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VILA DO CONDE,
PORTUGAL,
29-31 MARCH, 2016

11th Iberian Cosmology Meeting

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Are curvature singularities so bad? some counterexamples

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Based on arXiv: 1504.07015 (EPJC), 1507.07763 (PRD), 1508.03272 (PRD), 1511.03755 (PRD), and 1602.01798 (CQG)

IberiCos 2016, Vila do Conde, March 30th, 2016

Singularity theorems

- The theorems on singularities (Hawking, Penrose) tell us that space-time singularities are unavoidable in the context of GR.
- Fundamentally different from singularities on the fields living on a fixed space-time background! (e.g. Coulomb's divergence).
- If space-time breaks down when a singularity, how can we even speak of a singularity as something occurring at some "location"?.
- It is hard to rigorously capture the intuitive notion of a singularity (the theorems on singularities offer little clue about this).

Curvature divergences vs geodesic completeness

- If we see space-time singularities as indicative of a physically troublesome region, a natural guess is that something is going on ill with the geometry.
  - The blow up of curvature scalars (curvature divergences) tell us that we have a space-time singularity.
  - Then if you want non-singular space-time time, just make all the curvature invariants finite.
- A standard strategy is to propose a line element with known finite curvature scalars and drive the Einstein's equations back to find the action it derives from (through removal of any of the hypothesis): from Bardeen solution to a huge literature on the subject (Review: 0802.0330 [gr-qc]). But...
- Nothing in the singularity theorems speak of curvature invariants!.

- A more powerful characterization of space-times containing singularities is provided by the notion of geodesic completeness (Geroch, Penrose, Hawking,...).
- Geodesics describing null rays or time-like (physical) observers should be complete: in a consistent theory nothing can cease to exist suddenly or “emerge” from nowhere.

- That observers may experience intense tidal forces or large deformations is secondary as long as they exist.

- Thus:
  - In a singular space-time there exist geodesic curves which cannot be extended to arbitrarily large values of the affine parameter (i.e., they start or terminate at some finite value).
  - In a non-singular space-time geodesics can be extended to arbitrarily large values of the affine parameter (geodesic completeness), no matter the behaviour of the curvature invariants.
- The widespread identification between curvature divergences and space-time singularities comes from the fact that in many cases of interest, those space-times showing curvature divergences are also geodesically incomplete.
- Assumptions/approach of our research:
  - Fundamental criterium for space-time singularities is geodesic completeness.
  - Space-time singularities are an artifact of the classical GR description, which would break down at high curvature/short-scales: modified gravity.
  - The bulk of "quantum gravity" effects can be captured by some effective theory of gravity in which singularities are avoided.

Background geometry

- Consider the following electrovacuum geometry (derived first in 1207.6004 [gr-qc] (PRD))

$$ds^2 = -A(x)dv^2 + \frac{2}{\sigma_+}dvdx + r^2(x)d\Omega^2$$

where

$$\begin{aligned} A(x) &= \frac{1}{\sigma_+}\left[1 - \frac{r_S}{r}\frac{(1+\delta_1 G(r))}{\sigma_-^{1/2}}\right] \; ; \; \delta_1 = \frac{1}{2r_S}\sqrt{\frac{r_q^3}{l_\varepsilon}} \; ; \; \sigma_\pm = 1 \pm \frac{r_c^4}{r^4(x)}; \\ G(z) &= -\frac{1}{\delta_c} + \frac{1}{2}\sqrt{z^4-1}\left[f_{3/4}(z) + f_{7/4}(z)\right] \end{aligned}$$

where $r_c = \sqrt{l_\varepsilon r_q}$: "core" radius, l_ε some length scale, $r_q^2 = 2G_N q^2$: charge radius, $r_S = 2M_0$: Schwarzschild radius, $\delta_c \simeq -0.572$ a constant.

  - For $z = r/r_c \gg 1 \rightarrow$ Reissner-Nordström space-time:

$$A(x) \approx 1 - \frac{r_S}{r} + \frac{r_q^2}{2r^2} + O\left(\frac{1}{r^4}\right)$$

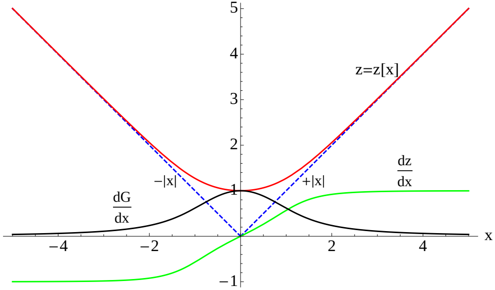
  - For $z \simeq 1$ (N_q: number of charges):

$$A(x) \simeq \frac{N_q}{4N_c}\frac{(\delta_1 - \delta_c)}{\delta_1\delta_c}\sqrt{\frac{r_c}{r-r_c}} + \frac{N_c - N_q}{2N_c} + \ldots$$

- From $\left(\frac{dr}{dx}\right)^2 = \frac{\sigma_-}{\sigma_+^2}$ we find

$$r^2(x) = \frac{x^2 + \sqrt{x^4 + 4r_c^4}}{2}$$

which has a minimum at $x = 0$. This is reminiscent of a wormhole geometry.



- Curvature scalars at the wormhole throat can be all finite ($\delta_1 = \delta_c$) or divergent ($\delta_1 \neq \delta_c$), but WH structure persists in all spectrum of mass and charge.

Theories and approach

- This space-time is an exact solution of quadratic gravity [1207.6004 [gr-qc] (PRD)]:

$$S_{Quad} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R + l_\varepsilon^2(aR^2 + R_{\mu\nu}R^{\mu\nu})\right] - \frac{1}{16\pi}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

- and also of Born-Infeld gravity [1311.0815 [hep-th] (PRD)]:

$$S_{BI} = \frac{1}{\kappa^2\varepsilon}\int d^4x\left[\sqrt{-|g_{\mu\nu} - l_\varepsilon^2 R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{-g}\right] - \frac{1}{16\pi}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

- Standard electromagnetic Maxwell field as matter $\rightarrow$ energy conditions are satisfied.
- Formulated in the Palatini approach: metric and connection as independent fields:
  - Second-order field equations.
  - Vacuum equations are Minkowski or (A)dS solutions $\rightarrow$ no extra propagating dofs.
  - In GR (and Lovelock), metric and Palatini formulations coincide.

Geodesics

- Parameterize a geodesic curve $\gamma^\mu = x^\mu(\lambda)$ with tangent vector $u^\mu = \frac{dx^\mu}{d\lambda}$ and affine parameter λ:

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

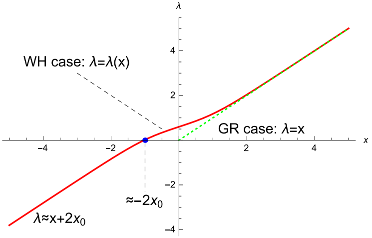
- Comments:
  - The metric defines a natural connection (Christoffel) and defines a set of geodesics.
  - The independent connection can be used to define a different set of geodesics.
  - Assuming the EEP and since matter is not coupled directly to the independent connection we assume geodesics to be those of the metric.
- By spherical symmetry there are two conserved quantities: $L = r^2 d\varphi/d\lambda$ (angular momentum per unit mass) and $E = Adt/d\lambda$ (total energy per unit mass).
- Rewrite the geodesic equation in terms of the geodesic tangent vector

$$\frac{1}{\sigma_+^2}\left(\frac{dx}{d\lambda}\right)^2 = E^2 - A\left(\kappa + \frac{L^2}{r^2(x)}\right)$$

where $k = 0(1)$ for null (time-like) geodesics.

- Radial null geodesics ($k=0, L=0$) integrate this equation as

$$\pm E \cdot \lambda(x) = \begin{cases} {}_2F_1[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{r_c^4}{r^4}]r & \text{if } x \geq 0 \\ 2x_0 - {}_2F_1[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{r_c^4}{r^4}]r & \text{if } x \leq 0 \end{cases}$$

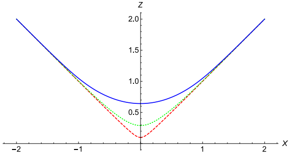


- The affine parameter $\lambda(x)$ extends over the real axis and the space-time, no matter the behaviour of curvature scalars.
- In GR: $r(\lambda) = \pm E\lambda$, the affine parameter is only defined on the positive/negative side of the real axis because $r(\lambda)$ is positive.
- Null (with $L \neq 0$) and time-like geodesics are complete as well no matter the behaviour of curvature scalars.

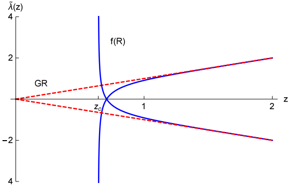
- $f(R) = R - \gamma R^2$ gravity with Born-Infeld electrodynamics

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R) + \frac{1}{4\pi}\int d^4x\sqrt{-g}\beta^2\left(1-\sqrt{1+\frac{F_{\mu\nu}F^{\mu\nu}}{2\beta^2}}\right),$$

- Wormhole structure:



$\rightarrow$ yields complete geodesics.



Infinite affine time $\rightarrow$ these wormholes lie beyond the reach of any observer or signal!.

