

Cosmology with Supernovae

Lecture 2

Bruno Leibundgut

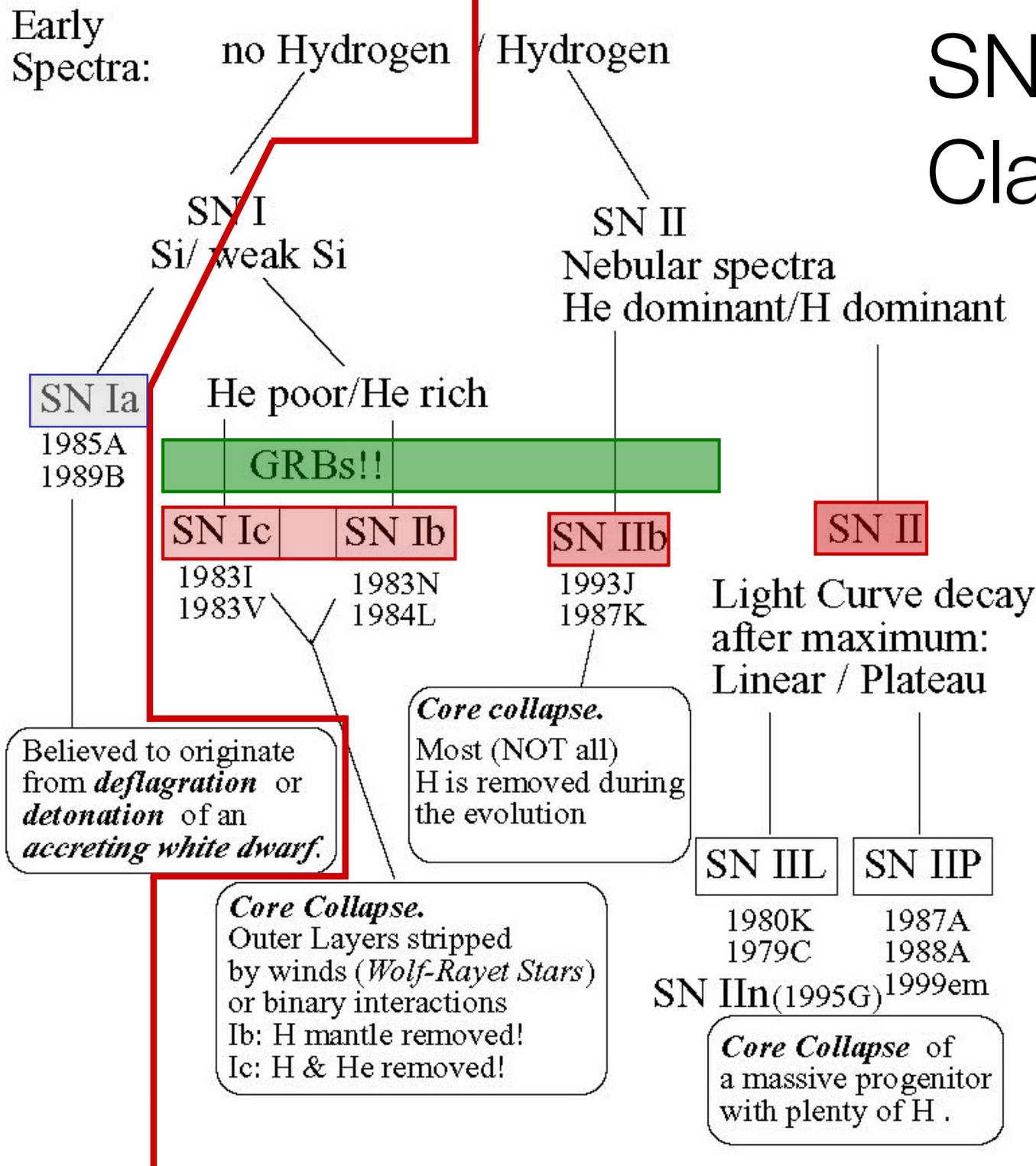
Programme

- Lecture 1
 - Hubble Constant
 - Importance of H_0
 - Measurements of H_0
 - local → distance ladder
 - global → gravitational lensing, cosmic microwave background
 - H_0 today and tomorrow

Programme

- Lecture 2
 - Tests of General Relativity
 - Expansion
 - time dilation
 - Distance duality
 - relation between luminosity distance and angular size distance
 - Cosmological parameters
 - Evidence for acceleration
 - Future of SN cosmology

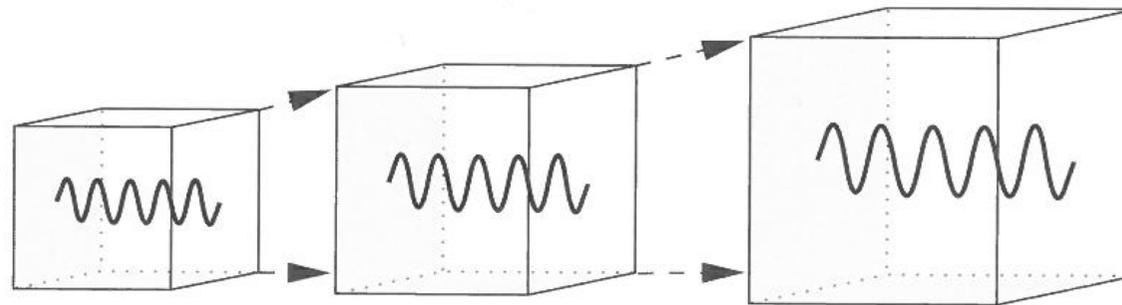
SN Classification



Time Dilation

In an expanding universe the time appears dilated for a distant object

- redshifts!



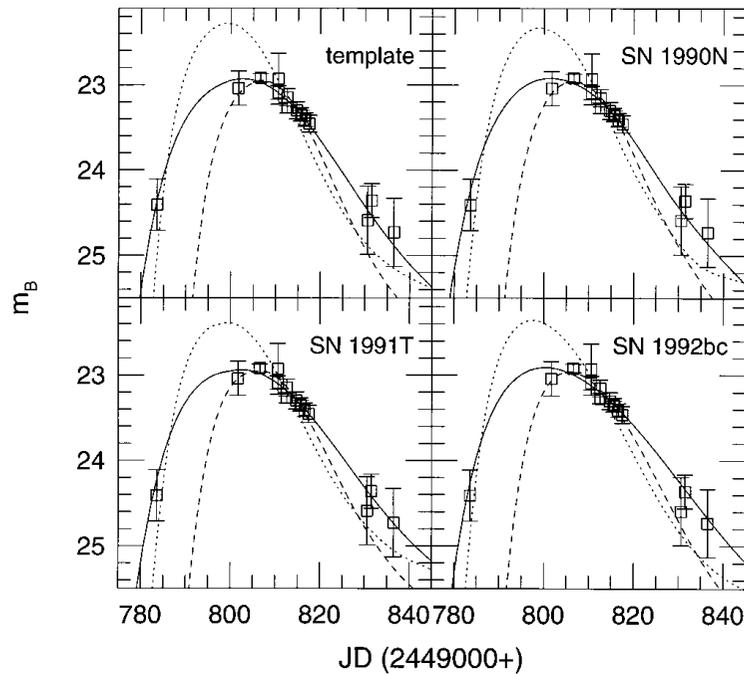
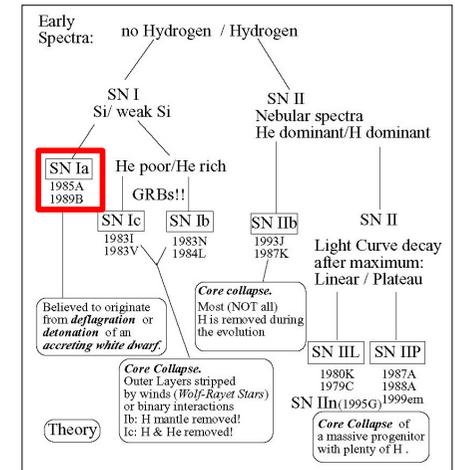
Find a clock ticking at a significant redshift

