modes

- β Cep type pulsations
- p and g modes
- excited by iron opacity bump at $\log(T) = 5.3$

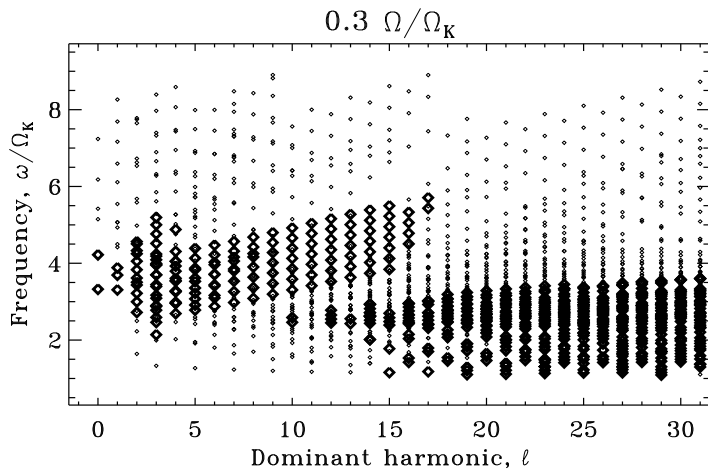
Frequencies



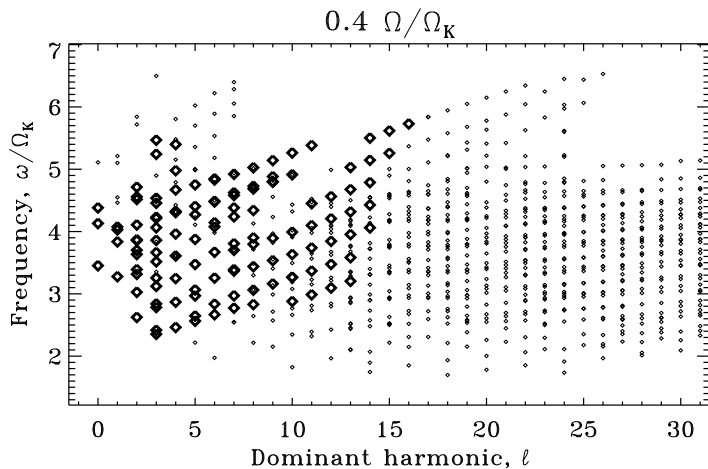
Frequencies



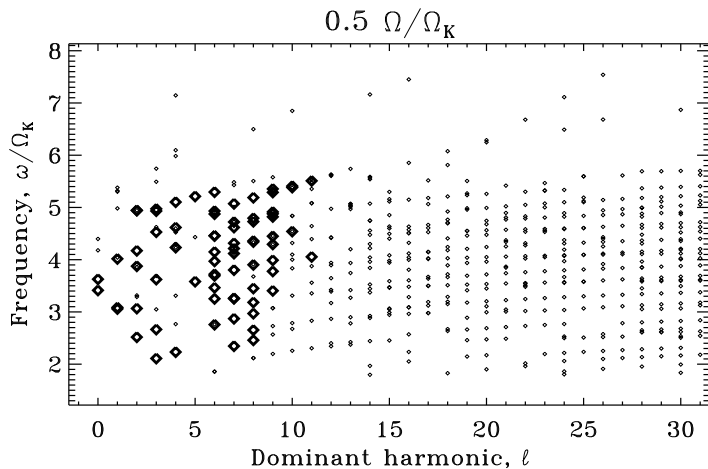
Frequencies



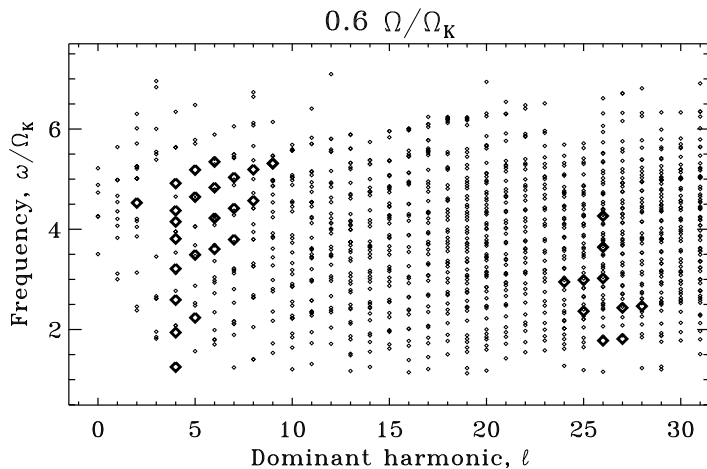
Frequencies



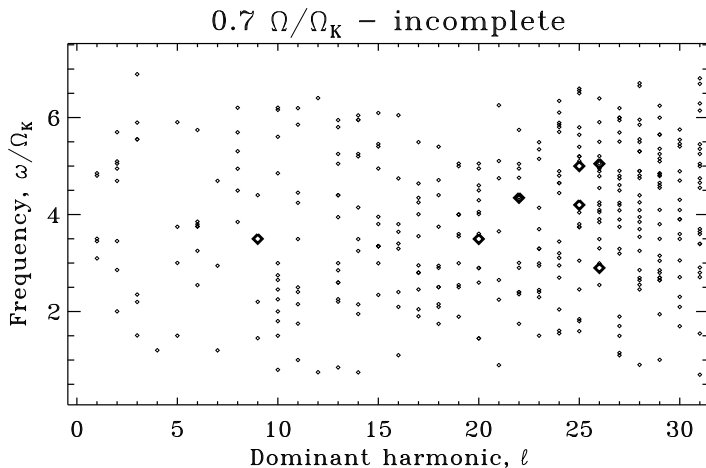
Frequencies