Final remarks

- Geodesics in these space-times are null and time-like complete for all spectrum of mass and charge, i.e., no matter the behaviour of the curvature scalars.
- Not *“designed”*: they arise in reasonable extensions of GR where matter satisfies the energy conditions.
- Curvature divergences and tidal forces? describe physical observers as a congruence of geodesics (Tipler, Krolak, Nolan, etc).
- Causal contact among the constituents making up an extended object crossing the wormhole throat is never lost [1602.01798 [hep-th](CQG)]
- The problem of the scattering of waves off the wormhole is well posed $\longrightarrow$ no absolutely destructive effects happen upon physical observers [1504.07015 [hep-th] (EPJC)].
- WH structure and geodesic completeness a general feature: also with anisotropic fluids [1509.02430 [hep-th]] and in higher dimensions [1507.07763 [hep-th] (PRD)].
- Bouncing cosmologies exist for these same theories [1406.1205 [hep-th] (PRD)].
- Palatini formulation of modified gravity is essential for these results [1412.4499 [hep-th] (PRD)].

THANK YOU FOR YOUR ATTENTION!

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Future singularities in cosmology

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March 29 2016

Based on
J. Beltran Jimenez, R. Lazkoz, DSG and V. Salzano, arXiv:1602.06211

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instituto de astrofísica e ciências do espaço

Outline

- FLRW cosmologies: late-time acceleration

- Singularities in General Relativity

- Classification of future singularities in cosmology

- Testing some singular models with data: Is the universe approaching the *doomsday*?

FLRW cosmologies

FLRW metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - k\, r^2} + d\Omega^2\right]$$

General Relativity:

FLRW equations in General Relativity

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2}\,,$$

$$\dot H = -4\pi G(p+\rho) + \frac{K}{a^2}\,,$$

For a perfect fluid with an equation of state, $w = p/\rho$

$$H = \frac{2}{3(1+w)(t-t_0)}\,,$$

$$a(t) \propto (t-t_0)^{\frac{2}{3(1+w)}}\,,$$

$$\rho \propto a^{-3(1+w)}\,,$$

$$\begin{aligned}\text{Radiation}: &\quad a(t) \propto (t-t_0)^{1/2}\,, \quad \rho \propto a^{-4}\,,\\ \text{Dust}: &\quad a(t) \propto (t-t_0)^{2/3}\,, \quad \rho \propto a^{-3}\,.\end{aligned}$$

FLRW cosmologies

Dark energy equation of state $$p = w\rho$$

$$\begin{aligned} &w > -1, && \text{quintessence fluid} \\ &w = -1, && \text{cosmological constant} \\ &w < -1, && \text{phantom fluid} \end{aligned}$$

Einstein gravity

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\ \sqrt{-g}\ R$$

+

Dark energy

$$S = -\int d^4x\ \sqrt{-g}\ \Lambda$$

$$S = \int d^4x\ \sqrt{-g}\ \left(\pm\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

$$S = -\frac{1}{4}\int d^4x\ \sqrt{-g}\ F_{\mu\nu}F^{\mu\nu}$$

$$S = \int d^4x\sqrt{-g}\ F(R)$$

$$S = \int d^4x\sqrt{-g}\ F(R,T)$$

E. J. Copeland, M. Sami, and S. Tsujikawa. Dynamics of Dark Energy. Int. J. Mod. Phys. D, 15:1753–1935, 2006, arXiv:hep-th/0603057S. Nojiri and S. D. Odintsov, eConf C0602061, 06 (2006); hep-th/0601213; arXiv: 0807.0685; S.~Capozziello and M.~De Laurentis, Phys. Rept. 509, 167 (2011), arXiv:1108.6266; S. Capozziello and V. Faraoni, Beyond Einstein Gravity, Fundamental Theories of Physics Vol. 170, Springer Ed., Dordrecht (2011); V. Sahni and A. Starobinsky. Reconstructing Dark Energy. Int. J. Mod. Phys. D, 15:2105–2132, 2006, arXiv:astro-ph/0610026......

Singularities in General Relativity

Geodesics completeness:

A regular spacetime is defined as far as its geodesics are complete, i.e. as far as the geodesics go smoothly through the *singularity*.

$$\frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda} = 0$$

Geodesic deviation (Tipler and Krolak criteria):

Even if geodesics are regular through the singularity, the infinitesimal distance between them may diverge, affecting the tidal forces. In order to account the possible effects on finite volumes, one may study the following integrals:

$$T(u) \equiv \int_0^\lambda \mathrm{d}\lambda' \int_0^{\lambda'} \mathrm{d}\lambda'' R_{ij} u^i u^j,$$
$$K(u) \equiv \int_0^\lambda \mathrm{d}\lambda''' R_{ij} u^i u^j.$$

If the tidal forces are strong enough (above integrals diverge), and the volume shrinks to zero, the singularity is said to be strong.

Future Singularities in Cosmology

Classification of singularities

S. Nojiri, S. D. Odintsov and S. Tsujikawa, Phys. Rev. D 71, 063004 (2005)
L. Fernndez-Jambrina, Phys. Rev. D 90, 064014 (2014)

- Type I ("Big Rip singularity"): For $t \to t_s$, $a \to \infty$ and $\rho \to \infty$, $|p| \to \infty$. Time-like geodesics are incomplete.
  R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003).

- Type II ("Typical Sudden singularity"): For $t \to t_s$, $a \to a_s$ and $\rho \to \rho_s$, $|p| \to \infty$. Geodesics are not incomplete. This is classified as a weak singularity.
  J. D. Barrow, Class. Quant. Grav. **21**, L79 (2004)

- Type III ("Big freeze"): For $t \to t_s$, $a \to a_s$ and $\rho \to \infty$, $|p| \to \infty$. No geodesics incompleteness. They can be weak or strong.
  M. Bouhmadi-Lopez, P. F. Gonzalez-Diaz and P. Martin-Moruno, Phys. Lett. B **659**, 1 (2008).

- Type IV ("Generalized Sudden singularity"): For $t \to t_s$, $a \to a_s$ and $\rho \to \rho_s$, $p \to p_s$ but higher derivatives of Hubble parameter diverge. They are weak singularities.
  M. P. Dabrowski, K. Marosek and A. Balcerzak, Mem. Soc. Ast. It. **85**, no. 1, 44 (2014);

- Type V ("w-singularities"): For $t \to t_s$, $a \to \infty$ and $\rho \to 0$, $|p| \to 0$ and $w = p/\rho \to \infty$. These singularities are weak
  M. P. Dabrowski and T. Denkieiwcz, Phys. Rev. D **79**, 063521 (2009).

Future Singularities in Cosmology

Some singular models

J. Beltran Jimenez, R. Lazkoz, DSG and V. Salzano, arXiv:1602.06211

We parametrise the Hubble rate in such a way that each model may contain a singularity.

our	$N.O.T.$	$H(x)$	$a(x)$	a	H	$\dot{H}$	$\ddot{H}$	ρ	p	$w_{\rm eff}$
A	I	$\frac{2}{3x}+\frac{2n}{3(1-x/x_s)}$	$a_0 x^{\frac{2}{3}}\,(x_s-x)^{-\frac{2}{3}\cdot n x_s}$	∞	∞	∞	∞	∞	$-\infty$	$w_s<0$
B	III	$\frac{2}{3x}+\frac{2n}{3\sqrt{1-x/x_s}}$	$a_0 x^{\frac{2}{3}}\,\exp\left[-\frac{4}{3}n\sqrt{x_x\,(x_s-x)}\right]$	a_s	∞	∞	∞	∞	$-\infty$	$-\infty$
C	III	$\frac{2}{3x}-\frac{2n}{3}\log\left(1-\frac{x}{x_s}\right)$	$a_0 x^{\frac{2}{3}}\,\exp\left[-\frac{2}{3}n\,(x-x_s)\,(-1+\log\left[1-x/x_s\right])\right]$	a_s	∞	∞	∞	∞	$-\infty$	$-\infty$
D	II	$\frac{2}{3x}+\frac{2n}{3}\sqrt{1-\frac{x}{x_s}}$	$a_0\,(x/x_s)^{\frac{2}{3}}\,\exp\left[-\frac{4}{9}\cdot n x_s\,(1-x/x_s)^{\frac{3}{2}}\right]$	a_s	$H_s>0$	$-\infty$	$-\infty$	ρ_s	∞	∞
E	IV	$\frac{2}{3x}+\frac{2n}{3}\left(1-\frac{x}{x_s}\right)^{3/2}$	$a_0\,(x/x_s)^{\frac{2}{3}}\,\exp\left[-\frac{4}{15}\cdot n x_s\,(1-x/x_s)^{\frac{5}{2}}\right]$	a_s	$H_s>0$	$\dot{H}_s<0$	∞	ρ_s	0	0

where $x = t/t_0, \quad x_s = t_s/t_0$

Data

Hubble parameter from early-type galaxies

M. Moresco, MNRAS 450 (2015) 1.

$$\chi^2_H = \sum_{i=1}^{24} \frac{\left(H(x_i,\boldsymbol{\theta}) - H_{obs}(x_i)\right)^2}{\sigma^2_H(x_i)} \,,$$