→ light curve of a type Ia supernova

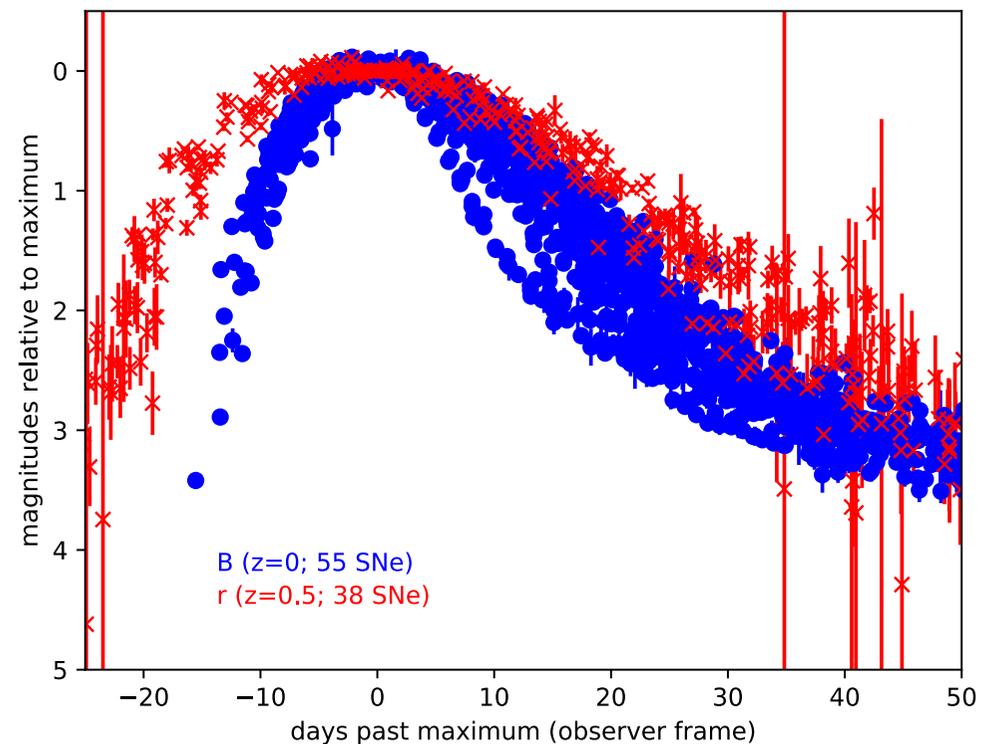
Time Dilation in SNe Ia

Uniform light curve shapes in a given filter

→ Distant supernovae should show a 'slower' light curve



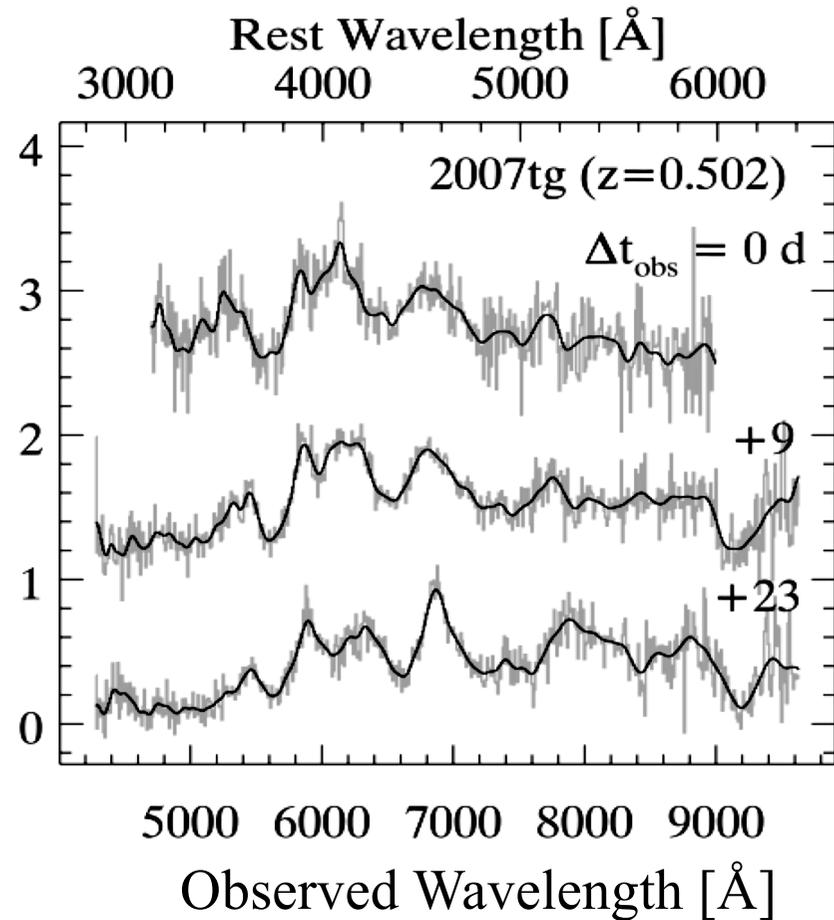
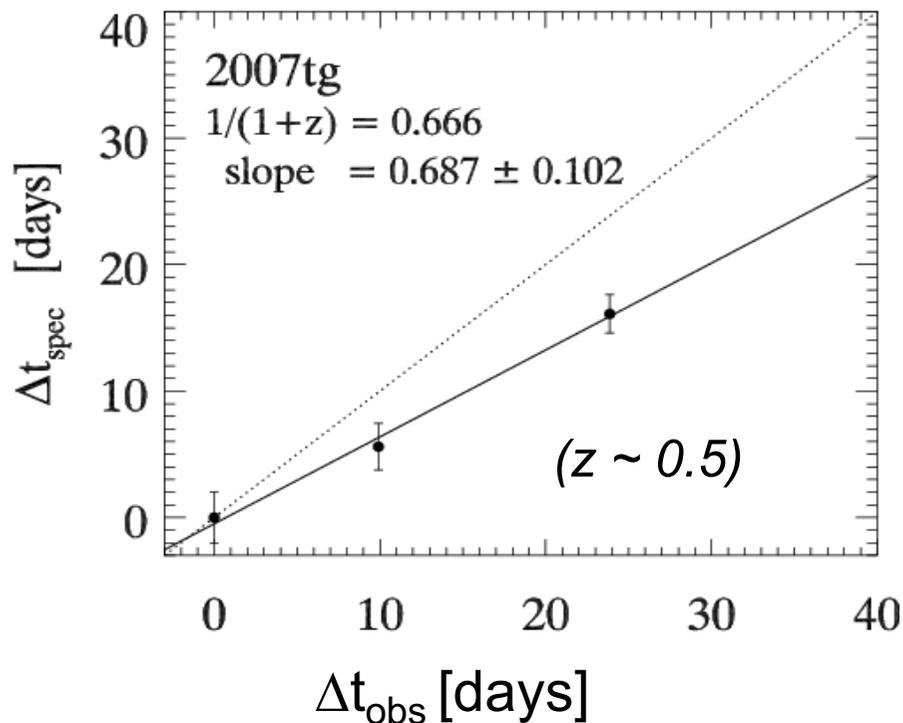
Leibundgut et al. 1996



Bruno Leibundgut

Time dilation

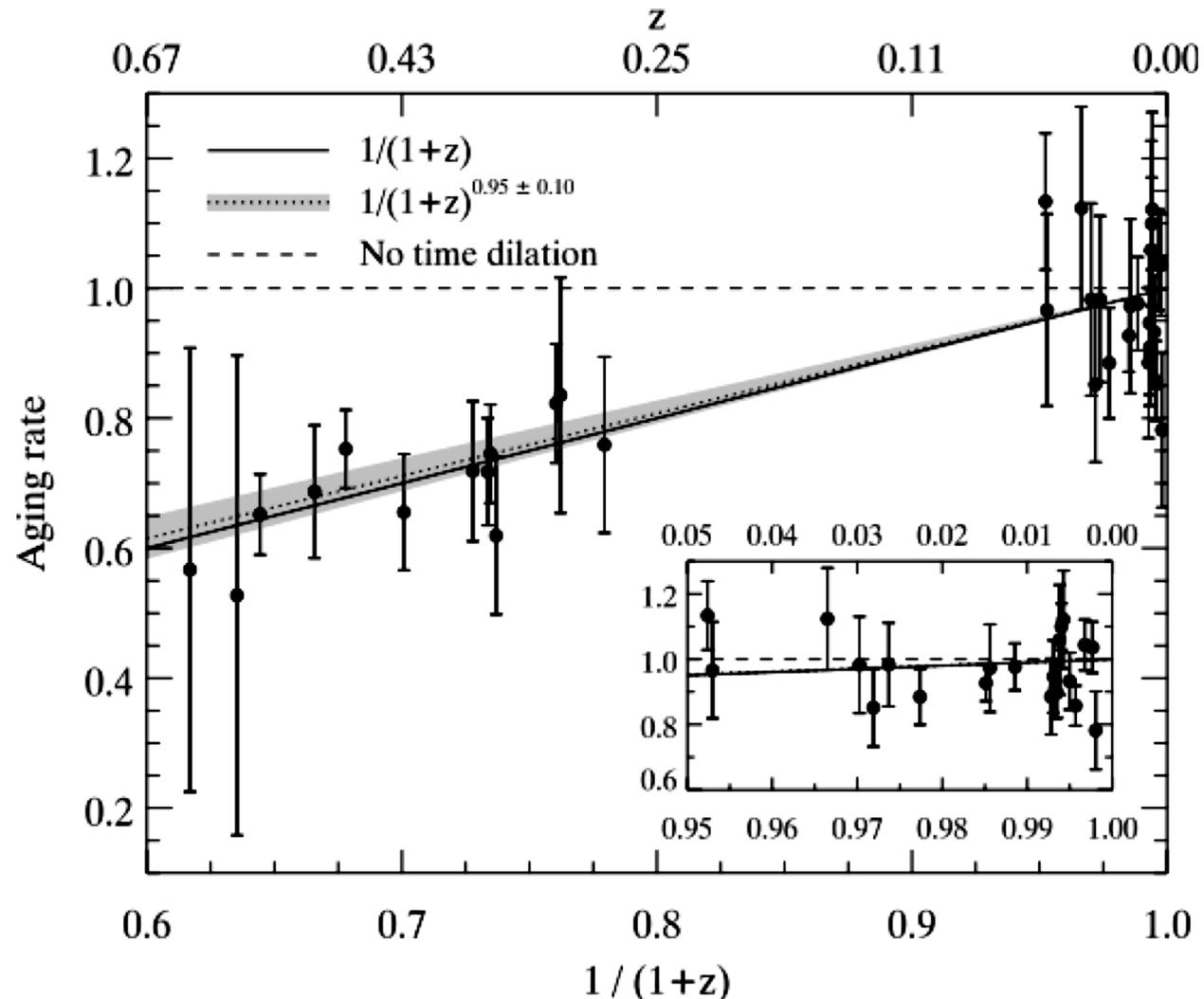
Spectroscopic clock in the distant universe



Blondin et al. (2008)

Time Dilation

'Tired Light' can be excluded beyond doubt ($\Delta\chi^2=120$)



Blondin et al. (2008)

Luminosity Distances

- The rate of the photon arrivals is reduced by a factor $\frac{a(t_1)}{a(t_0)} = \frac{1}{1+z}$ and the energy of the photons ($E = h\nu$) is also reduced by a factor $(1 + z)$ (remember luminosity L is energy per time)

$$l = \frac{L}{4\pi x_1^2 a^2(t_0)(1+z)^2}$$

- Set $D_L = x_1 a(t_0)(1+z)$ and we recover the equation for the luminosity distance $l = \frac{L}{4\pi D_L^2}$

Angular size distance

- A different method is to measure the angle of a distant object of known size $D_A = \frac{l}{\theta}$ (here l is the size of the object; θ the observed angle)
- Inspection of the metric (here we only need the $g_{\theta\theta}$ part), which gives $l = x_1 a(t_1)\theta$ and inserting this in the equation above yields $D_A = x_1 a(t_1)$ and with $\frac{a(t_1)}{a(t_0)} = \frac{1}{1+z}$ we find $\frac{D_L}{D_A} = (1+z)^2$.