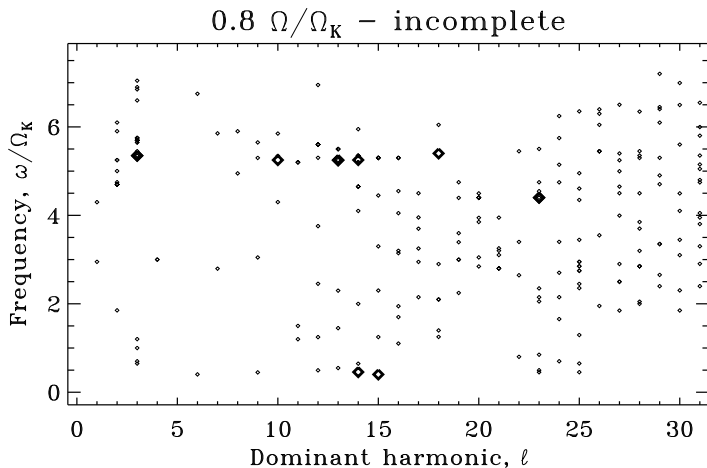
Frequencies



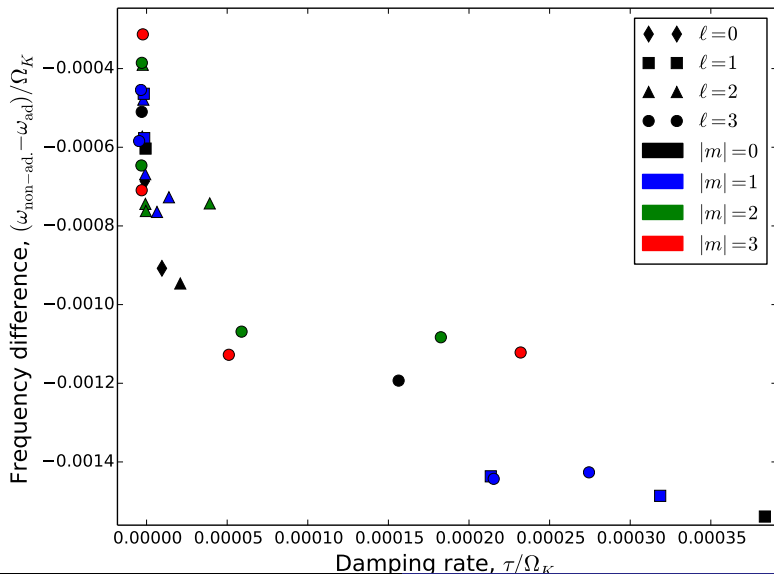
Frequencies

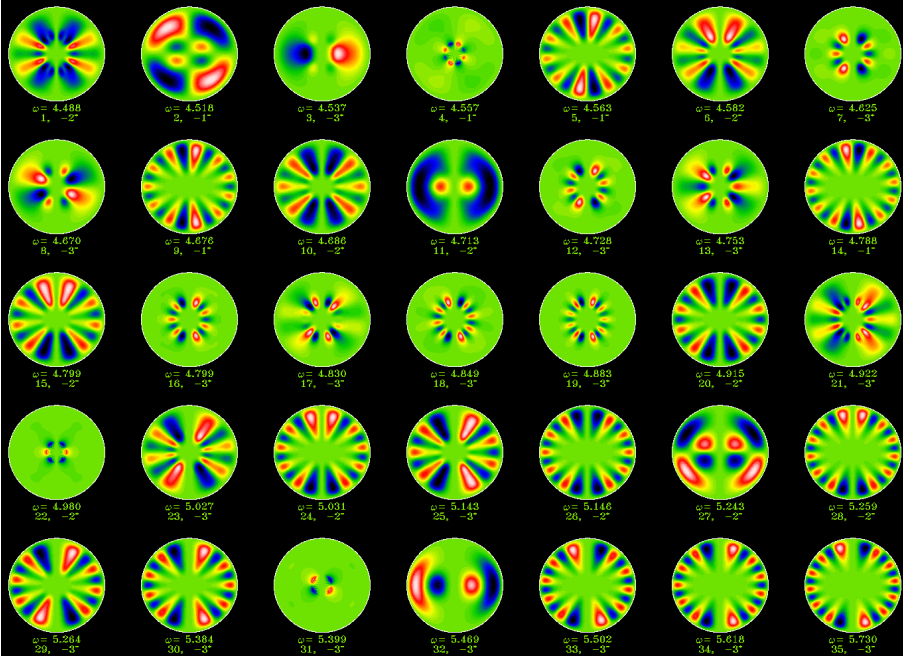


Frequencies

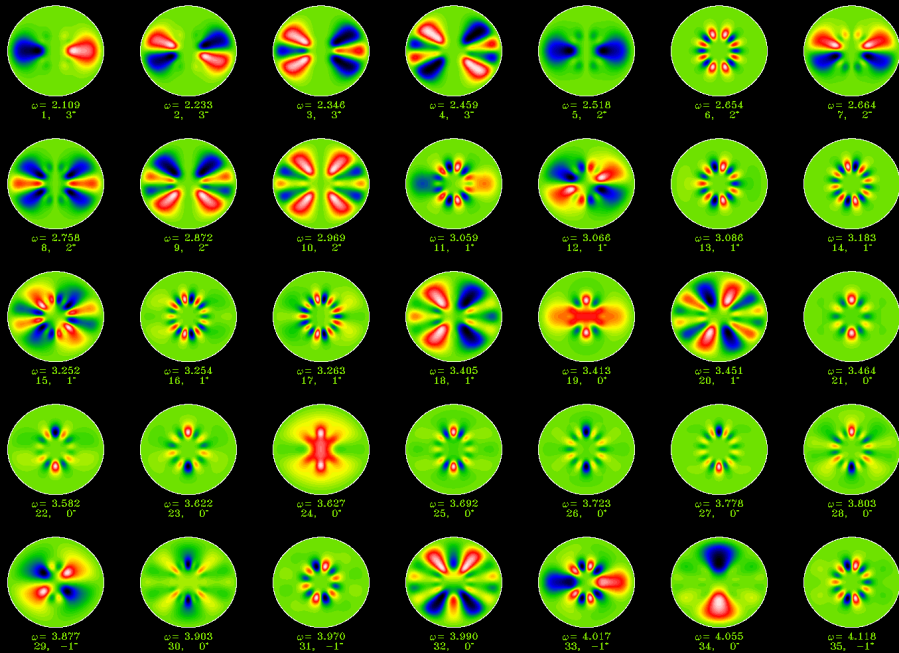


Damping rates

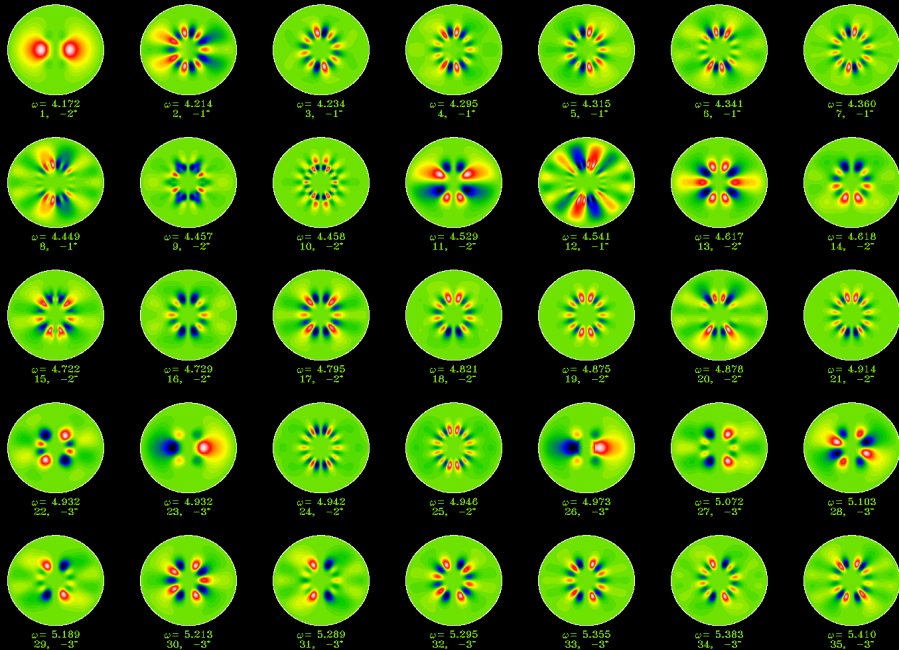




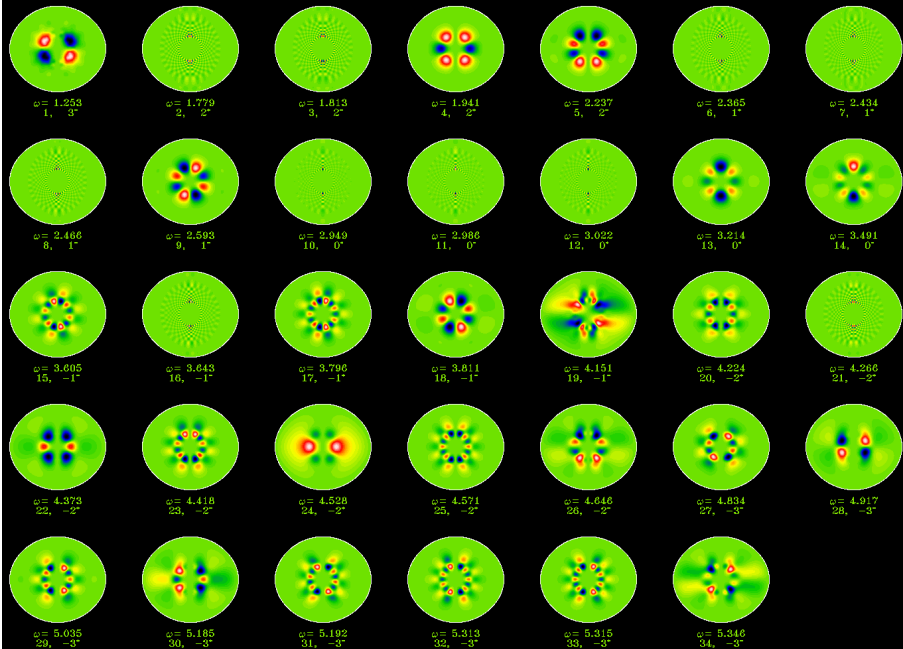
$$\Omega = 0.4 \Omega_k$$



$$\Omega = 0.5 \Omega_k$$

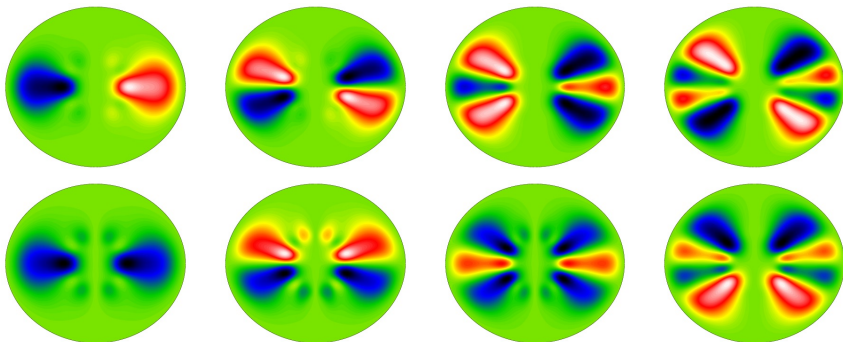