Here we use a gaussian prior on $H_0 = 69.6 \pm 0.7$

Baryon Acoustic Oscillations

C. Blacke, et al., MNRAS 425 (2012) 405.

$$\chi^2_{BAO} = \Delta\boldsymbol{\mathcal{F}}^{BAO} \cdot \mathbf{C}^{-1} \cdot \Delta\boldsymbol{\mathcal{F}}^{BAO} \,,$$

Alcock-Paczynski distortion parameter

$$F(z) = (1+z) D_A(z) \frac{H(z)}{c}$$

$$D_A(z,\boldsymbol{\theta}) = \frac{c}{(1+z)} \int_0^z \frac{\mathrm{d}\tilde{z}}{H(\tilde{z},\boldsymbol{\theta})}$$

Supernovae Ia

N. Suzuki, et al. (Union 2.1), ApJ 746 (2012) 85.

$$\chi^2_{SN} = \Delta\boldsymbol{\mathcal{F}}^{SN} \cdot \mathbf{C}^{-1} \cdot \Delta\boldsymbol{\mathcal{F}}^{SN} \,,$$

Distance modulus

$$\mu = 5\log_{10}[d_L(z,\boldsymbol{\theta})] + \mu_0$$

$$d_L(z,\boldsymbol{\theta}) = (1+z) \int_0^z \frac{H_0}{H(\tilde{z},\boldsymbol{\theta})} \mathrm{d}\tilde{z}$$

Total chi^2 to be minimized: $\chi^2 = \chi^2_H + \chi^2_{H_0} + \chi^2_{SN} + \chi^2_{BAO}$

Results

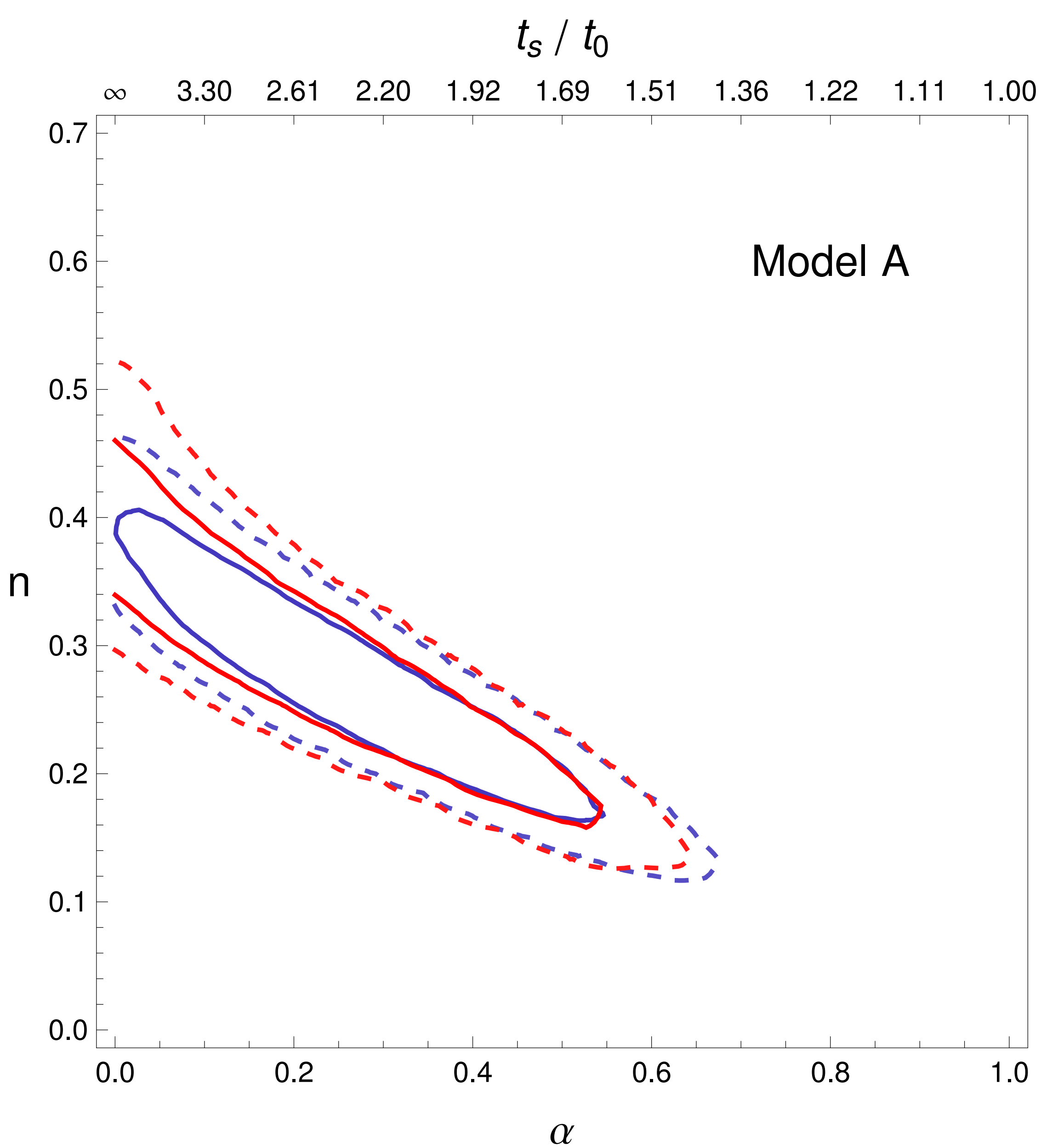
Markov Chain Monte Carlo. Free parameters: $n,\ t_0,\ x_s = 1 - \log \alpha_s$

id.	Ω_m			H_0 km s^{-1} Mpc^{-1}	t_0 Gyr	$w_{\rm eff,0}$	$\mathcal{B}_{ij}$	$\log \mathcal{B}_{ij}$
ΛCDM	$0.30^{+0.03}_{-0.03}$			$69.6^{+0.7}_{-0.7}$	$13.57^{+0.33}_{-0.31}$	-0.70	1	0

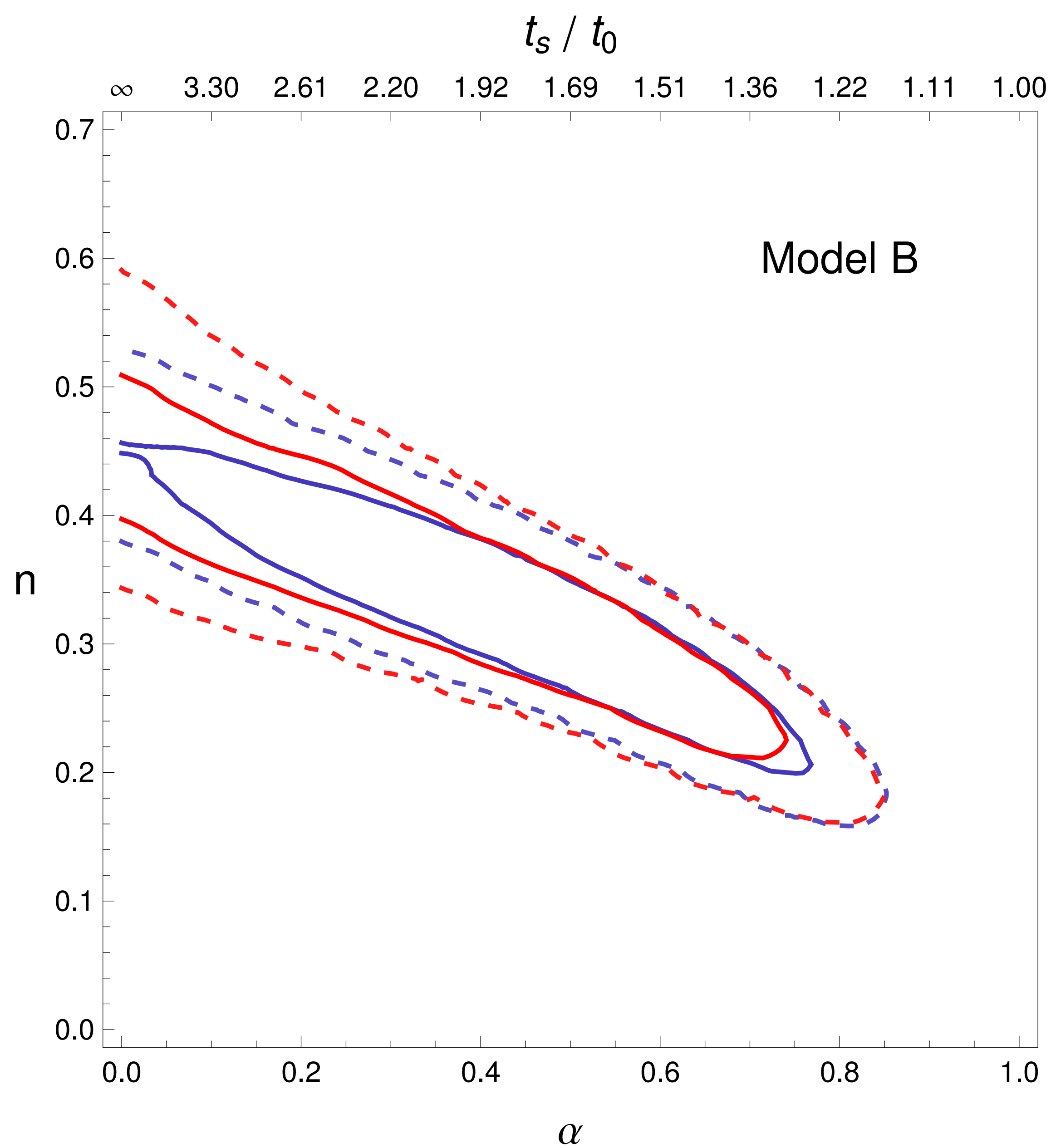
id.	Ω_m	w_0	w_a	H_0 km s^{-1} Mpc^{-1}	t_0 Gyr	$w_{\rm eff,0}$	$\mathcal{B}_{ij}$	$\log \mathcal{B}_{ij}$
CPL	$0.36^{+0.05}_{-0.08}$	$-0.93^{+0.25}_{-0.25}$	$-1.71^{+2.18}_{-3.12}$	$69.5^{+0.7}_{-0.7}$	$13.29^{+0.39}_{-0.32}$	-0.60	1.9	0.63

