Searching for Lorentz Invariance Violation in Theories with Modified Dispersion Relations

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Lorentz Invariance Violation & Modified Dispersion Relations

- Metric: Choose coords. s.t. $g^{\alpha\beta} = \eta^{\alpha\beta}$

$$S = \frac{1}{2}\int \sqrt{-g}d^4x \left[g^{\alpha\beta} \partial_\alpha\phi\partial_\beta\phi + \left(g^{\alpha\beta} + \tau^{\alpha\beta} \right) \partial_\alpha\psi\partial_\beta\psi \right]$$

- Symmetric tensor $\neq \eta^{\alpha\beta}$

Modified Dispersion Relations

$$p^2 = E^2 \;\to\; p^2 = E^2 \left[1 + f(E/E_{LV})\right]$$

$$f(E) \approx \chi_1 (E/E_{LV})^1 + \chi_2 (E/E_{LV})^2 + \mathcal{O}\left(E^3/E_{LV}^3\right)$$

General form of MDR

$$p^2 \approx E^2 \left[1 + \chi_n \left(\frac{E}{E_{LV}}\right)^n\right], \quad n = 1, 2$$

$$\Delta t = \frac{D_L}{c}\left(\frac{E}{E_{LV,n}}\right)^n$$

Limits on $E_{LV,n}$: $7.62 \cdot 10^{16}$ GeV ($n = 1$), $2.69 \cdot 10^{9}$ GeV ($n = 2$)

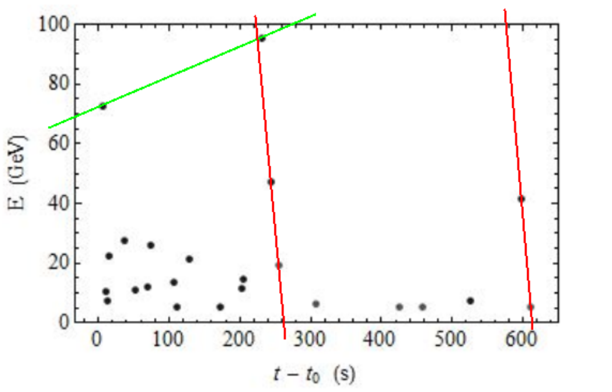
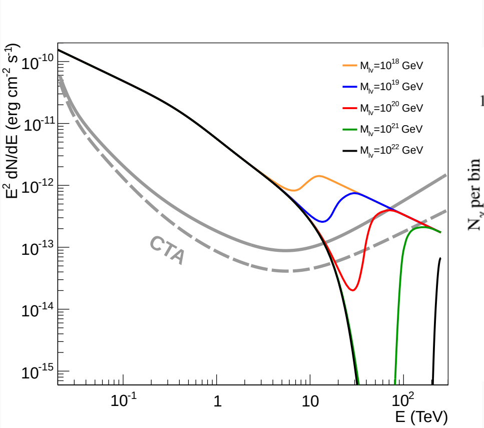


Figure 1: G.Amelino-Camelia, F.Fiore, D.Guetta, S.Puccetti, 2014

Cherenkov Telescope Array

Modified mean-free path: $\gamma\gamma \rightarrow e^+e^-$

$$x_{\gamma\gamma}^{-1} = \frac{1}{8E_\gamma^2\beta_\gamma}\int_{\epsilon_{min}}^{\infty} d\epsilon \frac{n(\epsilon)}{\epsilon^2}\int_{s_{min}}^{s_{max}(\epsilon,E_\gamma)} ds\,(s - m_\gamma^2 c^4)\,\sigma(s)$$



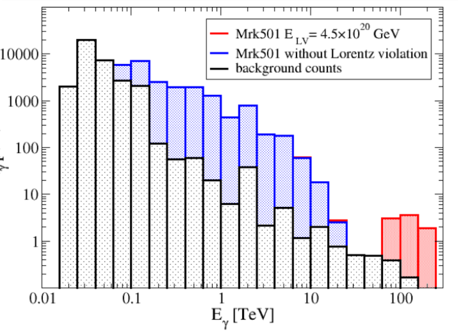


Figure 2,3: M.Fairbairn, NAN, J.Ellis, 2014

Lorentz Violation & the Equivalence Principle

- Modified Dispersion Relation:

$$E^2 = m^2 + p^2 + \frac{f^{(4)}}{E_{LV}}|p|^4$$

- Non-relativistic particle in a weak gravitational field:

$$H = m + \frac{p^2}{2m} + f^{(4)}\frac{p^4}{2mE_{LV}^2} + m\Phi(x)$$

- Hamilton's equations yields

$$\ddot{x}^i = -\frac{\partial\Phi}{\partial x^i}\left(1 + 6f^{(4)}\frac{m^2\dot{x}^i\cdot\dot{x}^i}{E_{LV}}\right)$$

Doubly Special Relativity & Rainbow Gravity

MDR from a Lorentz Violating model:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_0}{3}\frac{h_\pm^2(E)}{f_1^2(E)}\rho_c\left[\Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_k\left(\frac{a_0}{a}\right)^2\frac{1}{h_\pm^2(E)}\right]$$

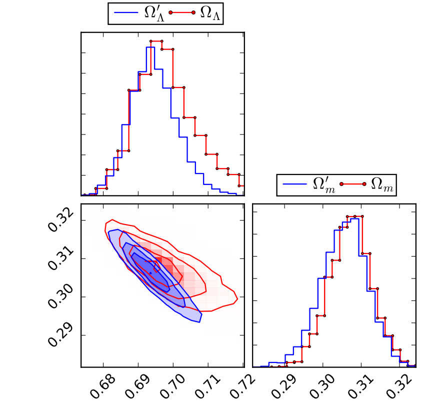


Figure 4: NAN, M.P.Dąbrowski, 2017

Thank you!