$$\Omega = 0.5 \Omega_k$$

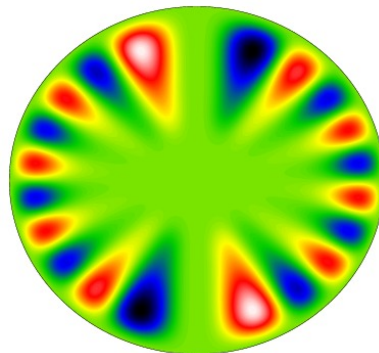
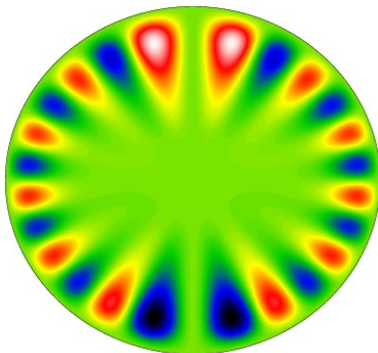


$$\Omega = 0.6 \Omega_k$$

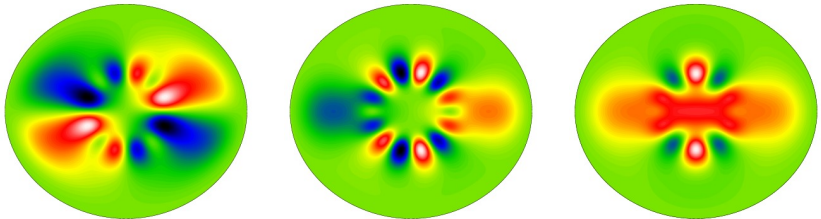
Island modes



Whispering gallery modes

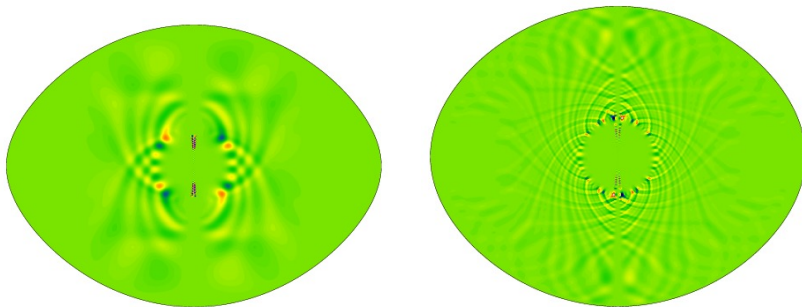


Mixed modes



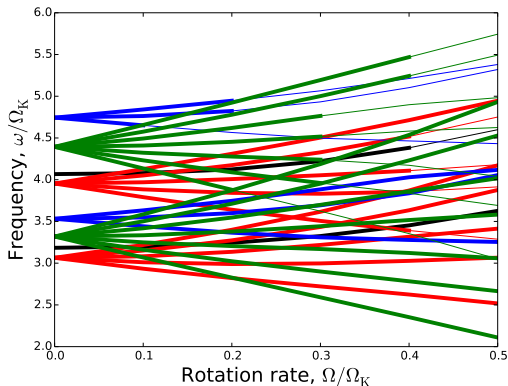
- see Ouazzani et al. (2015) for mixed modes in the adiabatic case