id.	n	α_s	$\frac{t_s}{t_0}$	$1/t_0$ km s^{-1} Mpc^{-1}	t_0 Gyr	$w_{\rm eff,0}$	$\mathcal{B}_{ij}$	$\log \mathcal{B}_{ij}$
Uniform prior								
A	$0.28^{+0.07}_{-0.06}$	$0.27^{+0.17}_{-0.16}$	$2.30^{+0.61}_{-0.58}$	$69.9^{+1.6}_{-1.7}$	$13.99^{+0.36}_{-0.32}$	$-0.72^{+0.12}_{-0.12}$	1.5	0.42
B	$0.34^{+0.07}_{-0.07}$	$0.42^{+0.21}_{-0.23}$	$1.86^{+0.50}_{-0.54}$	$69.4^{+1.7}_{-1.7}$	$14.10^{+0.35}_{-0.33}$	$-0.69^{+0.11}_{-0.11}$	1.6	0.48
C	$0.99^{+0.57}_{-0.39}$	< 0.28	> 2.28	$71.8^{+1.4}_{-1.4}$	$13.62^{+0.27}_{-0.27}$	$-0.91^{+0.22}_{-0.22}$	2.5	0.90
D	$0.72^{+0.11}_{-0.08}$	< 0.27	> 2.32	$66.6^{+1.9}_{-2.0}$	$14.70^{+0.45}_{-0.41}$	$-0.49^{+0.05}_{-0.05}$	13.5	2.60
E	$0.96^{+0.19}_{-0.14}$	< 0.10	> 3.33	$65.2^{+2.1}_{-2.2}$	$15.01^{+0.53}_{-0.47}$	$-0.44^{+0.06}_{-0.06}$	43.6	3.78
Logarithmic prior								
A	$0.23^{+0.19}_{-0.16}$	< 0.31	> 2.16	$69.8^{+1.9}_{-2.2}$	$14.03^{+0.45}_{-0.39}$	$-0.69^{+0.16}_{-0.16}$	2.0	0.67
B	$0.36^{+0.24}_{-0.24}$	< 0.47	> 1.74	$69.3^{+2.1}_{-2.3}$	$14.12^{+0.49}_{-0.41}$	$-0.74^{+0.14}_{-0.14}$	2.2	0.79
C	$1.82^{+0.76}_{-0.56}$	< 0.06	> 3.79	$71.7^{+1.4}_{-1.3}$	$13.64^{+0.25}_{-0.26}$	$-0.86^{+0.14}_{-0.14}$	1.8	0.57
D	$0.62^{+0.06}_{-0.05}$	< 0.05	> 3.98	$67.2^{+1.6}_{-1.5}$	$14.56^{+0.33}_{-0.33}$	$-0.54^{+0.03}_{-0.03}$	6.3	1.84
E	$0.85^{+0.17}_{-0.13}$	< 0.05	> 3.95	$65.8^{+2.2}_{-2.4}$	$14.87^{+0.56}_{-0.47}$	$-0.47^{+0.05}_{-0.05}$	25.4	3.24

Results

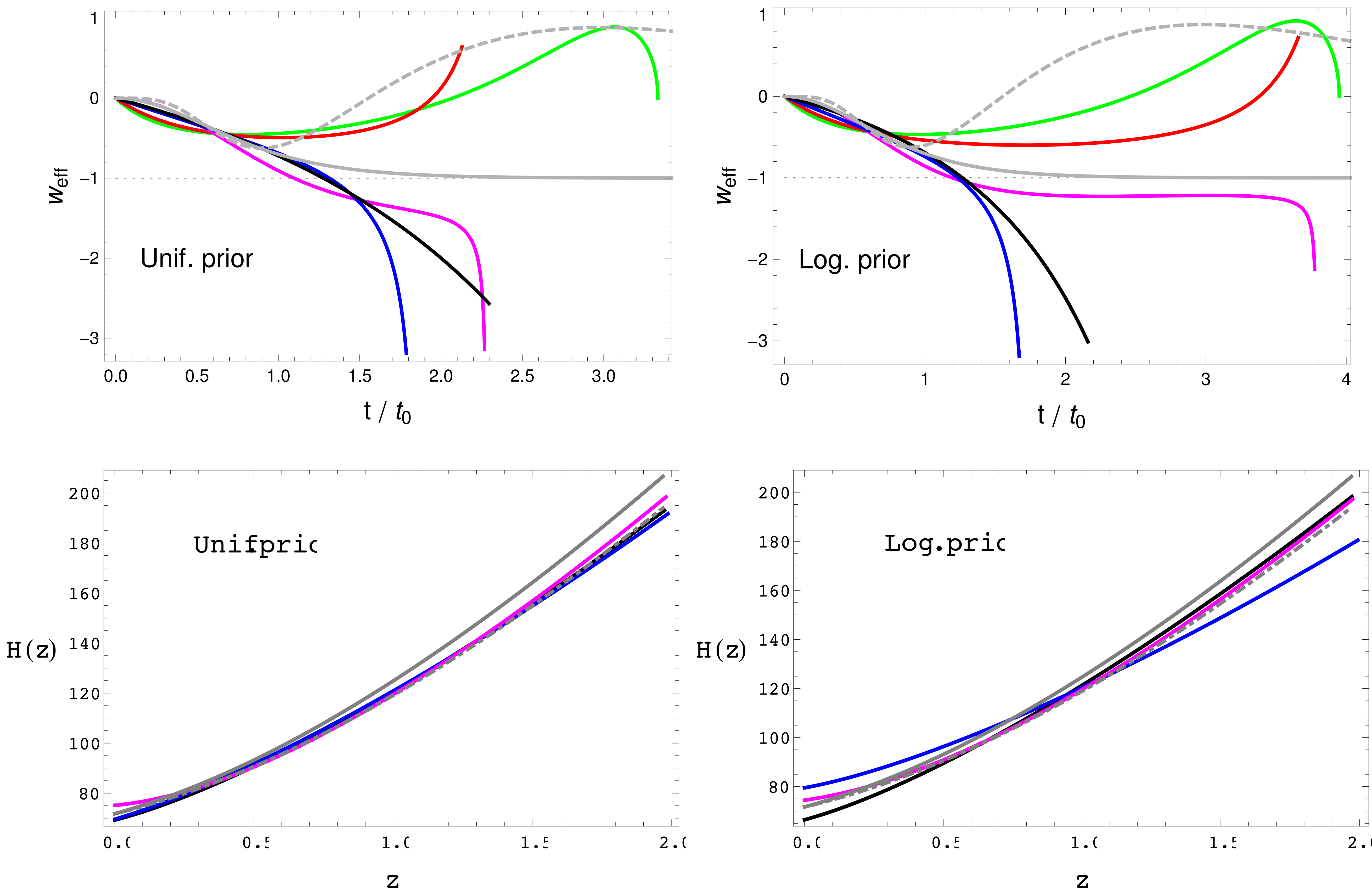


t_s / t_0
∞
3.30
2.61
2.20
1.92
1.69
1.51
1.36
1.22
1.11
1.00
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
n
Model A
0.0
0.2
0.4
0.6
0.8
1.0
α



t_s / t_0
∞
3.30
2.61
2.20
1.92
1.69
1.51
1.36
1.22
1.11
1.00
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
n
Model B
0.0
0.2
0.4
0.6
0.8
1.0
α

Results



Conclusions

- Two of the models (A and B) are as good as LCDM model, according to Jeffreys scale.

- The prior on α determines the higher bound on ts, going to infinity when assuming a logarithmic prior.

- We found that the proximity of the singularity to the present time has a mild dependence on the type of singularity for our parameterisations, but we can conclude that in all cases there is a consistent lower bound around 1.2 – $1.5t_0$.

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Stabilization of the sign of the gravitational constant

José Pedro Mimoso

Collaboration with F. S. Lobo and Nelson J. Nunes

Faculty of Science, University of Lisbon & IA

IberiCos 2016, March 30

JPM, FL, & NN thank the financial support of the IA grant UID/FIS/04434/2013.

General Aim

- We speculate on the possibility that the gravitational constant G might be negative in the past history of the universe and investigate a cosmological mechanism that stabilizes the positiveness of the sign of G.