Distance Duality

This is quite remarkable for high redshifts

- the physical distances differ for the same redshift!
- an object for which we could measure the angular size distance and the luminosity distance would give a different number of Mpc!
- a direct consequence of general relativity

$$\frac{D_L}{D_A} = (1 + z)^2$$

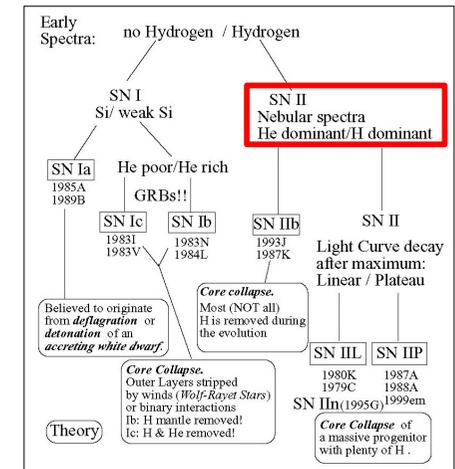
z	$\frac{D_L}{D_A}$
0.1	1.21
0.15	1.32
0.2	1.44
0.25	1.56
0.3	1.69
0.35	1.82

Distance Duality

- Now measured in several systems
 - galaxy clusters
 - Sunyaev-Zeldovich effect
 - gravitational lenses
- Type II Supernovae
 - use two different methods to the same object
 - Expanding Photosphere Method
 - equates luminosity distance with angular size distance
 - Standardizable Candle Method
 - pure luminosity distance

Distance to SN 2013eq ($z=0.041$)

- Use EPM and CSM to measure distance to same supernova
- EPM provides explosion date to be used by CSM



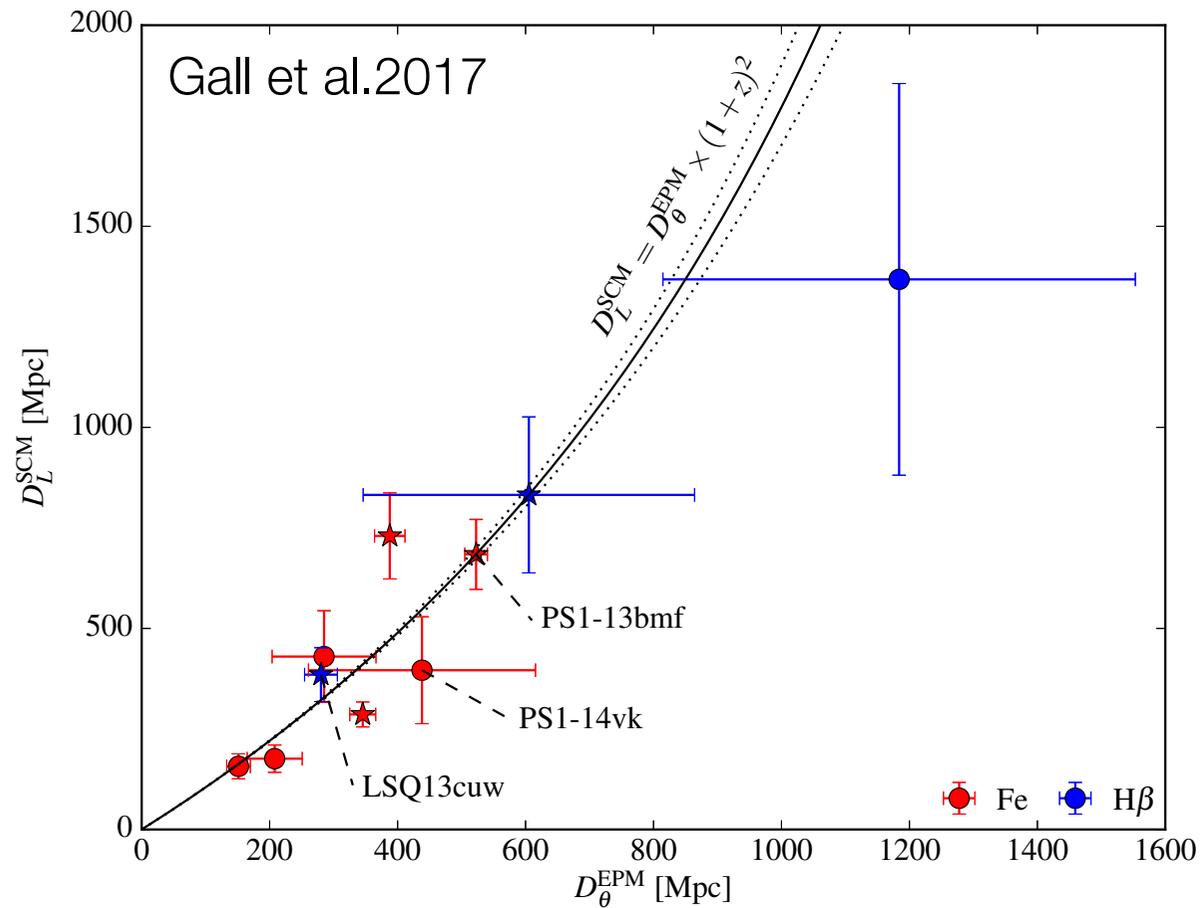
Gall et al. 2016

Dilution factor	Filter	D_L Mpc	Averaged D_L Mpc	t_0^* days*	Average t_0^* days*	t_0^\diamond MJD
H01	<i>B</i>	163 ± 45	151 ± 18	5.8 ± 10.5	4.1 ± 4.4	$56\,499.6 \pm 4.6$
	<i>V</i>	125 ± 22		-0.5 ± 5.4		
	<i>I</i>	165 ± 23		7.1 ± 6.0		
D05	<i>B</i>	177 ± 48	164 ± 20	4.7 ± 9.8	3.1 ± 4.1	$56\,500.7 \pm 4.3$
	<i>V</i>	136 ± 23		-1.3 ± 5.1		
	<i>I</i>	180 ± 25		5.9 ± 5.6		

Estimate of t_0 via	t_0^\diamond MJD	V_{50}^* mag	I_{50}^* mag	v_{50} km s ⁻¹	μ mag	D_L Mpc
EPM – H01	$56\,499.6 \pm 4.6$	19.05 ± 0.09	18.39 ± 0.04	4880 ± 760	36.03 ± 0.43	160 ± 32
EPM – D05	$56\,500.7 \pm 4.3$	19.06 ± 0.09	18.39 ± 0.04	4774 ± 741	35.98 ± 0.42	157 ± 31
Rise time – G15	$56\,496.6 \pm 0.3$	19.03 ± 0.05	18.39 ± 0.04	5150 ± 353	36.13 ± 0.20	168 ± 16

Distance Duality

First attempts inconclusive



Cosmological Parameters

Map the expansion history of the universe

- Type Ia supernovae provide the accurate *relative* distances
- Measurement independent of H_0
 - assumes no luminosity evolution of SNe Ia over time

Completely within the framework of FRW models

The Energy-Momentum Tensor

- Use the form for the ‘perfect fluid’

$$T^{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

The energy conservation requires that the covariant derivative

$$0 = T^{\mu\nu}_{;\mu} = \frac{\partial T^{0\mu}}{\partial x^\mu} + \Gamma^0_{\mu\nu} T^{\nu\mu} + \Gamma^\mu_{\mu\nu} T^{0\nu} = \frac{\partial T^{00}}{\partial t} + \Gamma^0_{ij} + \Gamma^i_{i0} T^{00} = \frac{c^2 d\rho}{dt} + 3 \frac{\dot{a}}{a} (p + \rho c^2)$$
$$c^2 \dot{\rho} + 3 \frac{\dot{a}}{a} (p + \rho c^2) = 0$$

Energy-Momentum Tensor

- A general form is an equation of state $p = \omega \rho c^2$. ω is the equation of state parameter.
- Inserting this into the conservation equation gives $\frac{\dot{\rho}}{\rho} = -3(1 + \omega) \frac{\dot{a}}{a}$ which integrates to $\log(\rho) = -3(1 + \omega) \log(a) + \text{const.}$
- Exponentiating yields $\rho \propto a^{-3(1+\omega)}$

Energy-Momentum Tensor

- The time (00) component of the Einstein equations is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\rho c^2 + 3p)$$

- As long as pressure and density are positive the universe decelerates $\ddot{a} < 0$.
- Acceleration requires $\rho c^2 + 3p < 0$ or $\omega < -\frac{1}{3}$.

Matter

- The pressure in matter is negligible compared to the mass content (think mc^2) and hence $\omega = 0$
- Thus $\rho_{matter} \propto a^{-3}$
- Inserting this in the Friedmann equation for a flat universe ($k=0$) provides the time dependence of the scale factor

$$a(t) \propto t^{\frac{2}{3}}$$

Radiation

- Radiation decreases with the volume (i.e. number of photons), but has one additional factor due to the redshift $\omega = \frac{1}{3}$ and hence $\rho_{rad} \propto a^{-4}$
- The time dependence here is now
$$a(t) \propto \sqrt{t}$$

Vacuum energy

- A special case is $\rho_{vacuum} = const.$
- In this case the density is associated to the vacuum
- Now the scale factor grows exponentially

$$a(t) \propto e^{Ht}$$

Friedmann equation (last time)

- We can put the various densities into the Friedmann equation

$$\frac{\dot{a}^2}{a^2} = H^2 = \frac{8\pi G}{3} \rho(t) = \frac{8\pi G}{3} (\rho_{matter} + \rho_{rad} + \rho_{vac}) - \frac{k}{a^2}$$

- We can define the critical density for a flat universe ($k = 0$) $\rho_{crit} = \frac{3H^2}{4\pi G}$ we can define the ratio to the critical density $\Omega = \frac{\rho}{\rho_{crit}}$

- Most compact form of Friedmann equation

$$1 = \Omega_{matter} + \Omega_{rad} + \Omega_{vac} + \Omega_k \text{ with } \Omega_k = -\frac{k}{a^2 H^2}$$