Rosette modes



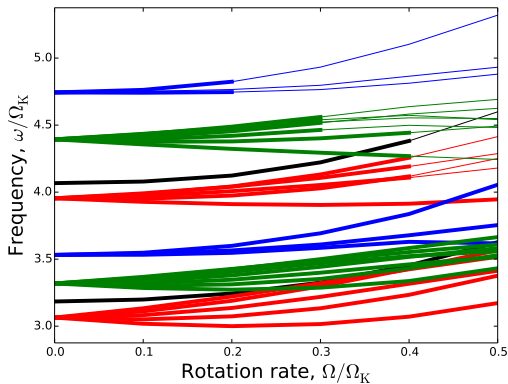
- also see Takata & Saio (2015) for non-adiabatic effects on Rosette modes and associated angular momentum transport

Multiplets – inertial frame



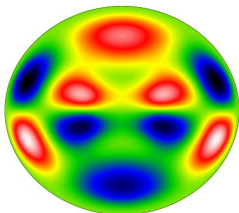
- prograde modes remain unstable longer
- Lee (2008) also found a preference for prograde modes

Multiplets – corotating frame

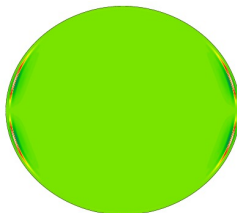


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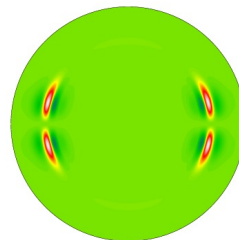
Work integral



Acoustic



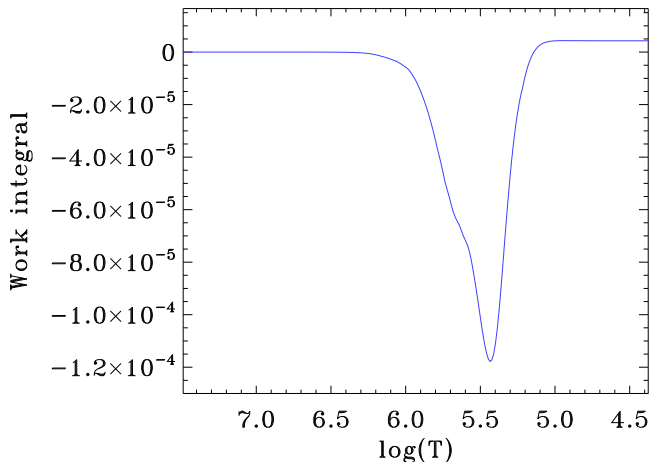
Work



Work (vs. $\log T$)

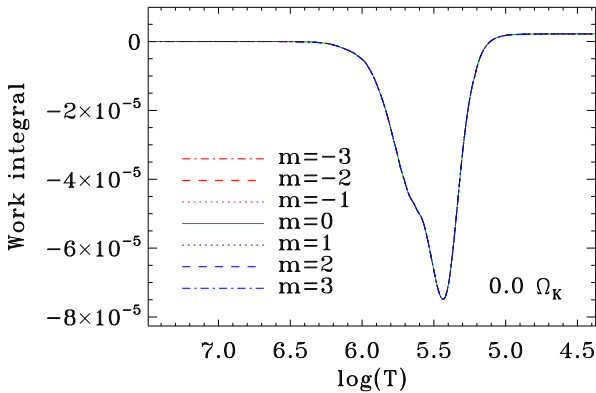
- red = driving regions
- blue = damping regions

Work integral



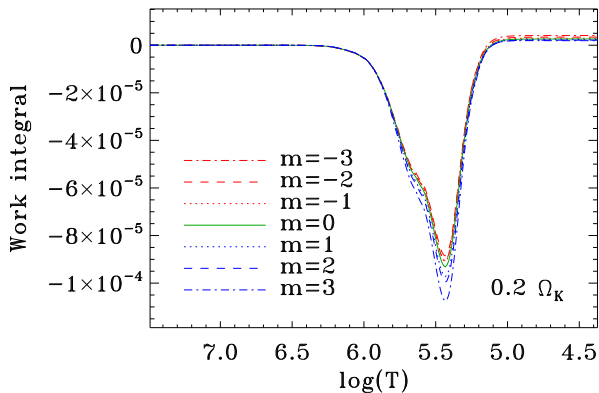
- obtained by integrating in horizontal direction + vertical anti-derivative

A multiplet



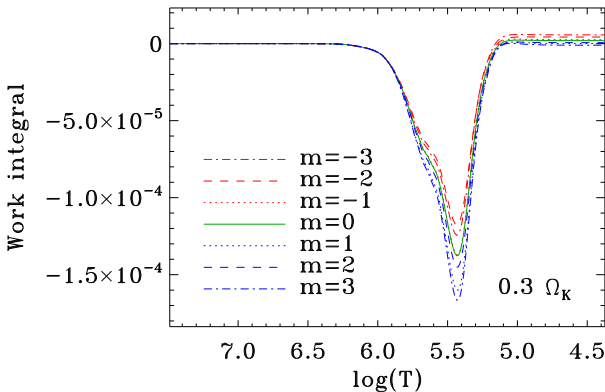
- rotation rate = 0.0 $\Omega_K, \varepsilon = 0$