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Newton and Einstein's G

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- Newton "De mundi systemate":
  - *(...) the forces are like the quantity of matter in each body (...)*
  - *Given that the centripetal action on the attracted body is, at a given distance, proportional to the quantity of matter of the latter, it is reasonable that it should also be proportional to the quantity of matter of the attracting body. Thus the action is mutual.*
- P. S. Laplace (Traité de Mécanique Céleste, 1799):

  Seemingly, for the first time an explicit gravitational constant

$$F = -k^2 \frac{m_1 m_2}{r^2} \tag{1}$$

  positive ... yet

- H. Cavendish (1798) Torsion Balance: 6.75×10^{-11} N m^2 /kg^2 (quite close to the present value of 6.67259×10^{-11} N m^2 /kg^2 .

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Motivation and Goals II

- Recently, several new measurements from respected research teams in Germany, New Zealand, and Russia have produced new values of G that wildly disagree (see http://www.npl.washington.edu/eotwash/bigG)

- Also, a controversial 2015 study by Anderson et al. [EPL 110, arXiv:1504.06604] suggested a periodic variation having a period of 5.9 years, similar to that observed in the length of day (LOD) measurements ...

- G. Rosi *et al* in Nature. 2014 Jun 26;510(7506):518-21,

  "*report the precise determination of G using laser-cooled atoms and quantum interferometry. We obtain the value* $G = 6.67191(99) \times 10^{-11}$ *m*^{3} *kg*$^{-1}$ *s*$^{-2}$*, with a relative uncertainty of 150 parts per million (the combined standard uncertainty is given in parentheses).*"

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- A. Einstein The general theory (1915)

$$G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}$$

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- P. M. A. Dirac (1938) "*The large numbers hypothesis*"

$$G \propto H \ ,$$

- Unification theories: H. Weyl (1919), Kaluza-Klein (1921), Einstein (himself) and collaborators
- Mach's principle (D. Sciama), and in the suite C. Brans and R. Dicke (1961) proposal of theory conveying the variation of G to fulf the latter goal
- Ggravitational fields near curvature singularities;
- First order approximation for the quantum theory of gravitational fields.
- Renormalization approaches to GR in the 1960s and 1970s equations of fourth or higher order, instead of second.

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Goals

- Consider general scalar-tensor gravity theories (extending BD theory)
- Envisage the possibility of G becoming negative
- Perform a study of the dynamics of cosmological models.
- Assess whether there is any cosmological mechanism that drives the sign of the gravitational constant to be positive and stabilizes the sign of G.

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- So, we consider a general scalar-tensor gravity theories given by the action

$$S = \int \left[\phi\, R - \frac{\omega(\phi)}{\phi}\, \phi_{,\mu}\phi^{,\mu} - 2U(\phi) + 16\pi L_m \right] \sqrt{-g}\; d^4x\ , \tag{4}$$

where a potential term $U(\phi)$ of cosmological nature is considered. (We shall also use $U(\phi) = \phi\,\lambda(\phi)$)

- The field equations are

$$\begin{aligned} R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta}\, R - \lambda(\phi)\, g_{\alpha\beta} &= \frac{\omega(\phi)}{\phi^2} \left[\phi_{;\alpha}\phi_{;\beta} - \frac{1}{2}\, g_{\alpha\beta}\phi_{;\gamma}\phi^{;\gamma} \right] \\ &+ \frac{1}{\phi}\, \left[\phi_{;\alpha\beta} - g_{\alpha\beta}\phi_{;\gamma}{}^{;\gamma} \right] + 8\pi\, \frac{T_{\alpha\beta}}{\phi} \end{aligned} \tag{5}$$

$$\Box\phi + \frac{2\phi^2\lambda'(\phi) - 2\phi\lambda(\phi)}{2\omega(\phi) + 3} = \frac{1}{2\omega(\phi) + 3} \left[8\pi\, T - \omega'(\phi)\phi_{;\gamma}\phi^{;\gamma} \right] \tag{6}$$

where $T \equiv T^c{}_c$ is the trace of the energy-momentum tensor, $T_\alpha{}^\beta$.

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FLRW equations

- For the FLRW models the field equations read

$$3\left(\frac{\dot a}{a}\right)^2 + 3\frac{\dot a}{a}\frac{\dot\phi}{\phi} + 3\frac{k}{a^2} = \lambda(\phi) + \frac{\omega(\phi)}{2}\frac{\dot\phi^2}{\phi^2} + 8\pi\frac{\rho}{\phi} \tag{7}$$

$$2\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot a}{a}\right) + 3\left(\frac{\dot a}{a}\right)^2 + 2\frac{\dot a}{a}\frac{\dot\phi}{\phi} + \frac{k}{a^2} = \lambda(\phi) - \frac{\omega(\phi)}{2}\frac{\dot\phi^2}{\phi^2} - 8\pi\frac{p}{\phi} - \frac{\ddot\phi}{\phi} \tag{8}$$

$$\begin{aligned} \ddot\phi + 3\frac{\dot a}{a}\dot\phi \quad & - \quad \frac{2\phi^2\lambda'(\phi) - 2\phi\lambda(\phi)}{2\omega(\phi)+3} = \\ & - \quad \frac{1}{2\omega(\phi)+3}\left[8\pi(3p-\rho) + \omega'(\phi)\dot\phi^2\right] \quad . \end{aligned} \tag{9}$$

Notice that the cosmological potential $U(\phi) = \phi\,\lambda(\phi)$ effectively reduces to a cosmological constant when $\lambda(\phi) = \lambda_0 =$ constant in this frame

Introduce the redefined variables

$$X = \phi\, a^2 \,, \qquad Y = \sqrt{\frac{2\omega(\phi)+3}{3}}\,\frac{\phi'}{\phi} \tag{10}$$

and use conformal time $\mathrm{d}\eta = \mathrm{d}t/a = \mathrm{d}\tilde{t}/\sqrt{X}$, where $\mathrm{d}\tilde{t} = \sqrt{\phi}\,\mathrm{d}t$.

- The generalized Friedmann equation

$$(X')^2 + 4\,k\,X^2 - (Y'\,X)^2 = 4M\;X\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}} + \frac{4}{3}\left(\frac{\lambda(\phi)}{\phi}\right)X^3 \tag{11}$$

- The scalar-field equation

$$\left[Y'\,X\right]' = M(4-3\gamma)\sqrt{\frac{3}{2\omega+3}}\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}} - \frac{2\,X^2}{\sqrt{2\omega(\phi)+3}}\left(\frac{\lambda(\phi)}{\phi} - \frac{\mathrm{d}\lambda}{\mathrm{d}\phi}\right) \tag{12}$$

- The generalized Raychaudhuri equation

$$X'' + 4\,k\;X = 3\,M(2-\gamma)\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}} + 2X^2\left(\frac{\lambda(\phi)}{\phi}\right) \tag{13}$$

where M is a constant defined by $M \equiv 8\pi\rho_0/3$.

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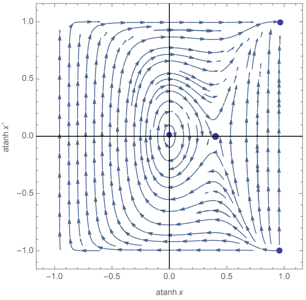
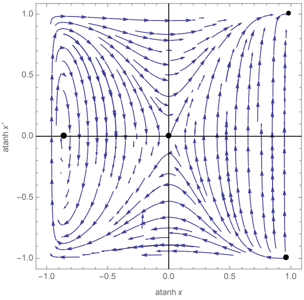
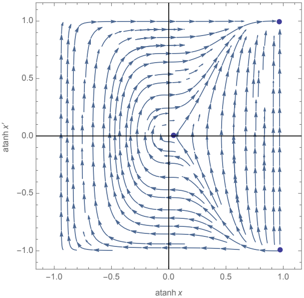
Qualitative analysis







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Towards the most general scalar-tensor theories of gravity: a unified approach in the language of differential forms