Dependence on scale parameter

For the different contents there were different dependencies for the scale parameter

$$\rho_{matter} \propto a^{-3}; \rho_{rad} \propto a^{-4}; \rho_{\Lambda} = const.$$

Combining this with the critical densities we can write the density as

$$\rho = \frac{3H_0^2}{8\pi G} \left[\Omega_{matter} \left(\frac{a_0}{a} \right)^3 + \Omega_{rad} \left(\frac{a_0}{a} \right)^4 + \Omega_{\Lambda} \right]$$

and the Friedmann equation

$$H^2 = H_0^2 [\Omega_{matter}(1+z)^3 + \Omega_{rad}(1+z)^4 + \Omega_{\Lambda} + \Omega_k(1+z)^2]$$

Lookback Time

- Consider

$$H = \frac{\dot{a}}{a} = \frac{da}{dt} \frac{1}{a} = \frac{1}{dt} \ln \left(\frac{a(t)}{a_0} \right) = \frac{1}{dt} \ln \left(\frac{1}{1+z} \right) = - \frac{1}{1+z} \frac{dz}{dt}$$

- Inserting into the Friedmann equation we find the equation for the time interval

$$dt = \frac{-dz}{H_0(1+z) \sqrt{\Omega_{matter}(1+z)^3 + \Omega_{rad}(1+z)^4 + \Omega_{\Lambda} + \Omega_k(1+z)^2}}$$

and integrating

$$t_0 - t_1 = \frac{1}{H_0} \int_0^{z_1} \frac{dz}{(1+z) \sqrt{\Omega_{matter}(1+z)^3 + \Omega_{rad}(1+z)^4 + \Omega_{\Lambda} + \Omega_k(1+z)^2}}$$

- Age in a matter dominated universe

$$(t_1 = 0, z = \infty) t_{0,matter} = \frac{1}{H_0} \int_0^{\infty} \left(\frac{dz}{(1+z)^{\frac{5}{2}}} \right) = \frac{2}{3H_0}$$

Distances (last time)

We can now also express the luminosity distance $D_L = a_0 x_1 (1 + z)$ in these terms

– from the metric for a light ray coming towards

us we have $\frac{dr}{cdt} = \frac{\sqrt{1-kx^2}}{a(t)}$ which turns into

$$\frac{a_0}{c} \frac{dx}{\sqrt{1-kx^2}} = (1+z)dt$$

– after integration we have (using dt from above)

$$\frac{a_0}{c} \int_0^{x_1} \frac{dx}{\sqrt{1-kx^2}} = \int_0^{z_1} \frac{dz}{H_0 \sqrt{\Omega_{matter}(1+z)^3 + \Omega_{rad}(1+z)^4 + \Omega_{\Lambda} + \Omega_k(1+z)^2}}$$

– solutions of the left side are $\frac{a_0}{c} \times \begin{cases} \frac{\arcsin(x_1 \sqrt{k})}{\sqrt{k}} & k > 0 \\ x_1 & k = 0 \\ \frac{\operatorname{arcsinh}(x_1 \sqrt{-k})}{\sqrt{-k}} & k < 0 \end{cases}$

Luminosity Distance

Putting this together with the appropriate trigonometric functions gives

$$D_L = a_0 x_1 (1 + z) = \frac{c(1+z)}{H_0 \sqrt{|\Omega_k|}} S \left(\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{\sqrt{\Omega_{matter}(1+z')^3 + \Omega_{rad}(1+z')^4 + \Omega_\Lambda + \Omega_k(1+z')^2}} \right)$$

$$\text{with } s(y) = \begin{cases} \sin(y) & k > 0 \\ y & k = 0 \\ \sinh(y) & k < 0 \end{cases}$$

We now have the luminosity distance as a function of today's measurements (H_0 , Ω 's) and the redshift z

With the equation of state parameter ω

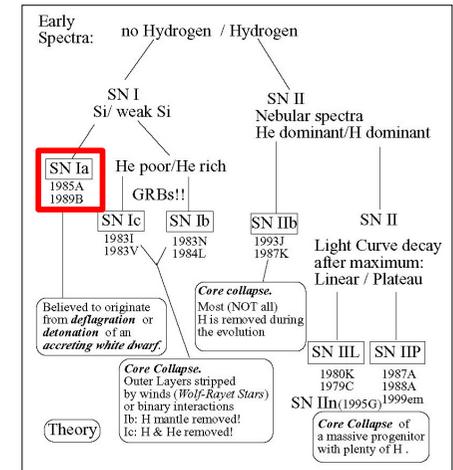
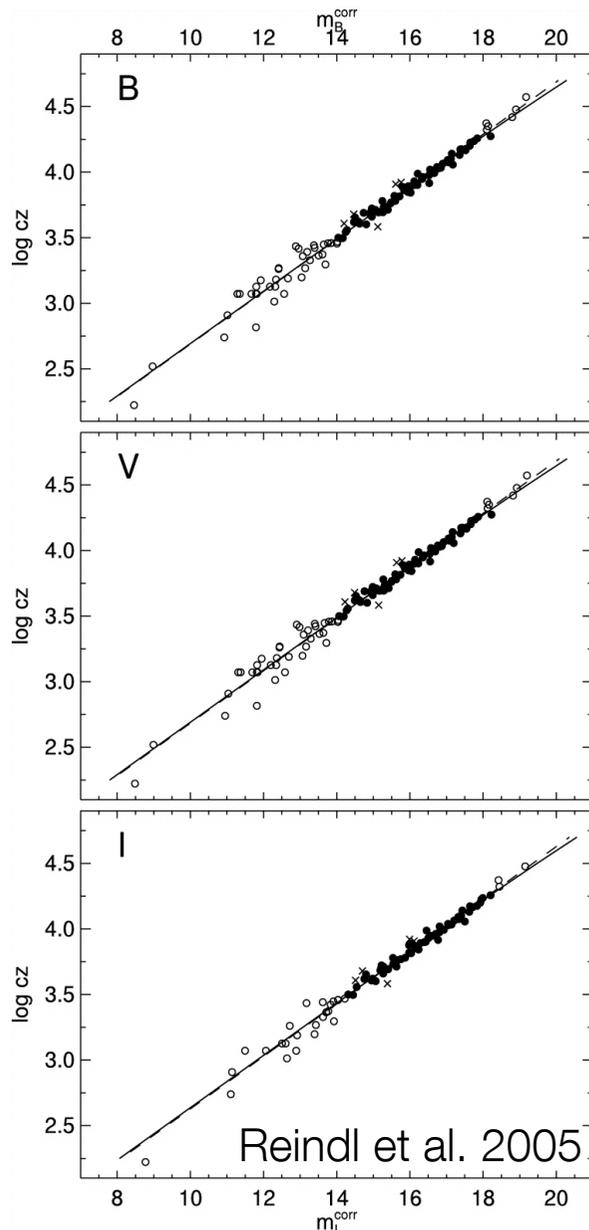
General luminosity distance

$$D_L = \frac{(1+z)c}{H_0 \sqrt{|\Omega_k|}} S \left\{ \sqrt{|\Omega_k|} \int_0^z \left[\Omega_k (1+z')^2 + \sum_i \Omega_i (1+z')^{3(1+\omega_i)} \right]^{-\frac{1}{2}} dz' \right\}$$

– with $\Omega_k = 1 - \sum_i \Omega_i$ and $\omega_i = \frac{p_i}{\rho_i c^2}$

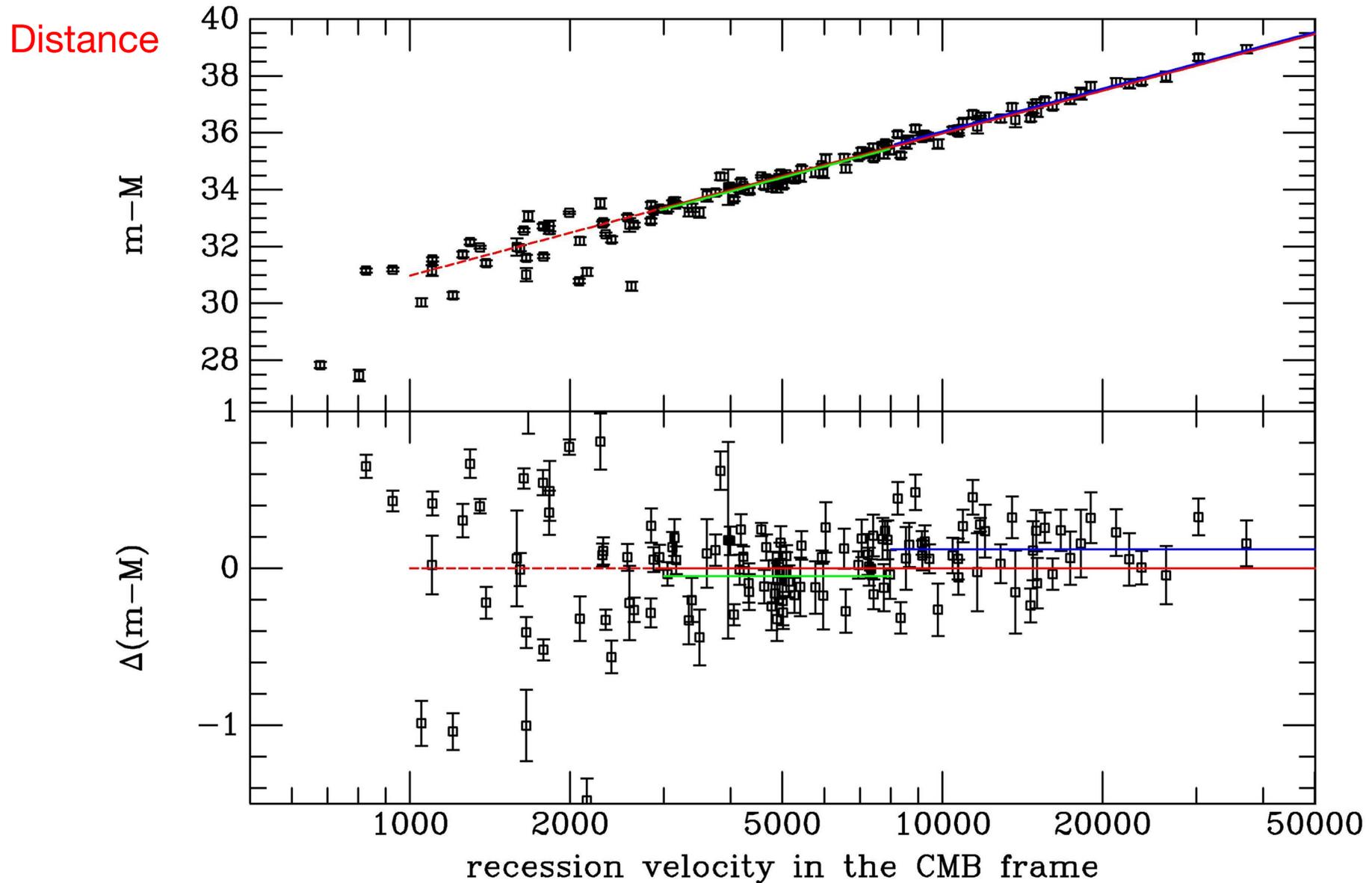
- $\omega_M = 0$ (matter)
- $\omega_R = 1/3$ (radiation)
- $\omega_\Lambda = -1$ (cosmological constant)