A multiplet



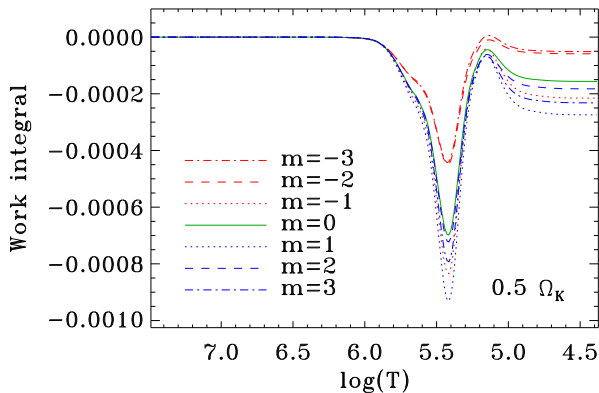
- rotation rate = $0.2 \Omega_K$, $\varepsilon = 1.9 \times 10^{-2}$

A multiplet



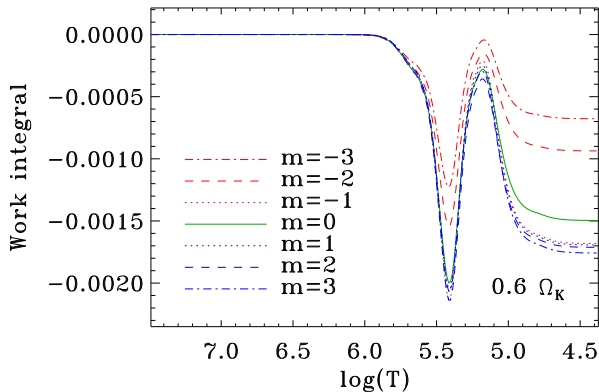
- rotation rate = $0.3 \Omega_K$, $\varepsilon = 4.3 \times 10^{-2}$

A multiplet



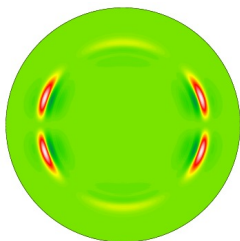
• rotation rate = $0.5 \Omega_K$, $\varepsilon = 11.2 \times 10^{-2}$

A multiplet

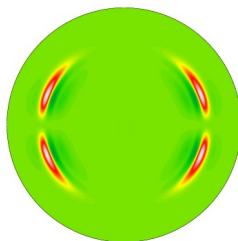


• rotation rate = $0.6 \Omega_K$, $\varepsilon = 15.5 \times 10^{-2}$

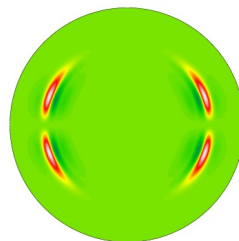
A multiplet



$m = 0$
Stable



$m = -2$
Excited



$m = 2$
Stable

- rotation rate = $0.4 \Omega_K$, $\varepsilon = 7.4 \times 10^{-2}$

Amplitude ratios

Previous works

- Daszyńska-Daszkiewicz et al. (2002, 2007), Townsend (2003)
 - non-adiabatic treatment
 - approximate treatment of rotation
- Reese et al. (2013) (see also Lignières et al. 2006, Lignières & Georgeot 2009)
 - full treatment of rotation
 - adiabatic calculations

Equations

- non-pulsating star:

$$\mathcal{I} = \iint_{\text{Vis.Surf.}} I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs.}} \cdot d\vec{S}$$

- pulsating star:

$$\begin{aligned} \delta\mathcal{I} &= \iint_{\delta\text{Vis.Surf.}} I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs.}} \cdot d\vec{S} \\ &+ \iint_{\text{Vis.Surf.}} \delta I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs.}} \cdot d\vec{S} \\ &+ \iint_{\text{Vis.Surf.}} I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs.}} \cdot \delta(d\vec{S}) \end{aligned}$$

Equations

First term

$$\iint_{\delta S} \dots \vec{e}_{\text{obs.}} \cdot d\vec{S} \propto \xi^2 \Rightarrow \text{negligible}$$

Second term

$$\delta I = I \cdot \left(\frac{\partial \ln I}{\partial \ln T_{\text{eff}}} \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} + \frac{\partial \ln I}{\partial \ln g_{\text{eff}}} \frac{\delta g_{\text{eff}}}{g_{\text{eff}}} \right) + \frac{\partial I}{\partial \mu} \delta \mu$$

- $\frac{\delta T_{\text{eff}}}{T_{\text{eff}}}$, $\frac{\delta g_{\text{eff}}}{g_{\text{eff}}}$, and $\delta \mu$ are deduced from the pulsation mode
- see next slide for I and its derivatives

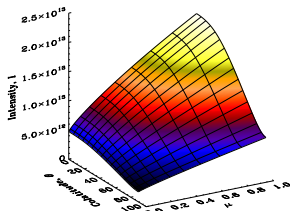
Third term

- $\delta(d\vec{S})$ is deduced from the Lagrangian displacement

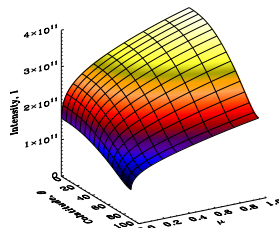
Intensities

$$I(T_{\text{eff}}, g_{\text{eff}}, \mu) = I_0(T_{\text{eff}}, g_{\text{eff}})h(\mu, T_{\text{eff}}, g_{\text{eff}})$$

- $I_0(T_{\text{eff}}, g_{\text{eff}})$ from blackbody spectrum
- $h(\mu, T_{\text{eff}}, g_{\text{eff}})$ from Claret (2000)
- bolometric, Strömgren, and Johnson-Cousins photometric bands