Jose María EZQUIAGA

Based on:
arXiv 1603.01269 by JME, J. GARCÍA-BELLIDO and M. ZUMALACÁRREGUI

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SUMMARY

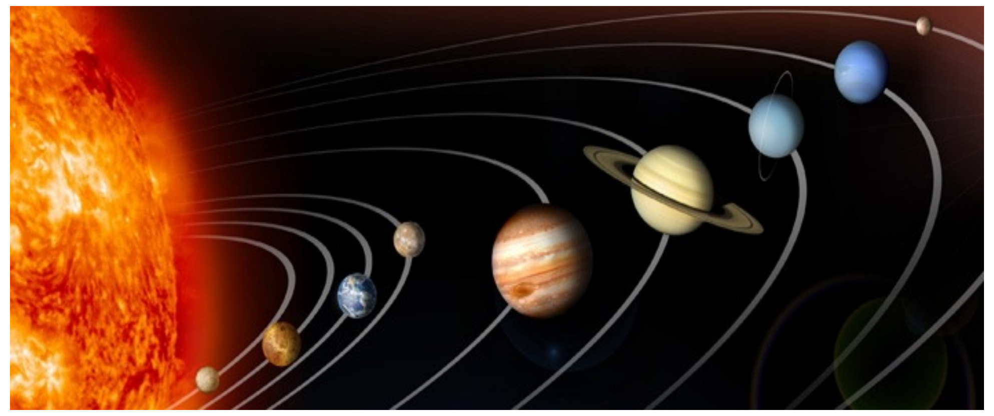
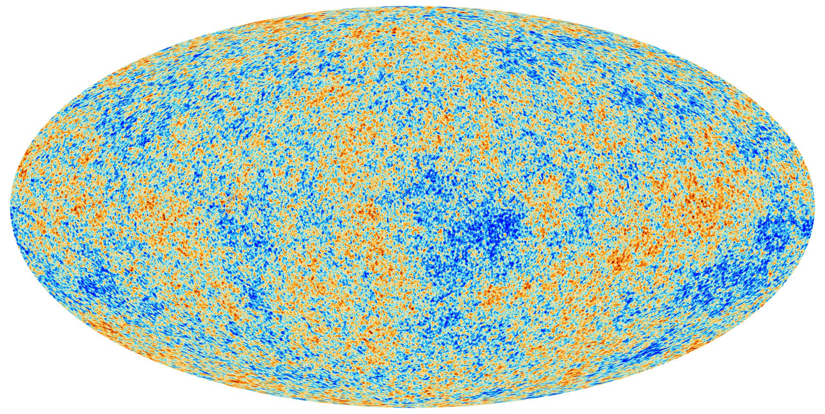
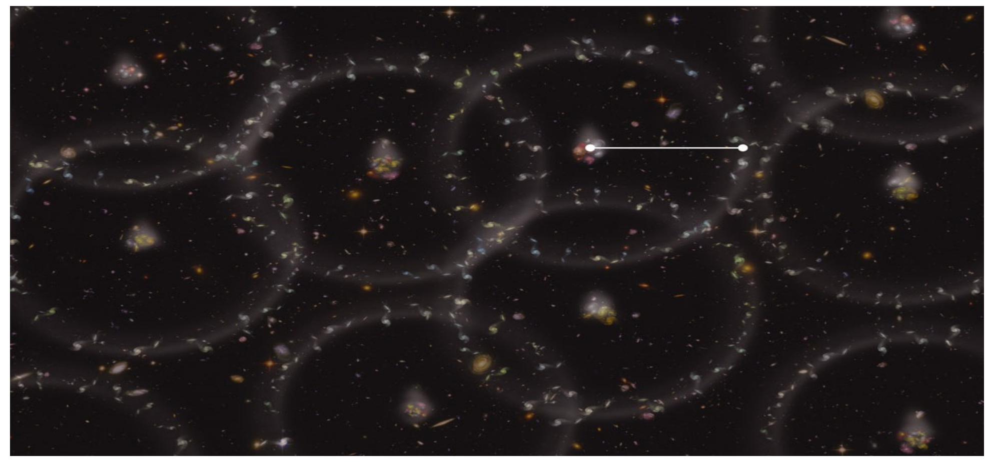
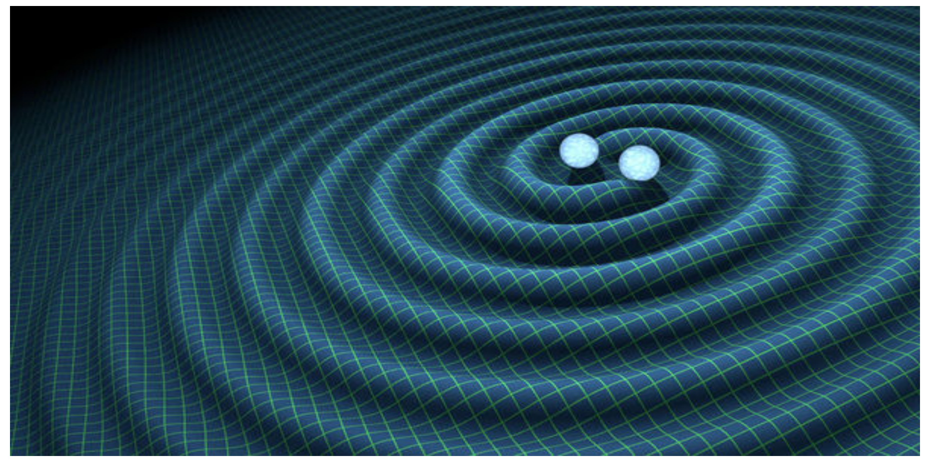
[arXiv 1603.01269]

- We have systematically explored the space of scalar-tensor theories of gravity using a **novel approach** based on **differential forms**.
- We have found a **finite** and **closed** basis of Lagrangians that describes general scalar-tensor theories. Among others, it contains Horndeski's and Beyond Horndeski's Lagrangians.
- In order to determine which combinations of Lagrangians give rise to second order equations of motion (e.o.m.), thus becoming automatically ghost free, we have computed the **e.o.m.**
- With the objective of distinguishing truly independent theories, we have determined all possible **exact forms** (total derivatives) and **antisymmetric identities** relating different Lagrangians.
- This new formulation has **interesting potential applications** due to its computational simplicity and systematic structure.

1 Scalar-Tensor Theories in a Nutshell

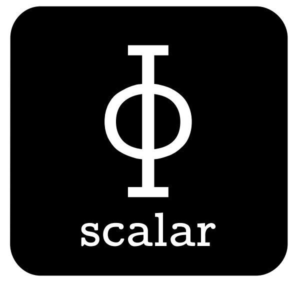
<u>Cosmological Considerations</u>:

- Epochs of Accelerated Expansion: Early and Late Universe
- Tests of General Relativity and ΛCDM

<u>Theoretical Considerations</u>:

- Simplest modification GR: add 1 degree of freedom


$+$


- Warning: Ostrogradski's Theorem

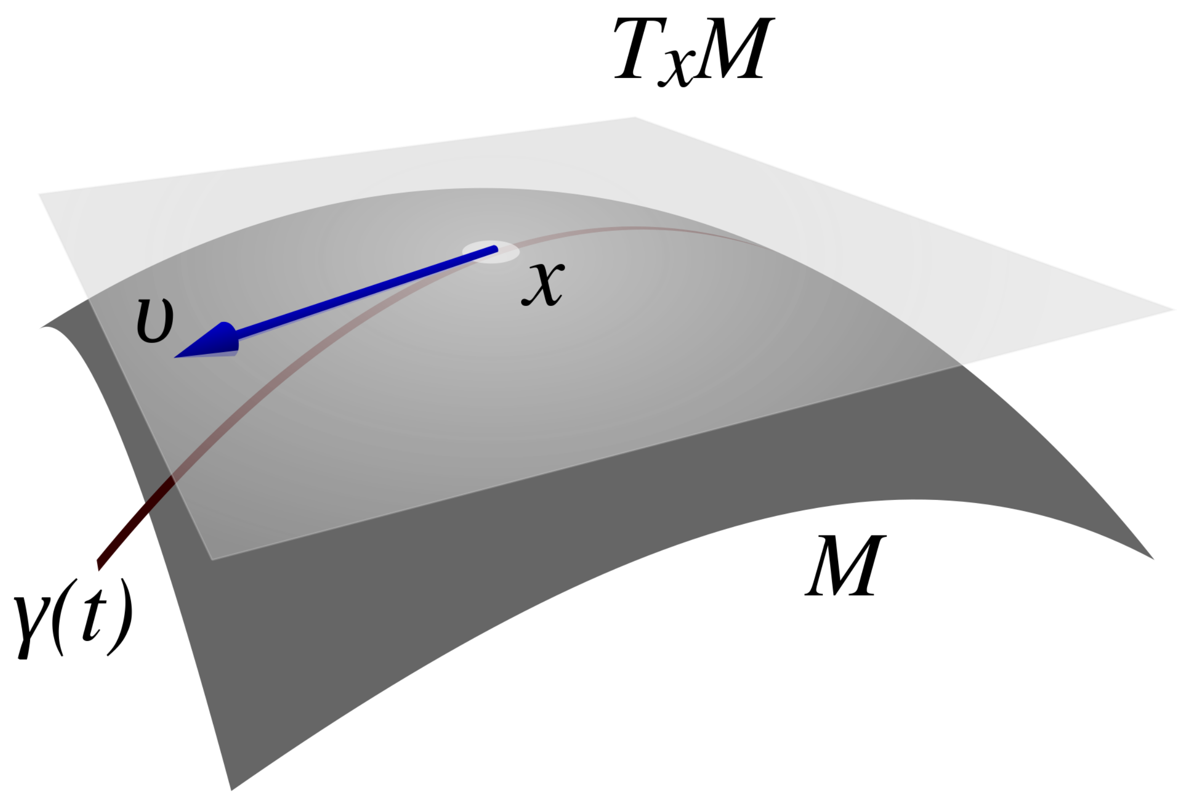
[Ostrogradski 1850]

E.o.m. with higher than 2 time derivatives induce linear instabilities in the Hamiltonian

- Most general second order e.o.m. in 4D given by **Horndeski**'s theory [Horndeski 1974]

(local+Diff. inv. theories)

2 Differential Forms and Gravity



- General Covariance (Diff Inv.) can be reinterpreted as the invariance under Local Lorentz Transformations (LLT) in the Tangent Space

- Needed to couple fermions to gravity!