SN Ia Hubble diagram



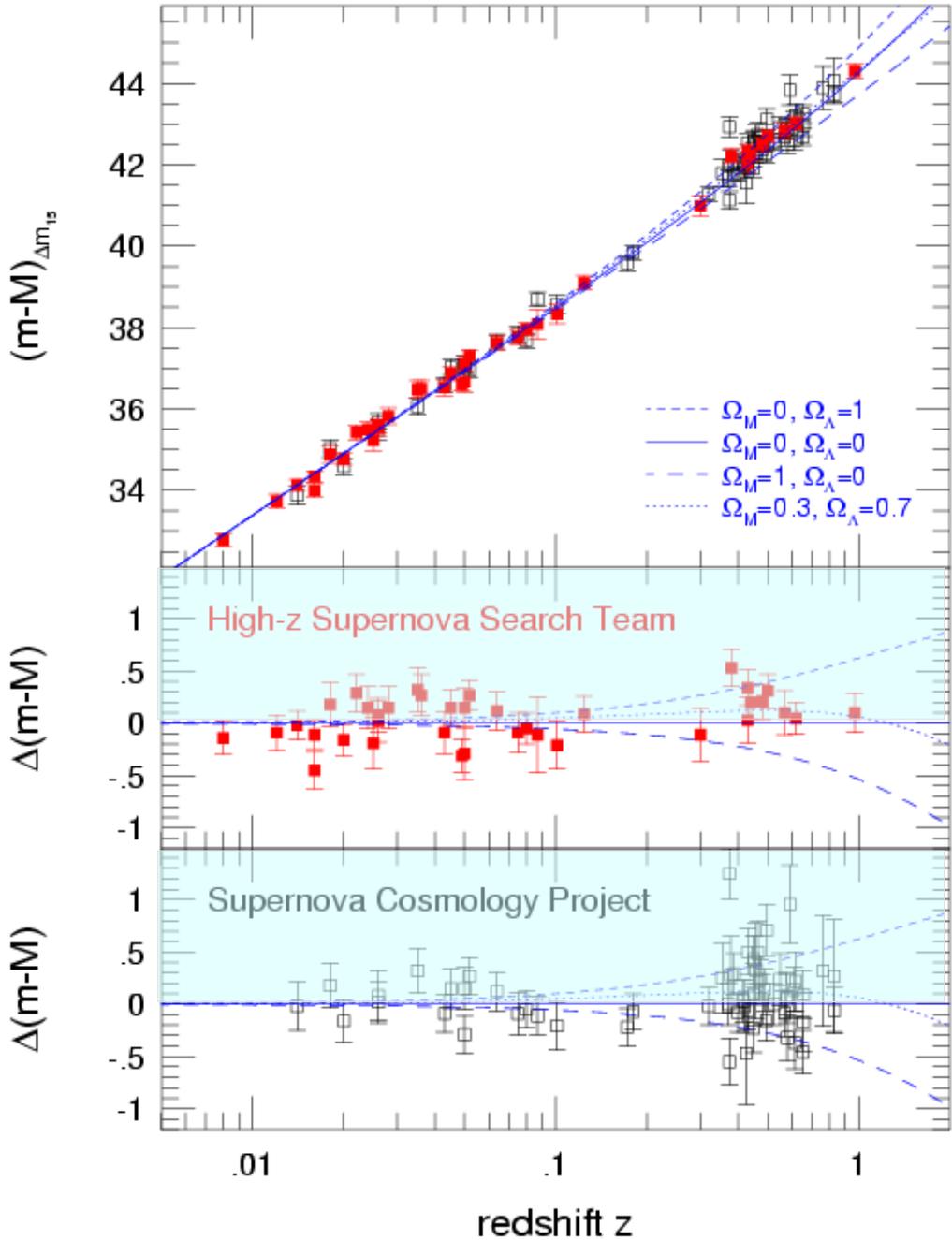
- Excellent distance indicators
- Experimentally verified
- Work of several decades
- Best determination of the Hubble constant

Distance indicator!



Expansion velocity
Bruno Leibundgut

The SN Hubble Diagram



If the observational evidence upon which these claims are based are reinforced by future experiments, the implications for cosmology will be incredible.