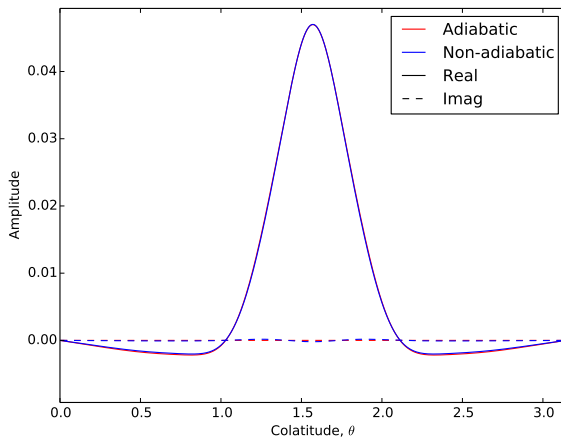


Bolometric

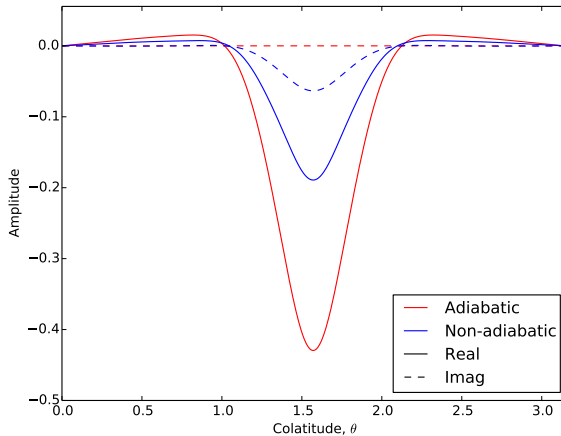


Strömgren, u

Various profiles

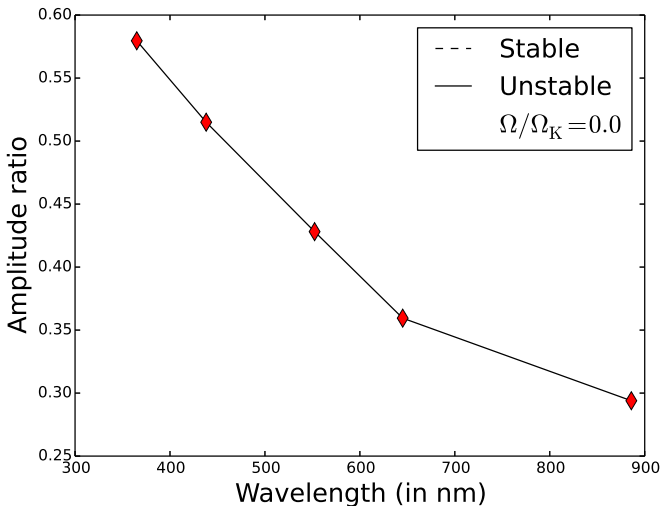
 ξ_r

Various profiles

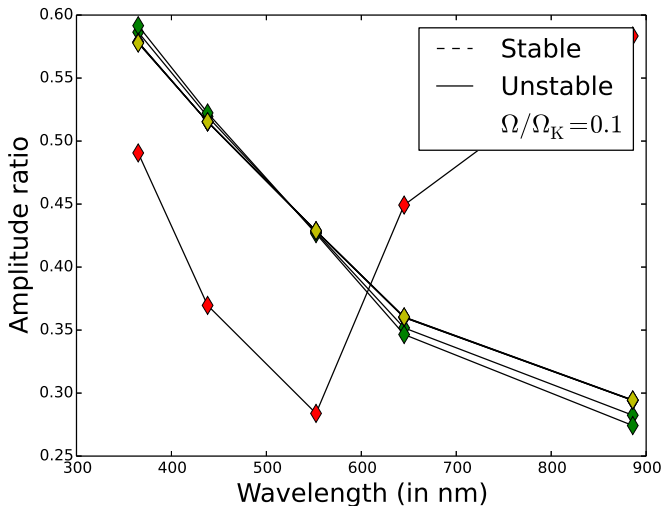


$$\delta T_{\text{eff}} / T_{\text{eff}}$$

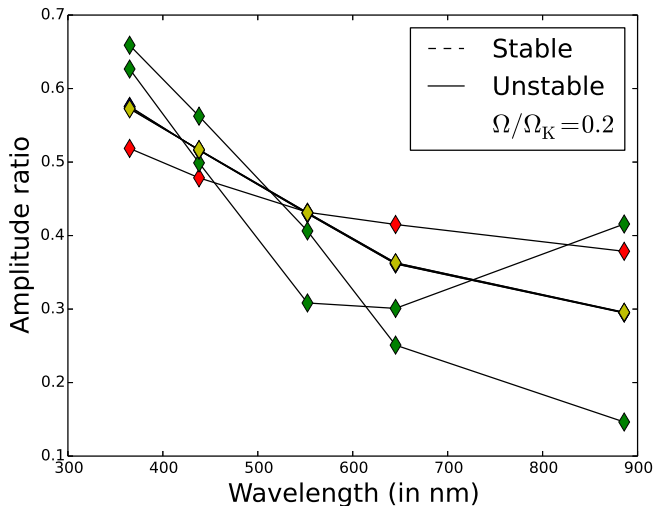
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



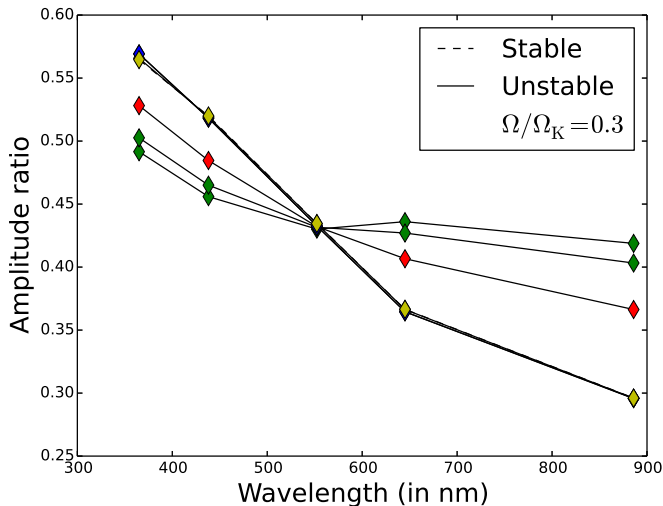
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



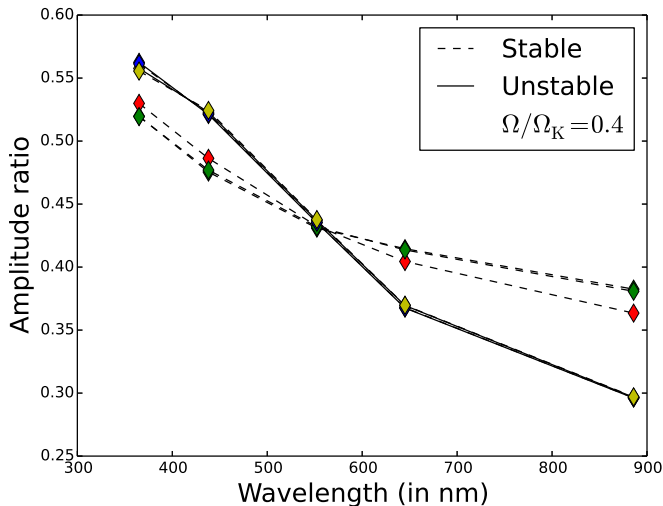
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