- In a pseudo-Riemannian manifold (usual spacetime without torsion and metric compatible)
  - Geometry (and Physics) is encoded in the vielbein θ^a and the 1-form connection $\omega^a{}_b$ $\Rightarrow$ $\boxed{\mathcal{R}^a{}_b}$

- Example: Lovelock's Theory [Lovelock 1971, Zumino 1986]

$$\boxed{\mathcal{L}_{(l)} = \bigwedge_{i=1}^{l} \mathcal{R}^{a_i b_i} \wedge \theta^\star{}_{a_1 b_1 \cdots a_l b_l}} \qquad \text{where} \qquad \theta^\star{}_{a_1 \cdots a_k} = \frac{1}{(D-k)!} \epsilon_{a_1 \cdots a_k a_{k+1} \cdots a_D} \theta^{a_{k+1}} \wedge \cdots \wedge \theta^{a_D}$$

Differential Forms Dictionary

$$\mathfrak{g} = g_{\mu\nu} dx^\mu \otimes dx^\nu = \eta_{ab}\theta^a \otimes \theta^b$$

[Nakahara, *Geometry, Topology and Physics*, 2003]

Metric Formalism

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$$
$$\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}(\partial_\mu g_{\alpha\nu} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\mu\nu})$$
$$R^\lambda{}_{\mu\nu\rho} = \partial_\nu \Gamma^\lambda{}_{\mu\rho} - \partial_\rho \Gamma^\lambda{}_{\mu\nu} + \Gamma^\gamma{}_{\mu\rho}\Gamma^\lambda{}_{\gamma\nu} - \Gamma^\gamma{}_{\mu\nu}\Gamma^\lambda{}_{\gamma\rho}$$
$$\nabla_\mu g_{\alpha\beta} = 0$$
$$\Gamma^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\nu\mu}$$

Vielbein Formalism

$$\theta^a = e^a{}_\mu dx^\mu$$
$$\omega^{ab} = \frac{1}{2}\left(\mathfrak{i}_{e^b}(d\theta^a) - \mathfrak{i}_{e^a}(d\theta^b) + \mathfrak{i}_{e^a}(\mathfrak{i}_{e^b}(d\theta_c))\theta^c\right)$$
$$\mathcal{R}^a{}_b = \mathcal{D}\omega^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b$$
$$\omega_{ab} = -\omega_{ba}$$
$$T^a = \mathcal{D}\theta^a = 0$$

- Invariant objects: η_{ab} and $\epsilon_{a_1 a_2 \cdots a_D}$
- Basic operations: wedge product, exterior differential, integration...
- Basic identities: Cartan structure eqs and Bianchi identities $T^a = \mathcal{D}\theta^a$, $\mathcal{R}^a{}_b = \mathcal{D}\omega^a{}_b$, $\mathcal{D}T^a = 0$ and $\mathcal{D}\mathcal{R}^a_b = 0$

3 A general basis for Scalar-Tensor Theories [arXiv 1603.01269]

- Define 1-forms with derivatives of the scalar field (at lowest order)

$$\boxed{\Psi^a \equiv \nabla^a\phi\nabla_b\phi\ \theta^b} \qquad \boxed{\Phi^a \equiv \nabla^a\nabla_b\phi\ \theta^b}$$

and construct a basis of Lagrangians invariant under LLT in a pseudo-Riemannian manifold.

$$\boxed{\mathcal{L}_{(lmn)} = \bigwedge_{i=1}^{l} \mathcal{R}^{a_i b_i} \wedge \bigwedge_{j=1}^{m} \Phi^{c_j} \wedge \bigwedge_{k=1}^{n} \Psi^{d_k} \wedge \theta^{\star}{}_{a_1 b_1 \cdots a_l b_l c_1 \cdots c_m d_1 \cdots d_n}}$$

$\longrightarrow$ Clear structure in terms of the number of fields: $p \equiv 2l + m + n \leq D$

$\longrightarrow$ *Finite* basis due to antisymmetry

$\longrightarrow$ Contains well-known theories, e.g. Horndeski and Beyond Horndeski

- Action of a general scalar-tensor theory: $$\boxed{S = \sum_{l,m,n}^{p\leq D} \int_{\mathcal{M}} \alpha_{lmn} \mathcal{L}_{(lmn)}}$$

- Examples: some 4D Lagrangians $\left(\eta \equiv \sqrt{-g} d^4x, \;\; {\Phi^n}_{\mu\nu} = \phi_{;\mu\alpha_1} {\phi^{;\alpha_1}}_{;\alpha_2} \cdots {\phi^{;\alpha_{n-1}}}_{;\nu}, \;\; [\Phi^n] \equiv {\Phi^n}_{\mu\nu}\, g^{\mu\nu}\right)$

$$\begin{aligned}
\mathcal{L}_{(001)} &= \Psi^a \wedge {\theta^\star}_a = \nabla_\mu \phi \nabla^\mu \phi \eta \equiv -2X\eta \\
\mathcal{L}_{(010)} &= \Phi^a \wedge {\theta^\star}_a = [\Phi]\eta \\
\mathcal{L}_{(110)} &= \mathcal{R}^{ab} \wedge \Phi^c \wedge {\theta^\star}_{abc} = (-2R^{\mu\nu} + R g^{\mu\nu})\Phi_{\mu\nu}\eta = -2(G^{\mu\nu}\Phi_{\mu\nu})\eta \\
\mathcal{L}_{(030)} &= \Phi^a \wedge \Phi^b \wedge \Phi^c \wedge {\theta^\star}_{abc} = ([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3])\eta \\
\mathcal{L}_{(200)} &= \mathcal{R}^{ab} \wedge \mathcal{R}^{cd} \wedge {\theta^\star}_{abcd} = (R_{\mu\nu\rho\gamma} R^{\mu\nu\rho\gamma} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2)\eta
\end{aligned}$$

- The basis is *closed* under exterior derivatives if contractions with the gradient field are included
  - Notation: over bar indicates contractions with $\nabla^a \phi$ e.g. $\mathcal{L}_{(0\bar{1}0)} = \nabla_a \phi \Phi^a \wedge {\theta^\star}_b \nabla^b \phi$
  - Additional Lagrangians: $\mathcal{L}_{(\bar{l}mn)}$ and $\mathcal{L}_{(l\bar{m}n)}$

4 Results

[arXiv 1603.01269]

- We compute the e.o.m. both for the scalar field ϕ and the vielbein θ^a for arbitrary dimensions

  ⟶ The calculations greatly simplifies

  ⟶ We find all possible Lagrangians with 2nd order e.o.m.

- We obtain all the exact forms (total derivatives) and antisymmetric algebraic identities

  ⟶ We determine the number of independent Lagrangians

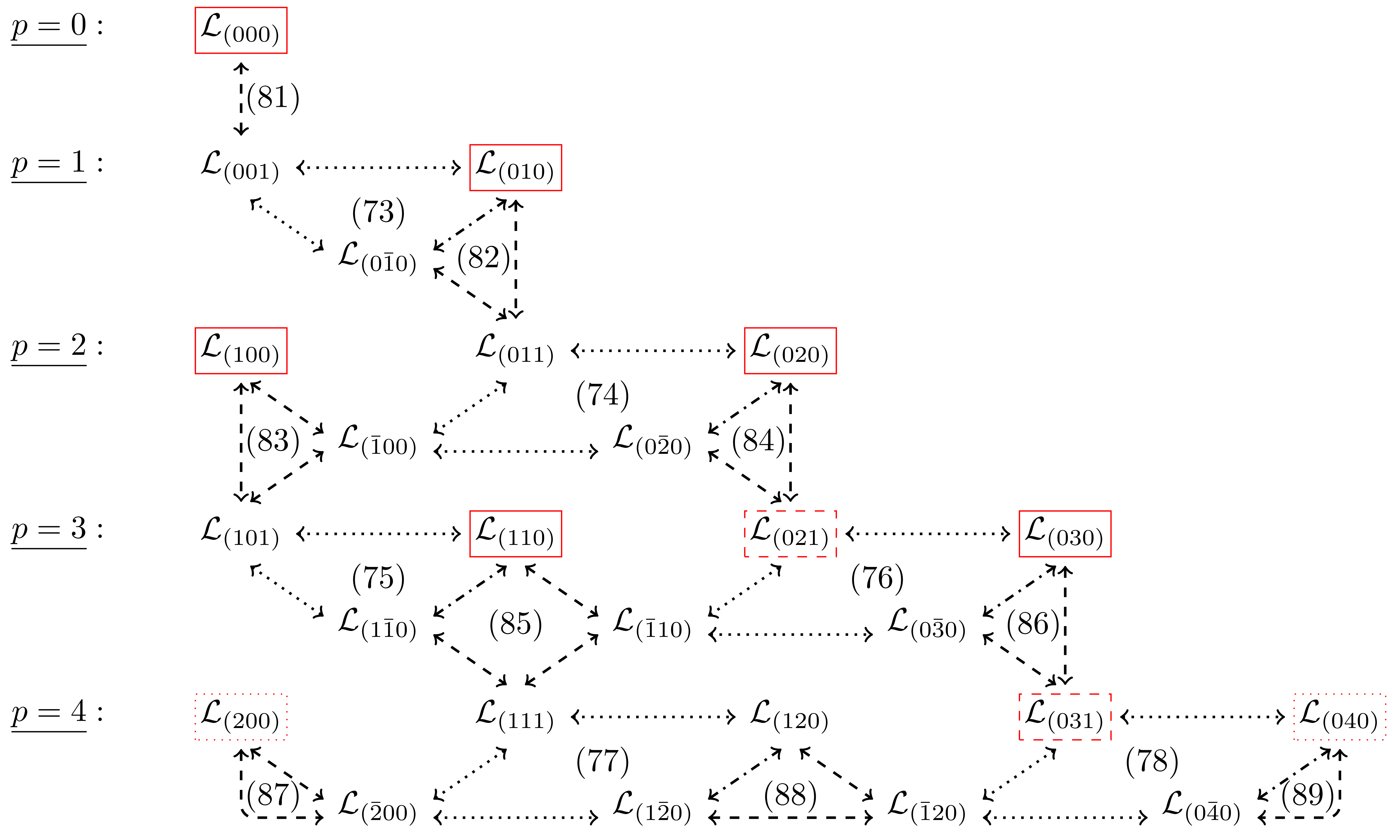
- Results in 4D:

  ⟶ There are 10 independent elements in the basis of Lagrangians

  ⟶ Only 4 independent linear combinations give rise to 2nd order e.o.m.