Preprint August 1999

Nobel Prize in Physics 2011



Saul Perlmutter



Brian Schmidt



Adam Riess

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

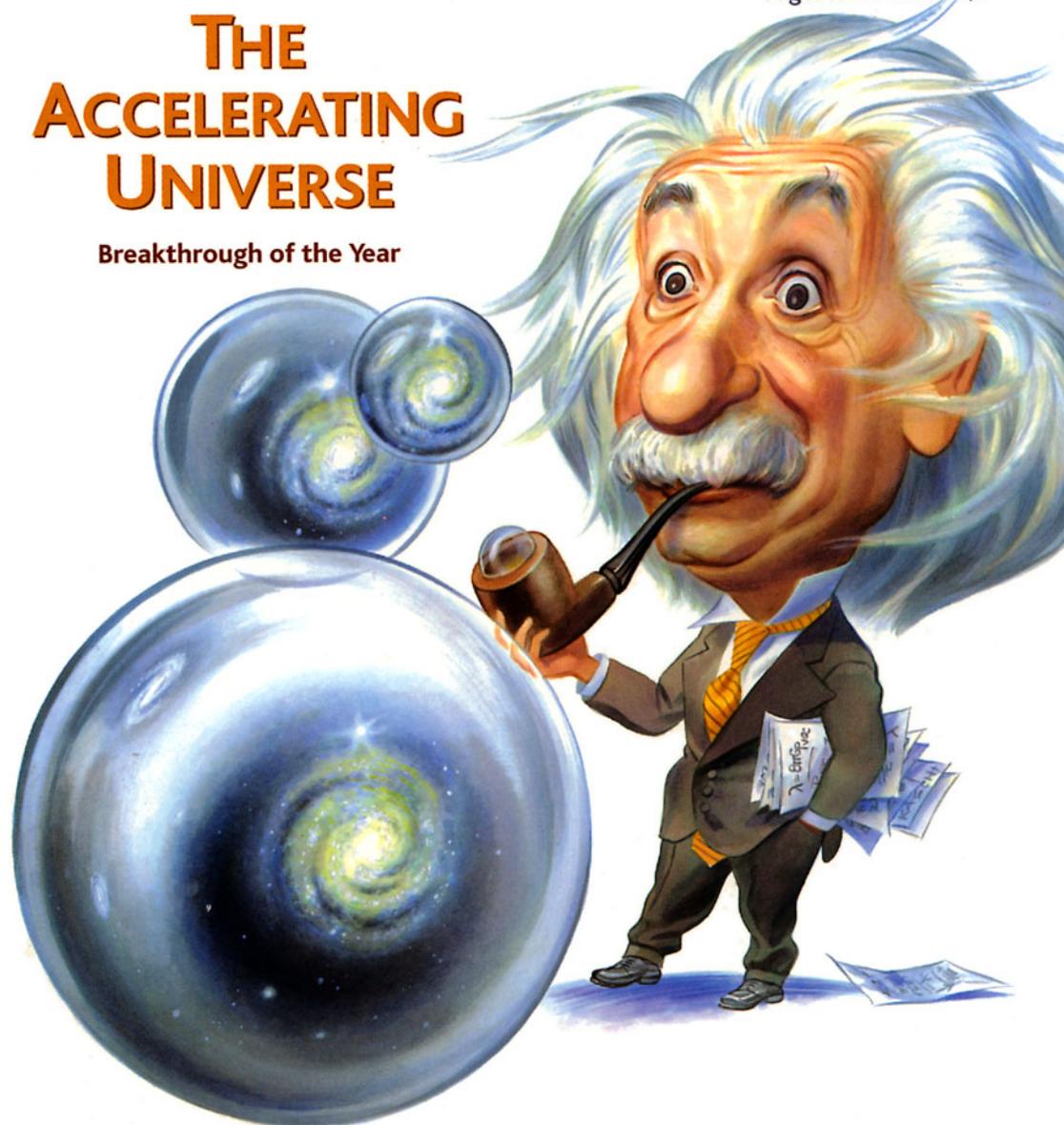
Science

18 December 1998

Vol. 282 No. 5397
Pages 2141-2336 \$7

THE ACCELERATING UNIVERSE

Breakthrough of the Year



Distan
a freely
This re

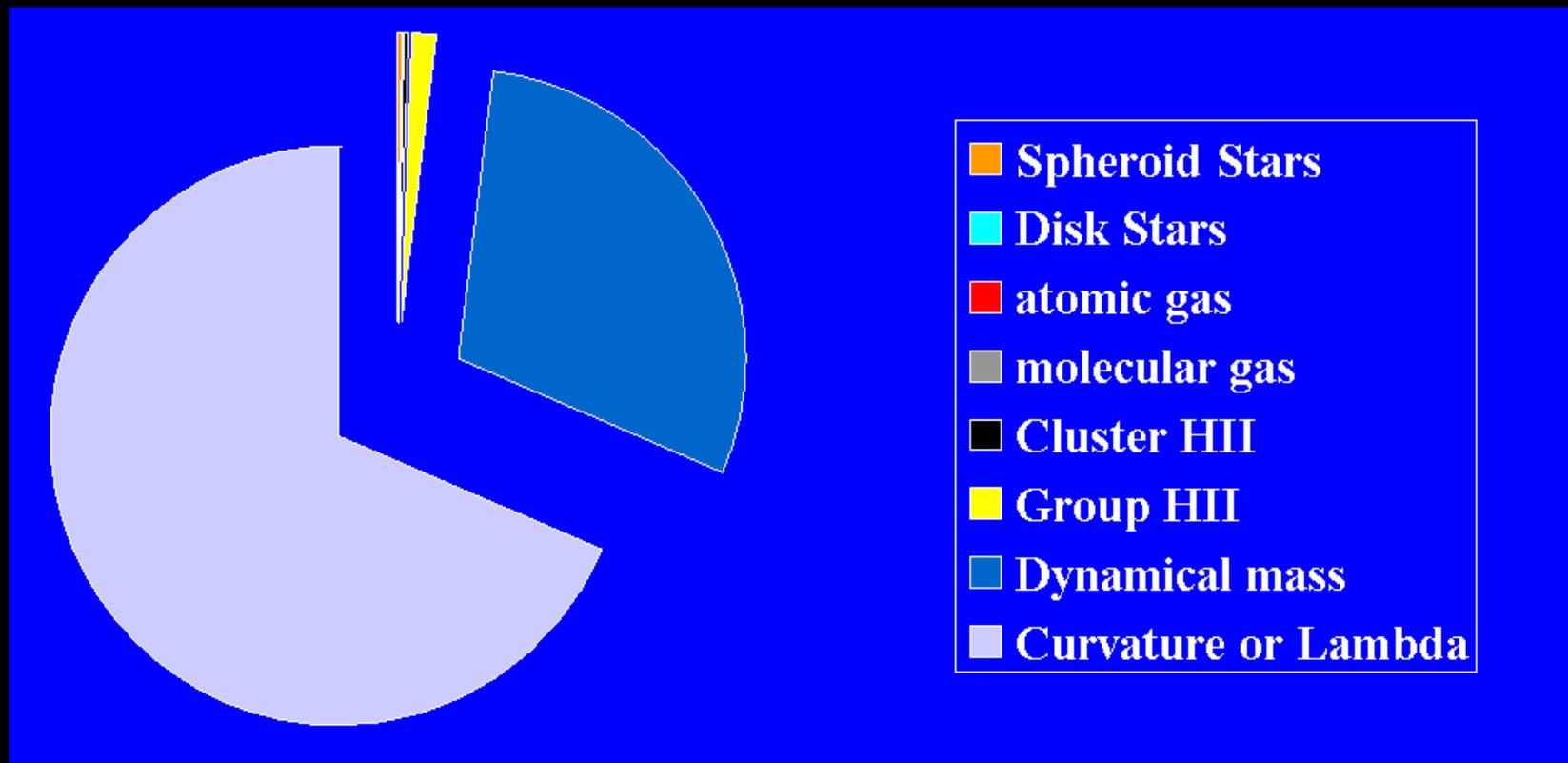
?

/ than in

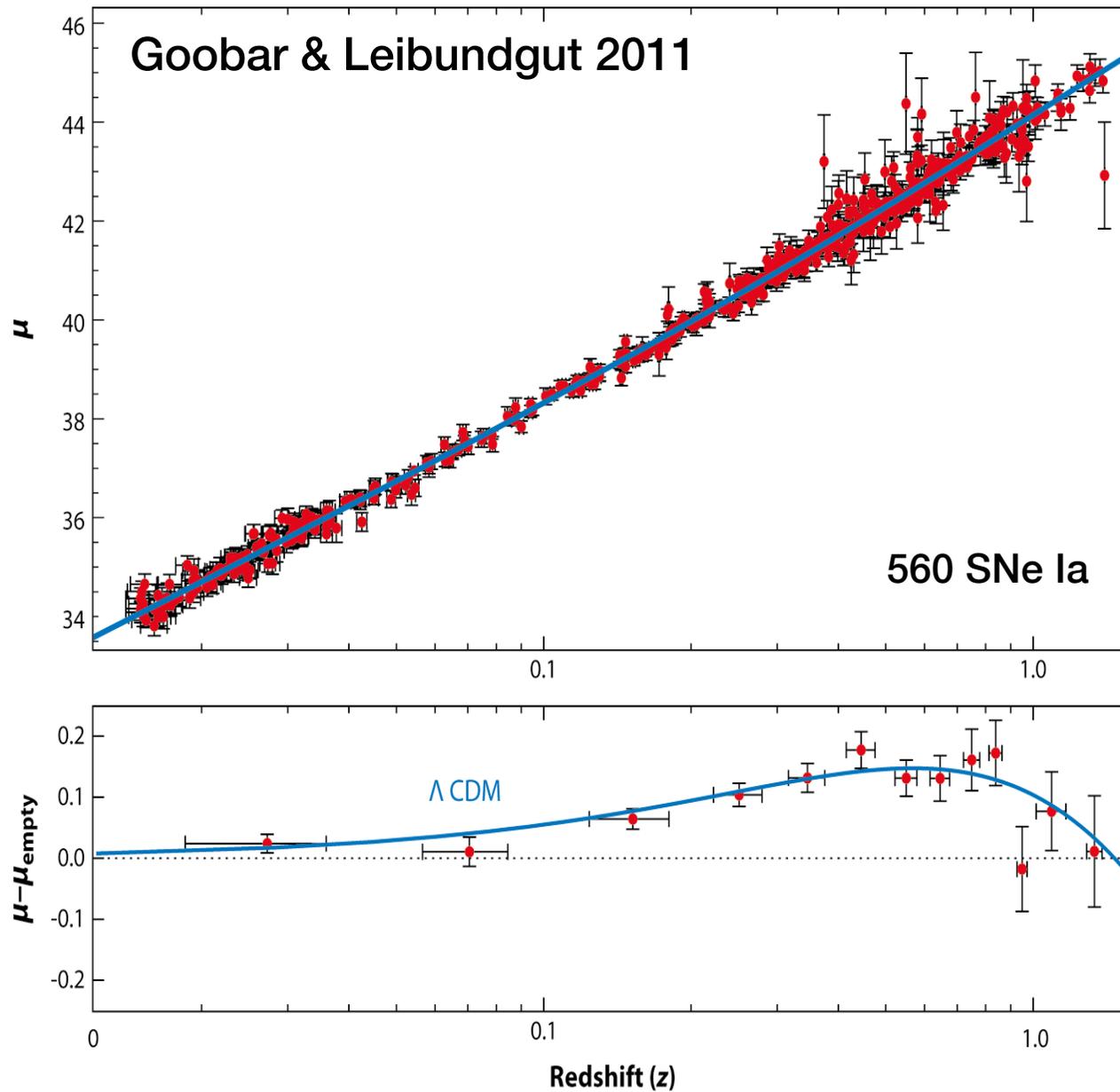
t

Contents of the universe

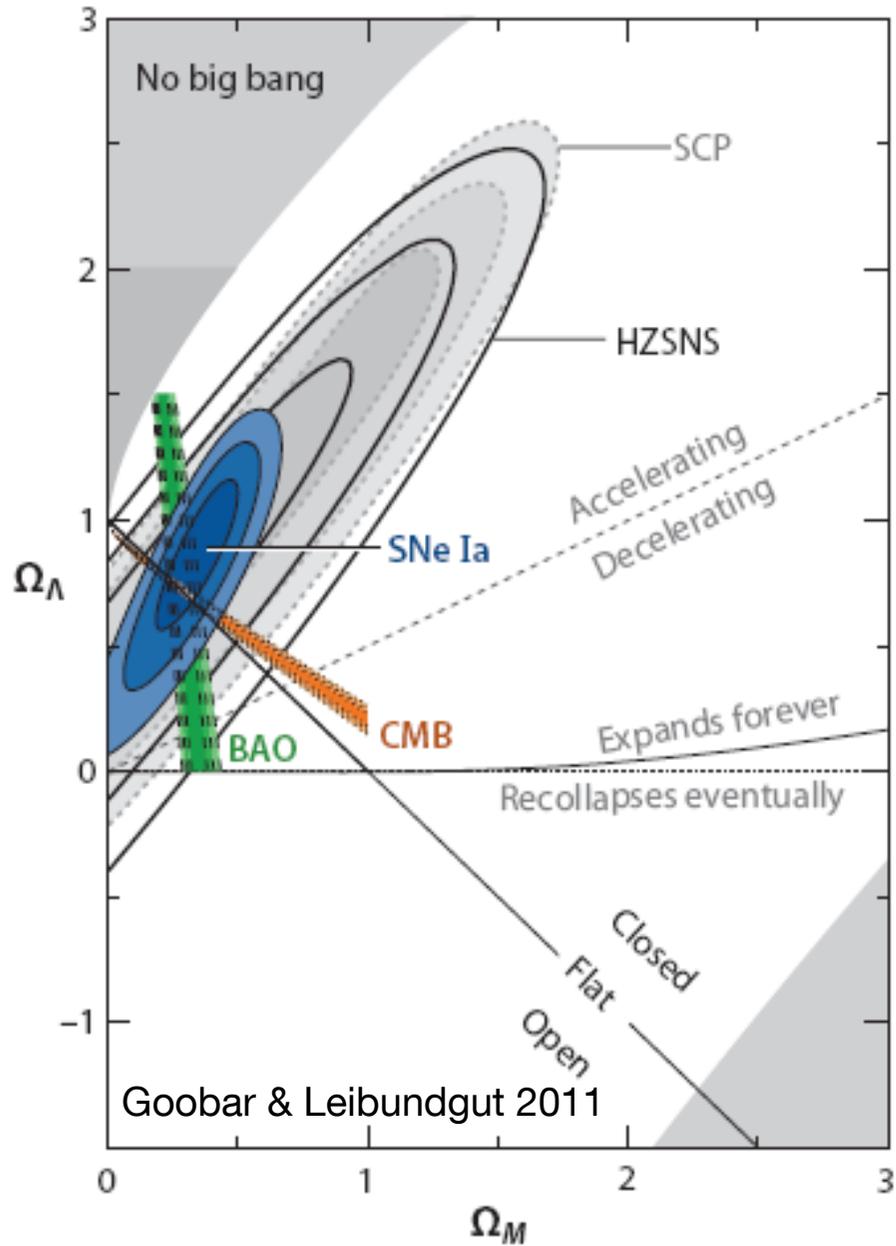
Dark Matter and Dark Energy are the dominant energy components in the universe.