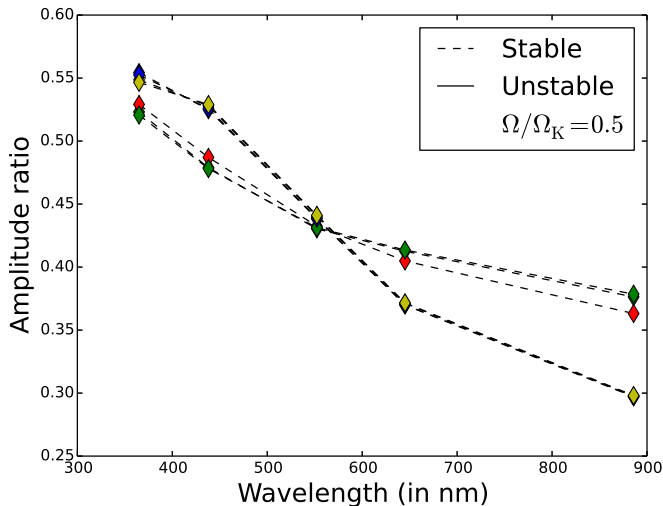
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



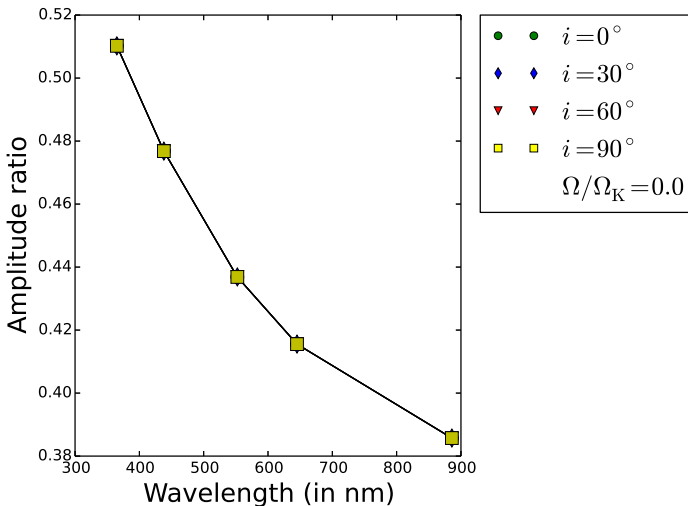
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



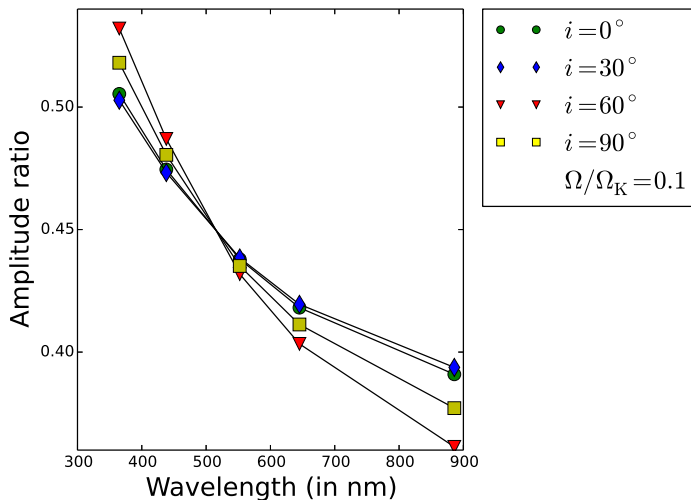
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



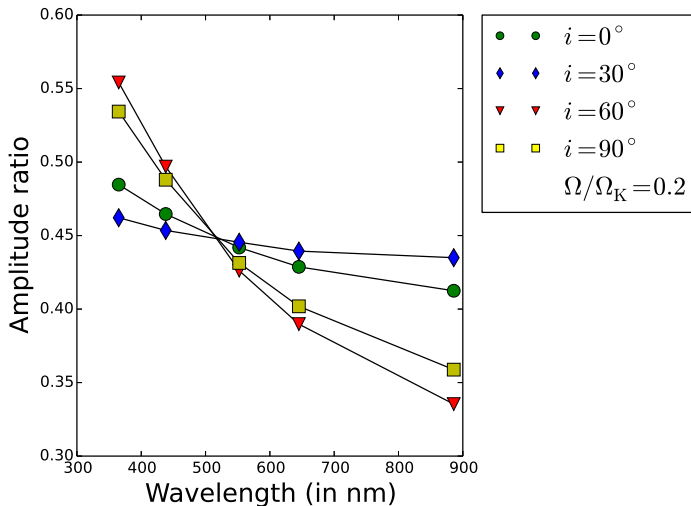
Amplitude ratios for an ($\ell = 2, m = 0$) mode



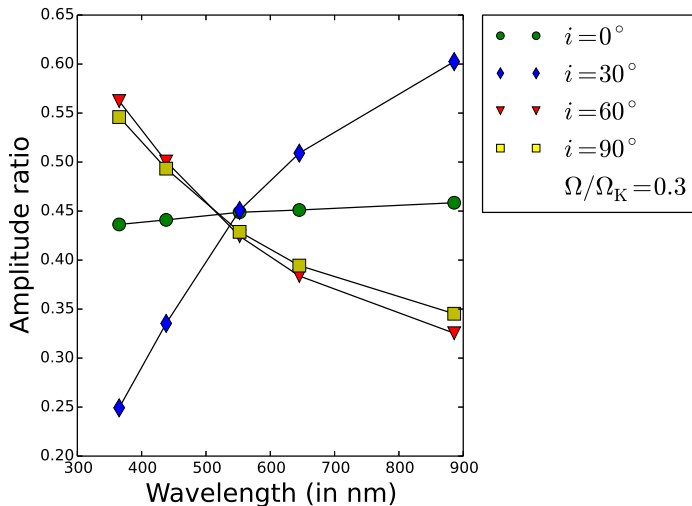
Amplitude ratios for an ($\ell = 2, m = 0$) mode



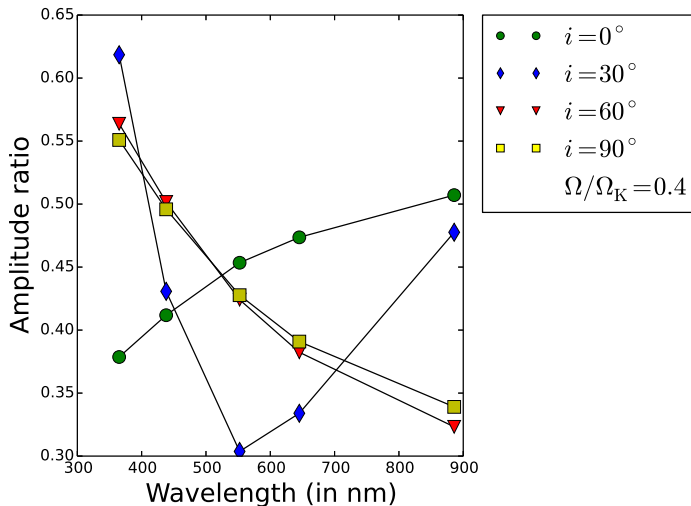
Amplitude ratios for an ($\ell = 2, m = 0$) mode