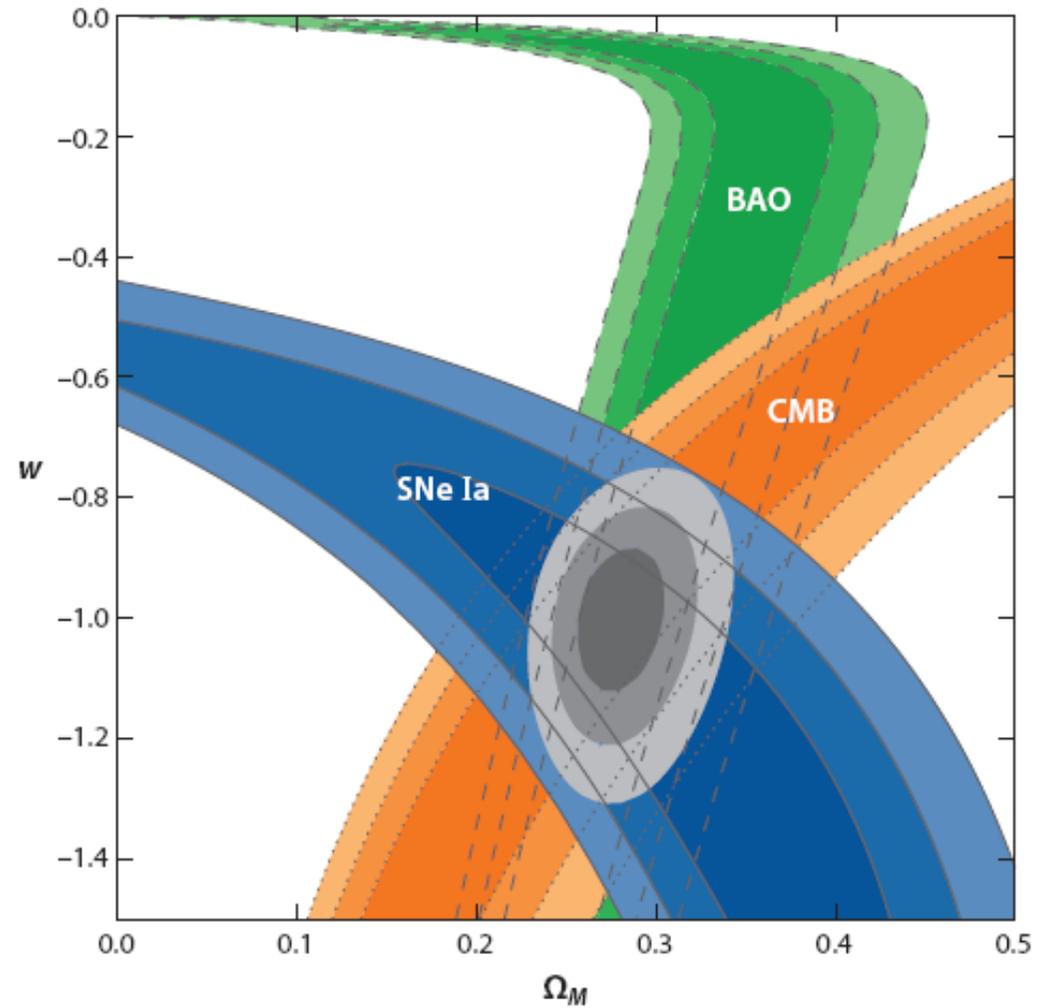
Supernova Cosmology



et voilà ...



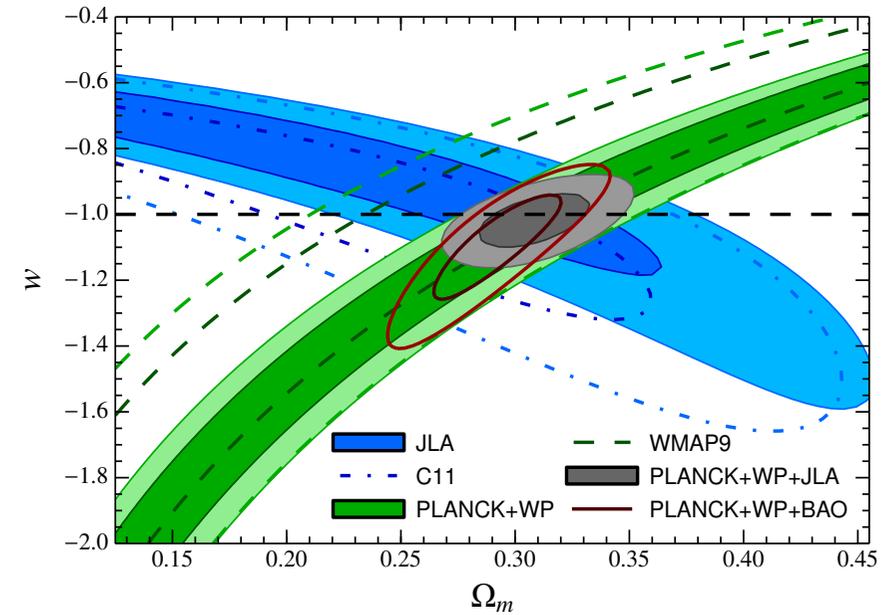
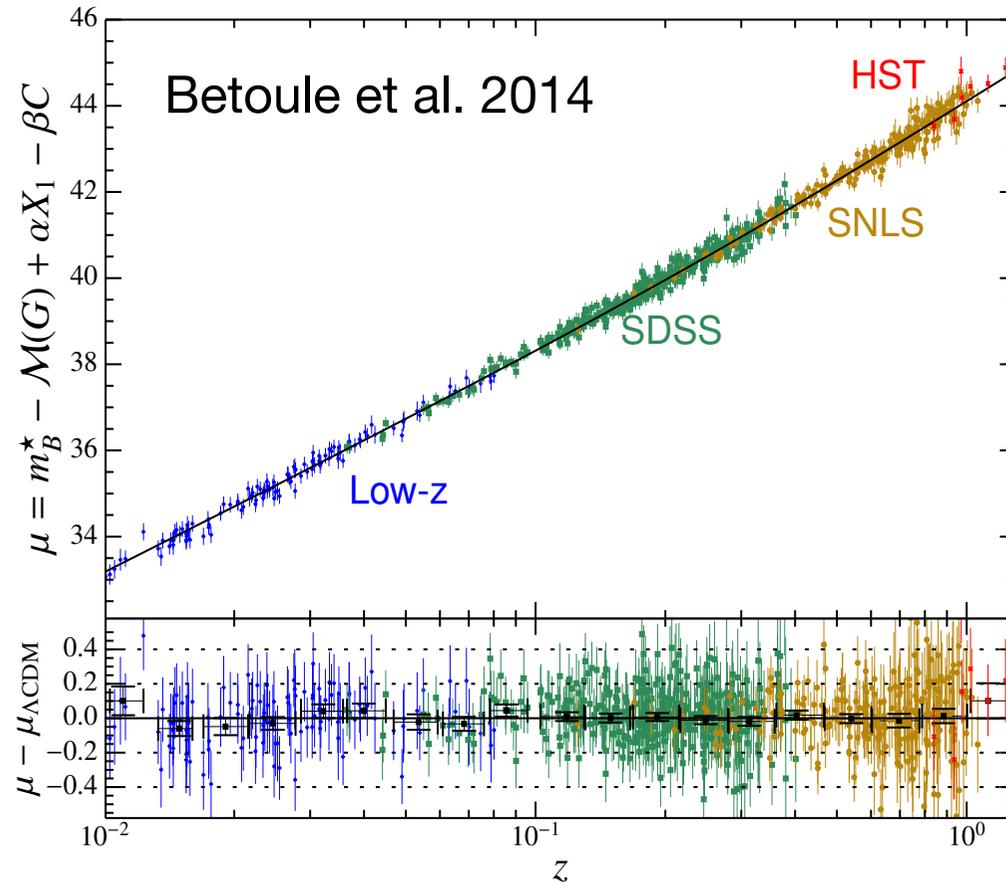
10 years of progress



Constant ω firmly established

N_{SN}	$\Omega_{\text{M}}(\text{flat})$	w (constant, flat)	Light curve fitter	Reference
115	$0.263^{+0.042+0.032}_{-0.042-0.032}$	$-1.023^{+0.090+0.054}_{-0.090-0.054}$	SALT	Astier et al. 2006
162	$0.267^{+0.028}_{-0.018}$	$-1.069^{+0.091+0.13}_{-0.083-0.13}$	MLCS2k2	Wood-Vasey et al. 2007
178	$0.288^{+0.029}_{-0.019}$	$-0.958^{+0.088+0.13}_{-0.090-0.13}$	SALT2	
288	$0.307^{+0.019+0.023}_{-0.019-0.023}$	$-0.76^{+0.07+0.11}_{-0.07-0.11}$	MLCS2k2	Kessler et al. 2009
288	$0.265^{+0.016+0.025}_{-0.016-0.025}$	$-0.96^{+0.06+0.13}_{-0.06-0.13}$	SALT2	
557	$0.279^{+0.017}_{-0.016}$	$-0.997^{+0.050+0.077}_{-0.054-0.082}$	SALT2	Amanullah et al. 2010
472		$-0.91^{+0.16 \pm 0.07}_{-0.20-0.14}$	SIFTO/SALT2	Conley et al. 2011
472	0.269 ± 0.015	$-1.061^{+0.069}_{-0.068}$	SALT2	Sullivan et al. 2011
580	0.271 ± 0.014	$-1.013^{+0.077}_{-0.073}$	SALT2	Suzuki et al. 2011
740	0.295 ± 0.034	-1.018 ± 0.057 CMB	SALT2	Betoule et al. 2014
		-1.027 ± 0.055 CMB+BAO		