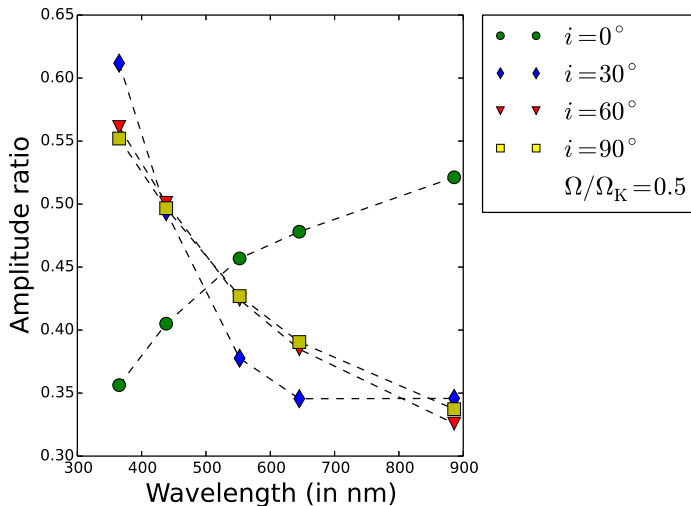
Amplitude ratios for an ($\ell = 2, m = 0$) mode



Amplitude ratios for an ($\ell = 2, m = 0$) mode



Amplitude ratios for an ($\ell = 2, m = 0$) mode



Line Profile Variations (LPVs)

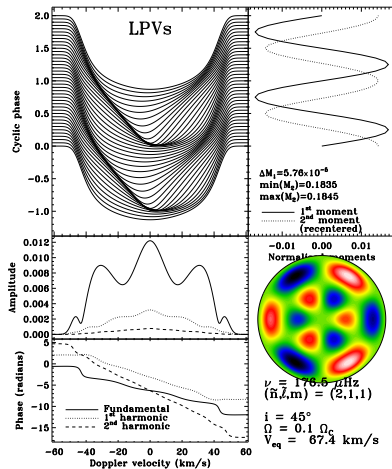
Previous works

- Clement (1994): 2D calculations
- Townsend (1997): the traditional approximation

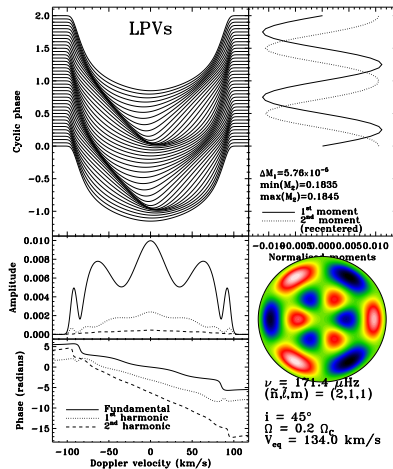
Description

- includes Doppler shifts and $\delta(\vec{dS})$
- δT_{eff} and δg_{eff} neglected
- use of blackbody spectrum (incl. gravity darkening)
- rudimentary description of limb darkening

Increasing rotation rates

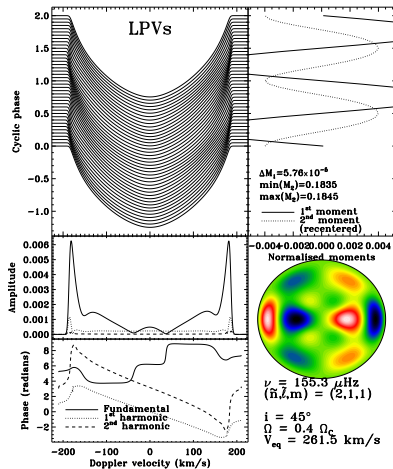


Increasing rotation rates



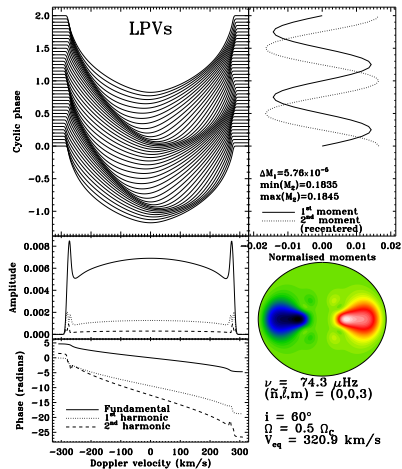
Non-adiabatic pulsations in ESTER models

Increasing rotation rates

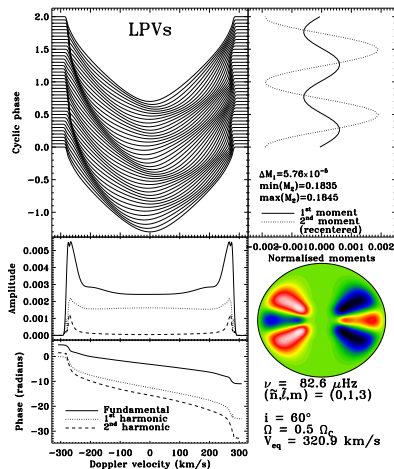


Non-adiabatic pulsations in ESTER models

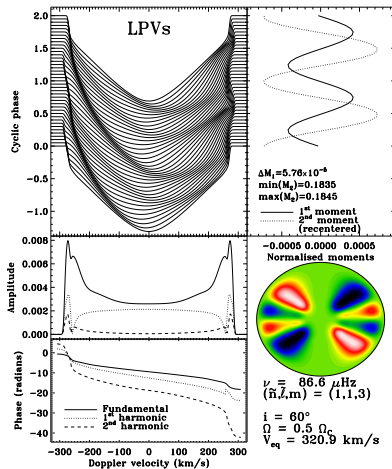
Increasing ℓ value



Increasing l value



Increasing l value



Conclusion

- important step forward:
 - can now predict which modes are unstable
 - can calculate amplitude ratios and LPVs

Prospects

- understand how rotation (de)stabilise modes
- what are the differences between prograde and retrograde modes
- include more realistic atmosphere
- identify modes