Status 2014

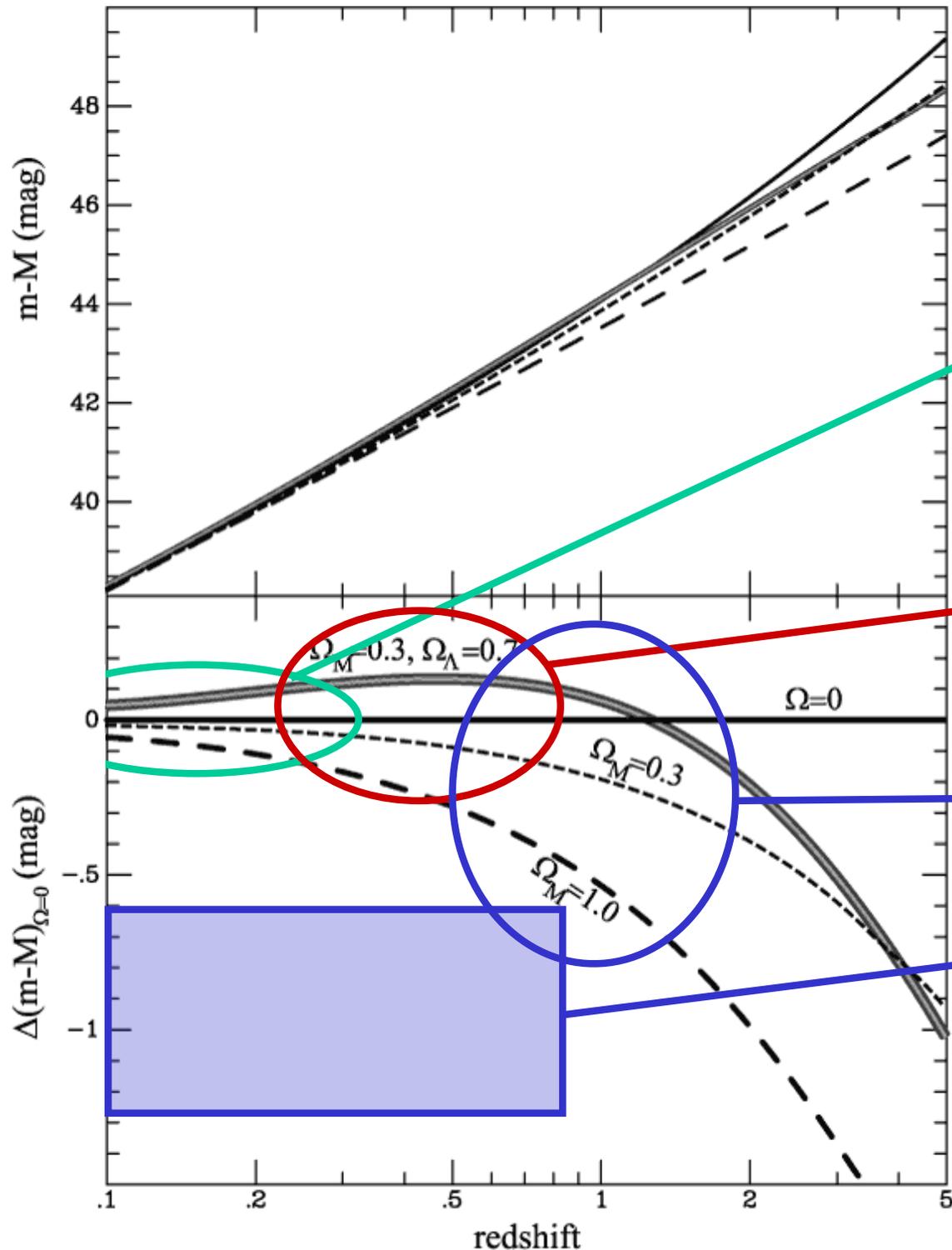


Systematic uncertainties

Current questions

- calibration
- reddening and absorption
 - detection
 - through colours or spectroscopic indicators
 - correction
 - knowledge of absorption law
- light curve fitting
- selection bias
 - sampling of different populations
- gravitational lensing
- brightness evolution

Where are we ...



SN Factory
Carnegie SN Project
SDSSII
Pan-STARRS1

ESSENCE
CFHT Legacy Survey
Dark Energy Survey

Higher-z SN Search
(GOODS, SH0ES)

Euclid/WFIRST/LSST

Plus the local searches:
LOTOSS, CfA, ESC, PTF

What next?

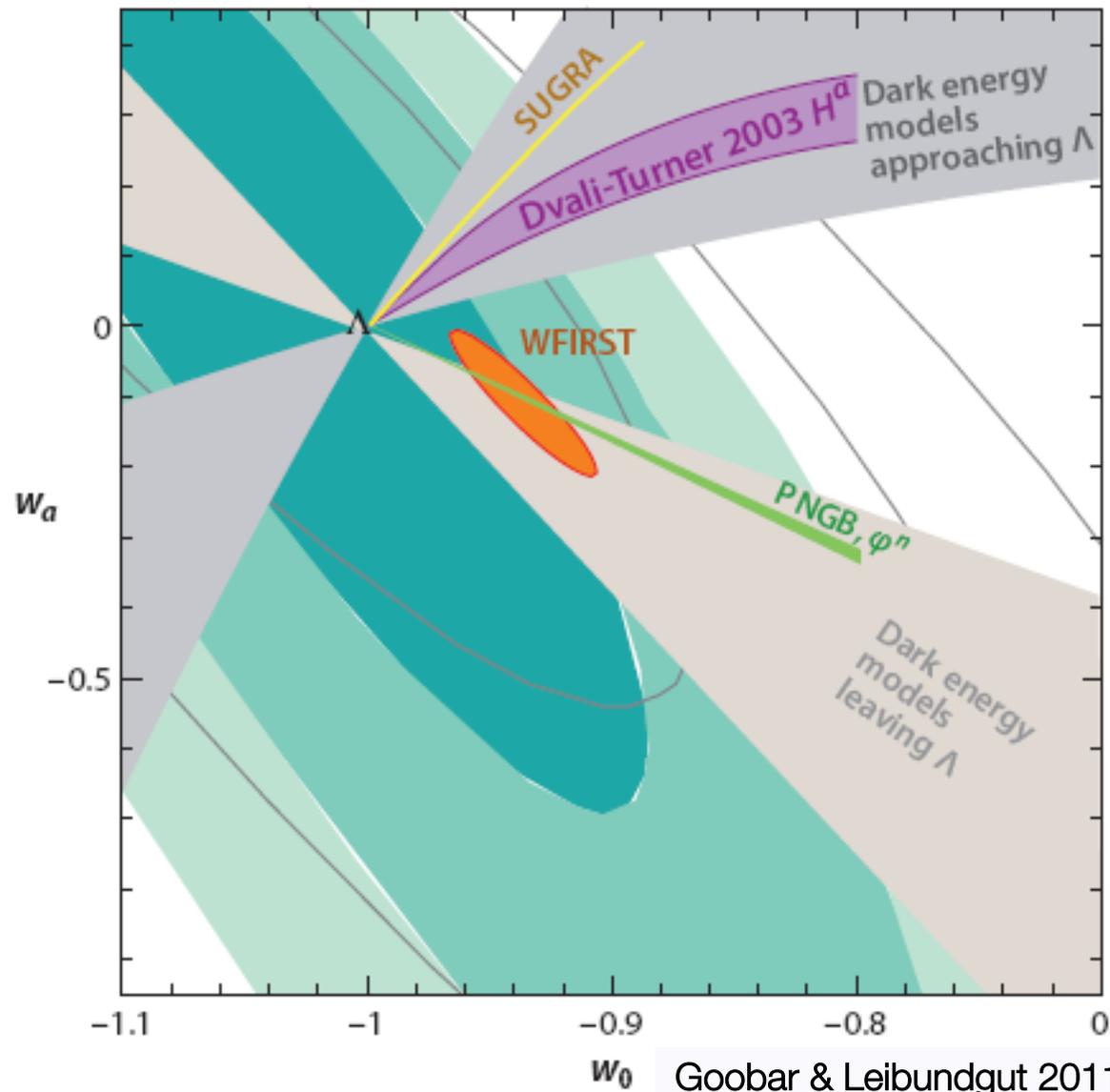
Already in hand

- >1000 SNe Ia for cosmology
- constant ω determined to 5%
- accuracy dominated by systematic effects

Missing

- good data at $z > 1$
 - light curves and spectra
- good infrared data at $z > 0.5$
 - cover the restframe B and V filters
 - move towards longer wavelengths to reduce absorption effects

Cosmology – more?



Goobar & Leibundgut 2011
(courtesy E. Linder and J. Johansson)

Speculations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Einstein's cosmological constant

No explanation in particle physics theories

Quintessence

Quantum mechanical particle field releasing energy into the universe

Signatures of high dimensions

Gravity is best described in theories with more than four dimensions

Phantom Energy

Dark Energy dominates and eventually the universe end in a (Big Rip)

Supernova Cosmology – do we need more?

Test for variable ω

- required accuracy $\sim 2\%$ in *individual* distances
- can SNe Ia provide this?
 - can the systematics be reduced to this level?
 - homogeneous photometry?
 - further parameters (e.g. host galaxy metallicity)
 - handle >100000 SNe Ia per year?

Euclid

- SNe Ia with IR light curves (deep fields)
 - restframe I ($z < 1.2$), J ($z < 0.8$) and H ($z < 0.4$)
- several thousand SNe to be discovered