  -This set can be associated with Horndeski's theory

- Relations among second order theories in 4D:



$$\begin{aligned}
\mathcal{L}_2^H[G_2] &= G_2\mathcal{L}_{(000)} \\
\mathcal{L}_3^H[G_3] &= G_3\mathcal{L}_{(010)} \\
\mathcal{L}_4^H[G_4] &= G_4\mathcal{L}_{(100)} + G_{4,X}\mathcal{L}_{(020)} \\
\mathcal{L}_5^H[G_5] &= G_5\mathcal{L}_{(110)} + \frac{1}{3}G_{5,X}\mathcal{L}_{(030)}
\end{aligned}$$

$\leftarrow\cdots\rightarrow$ $\mathcal{D}\mathcal{L}^{D-1}_{(lmn)}[G_i] = G_{i,\phi}\mathcal{L}_{(lm(n+1))} - G_{i,X}\mathcal{L}_{(l\overline{(m+1)}n)} + G_i\left(\mathcal{L}_{(l(m+1)n)} - m\mathcal{L}_{(\overline{(l+1)}(m-1)n)} - n\mathcal{L}_{(l(m+1)n)}\right)$

$\leftarrow - - \rightarrow$ $\mathcal{L}_{(lm1)} = -2l\mathcal{L}_{(\bar{l}m0)} - m\mathcal{L}_{(l\bar{m}0)} - 2X\mathcal{L}_{(lm0)}$

5 Discussion

[arXiv 1603.01269]

- **New formulation** for scalar-tensor theories in the language of differential forms.

- This approach **simplifies** the computations and allows for a **systematic** classification of general scalar-tensor theories and the relations among them.

- We have proven that **Horndeski**'s theory correspond to the most general Lagrangian in 4D invariant under LLT in a pseudo-Riemannian manifold and constructed with θ^a, $\mathcal{R}^{ab}$, Ψ^a and Φ^a

- The relations among second order theories can be used to connect, for instance, different covariantizations of **Galileons** theory.

6 Future Prospects

[arXiv 1603.01269]

- There are interesting potential applications of this new formalism:
  - Analyze phenomenological and theoretical properties of **concrete** scalar-tensor models.
  - Investigate the role of **fermions** in scalar-tensor theories of gravity.
  - Explore general **field redefinitions** and determine which of them leave this basis of Lagrangians unchanged
  - Systematically study scalar-tensor theories with **higher derivative** e.o.m.

Thank you

Find more details at

arXiv 1603.01269 by JME, J. GARCÍA-BELLIDO, M. ZUMALACÁRREGUI

or by e-mail

jose.ezquiaga@uam.es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Bridging cosmology and astrophysics with gravitational waves

Ippocratis Saltas

Institute of Astrophysics and Space Sciences, U. Lisbon

funded by FCT SFRH/BPD/95204/2013

based on

IDS, I. Sawicki, L. Amendola, M. Kunz, PRL 113 191101 (2013)

and work to appear very soon together with L. Amendola, M. Kunz, M. Motta, I. Sawicki

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Gravitational waves exist! But what can they teach us about the theory of gravity?

What drives the late-time acceleration of the universe? Is it dark energy, modified gravity, or Λ ?

This talk: The implications of large-scale modifications of gravity for the propagation of gravitational waves and vice versa

What is modified gravity?

Modifications of gravity will modify in one or the other way the propagation of the only degree of freedom in GR: the graviton

$$h''_{ij} + (2 + \alpha_M)Hh'_{ij} + c_T^2 k^2 h_{ij} + a^2 \mu^2 h_{ij} = a^2 \Gamma\gamma_{ij}$$

Horndeski:
The most general, second-order theory built out of the metric and a **scalar** ϕ

$$\alpha_M \equiv H(t)^{-1} \frac{d \ln M_p^2}{dt} \quad \text{and} \quad c_T^2 \neq 0$$

Einstein-Aether:
GR minimally coupled to a dynamical **vector** field with non-trivial vacuum-expectation value

$$c_T^2 \neq 0$$

Bi-metric gravity:
Most general theory constructed out of two interacting, **rank-two** fields

$$\mu^2 \neq 0 \quad \text{and} \quad \Gamma\gamma_{ij} \neq 0$$

The gravitational slip and its significance

The evolution of large-scale structures can be well-described by small, scalar inhomogeneities around an FLRW spacetime

$$ds^2 = -\left(1 + 2\,\Psi(t, \mathbf{x}\,)\right) dt^2 + a(t)^2 \left(1 + 2\,\Phi(t, \mathbf{x}\,)\right) d\mathbf{x}^2$$

Galaxies are test particles which' density fraction is related to that of dark matter through a bias function b(z,k)

$$\delta_b(z, k) = \; b(z, k)\; \delta_m(z, k)$$

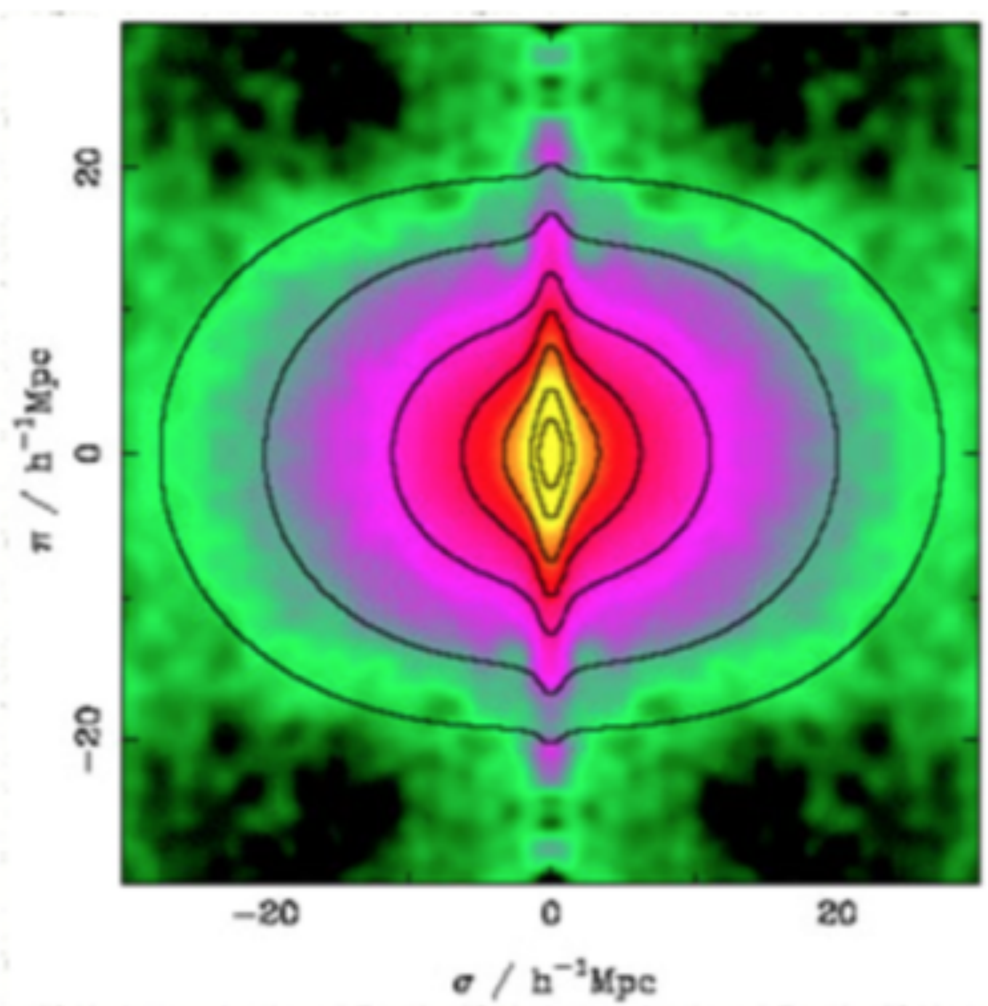
Neglecting any relativistic species, any genuine modification of gravity will source a gravitational slip

$$\mathbf{\Phi} \;-\; \mathbf{\Psi} \;= \mathbf{A}\left[\sigma_i\right] \cdot \mathbf{X}(t, k) \qquad\qquad \eta \equiv \frac{\mathbf{\Phi}}{\mathbf{\Psi}} \neq 1$$

Free model parameters

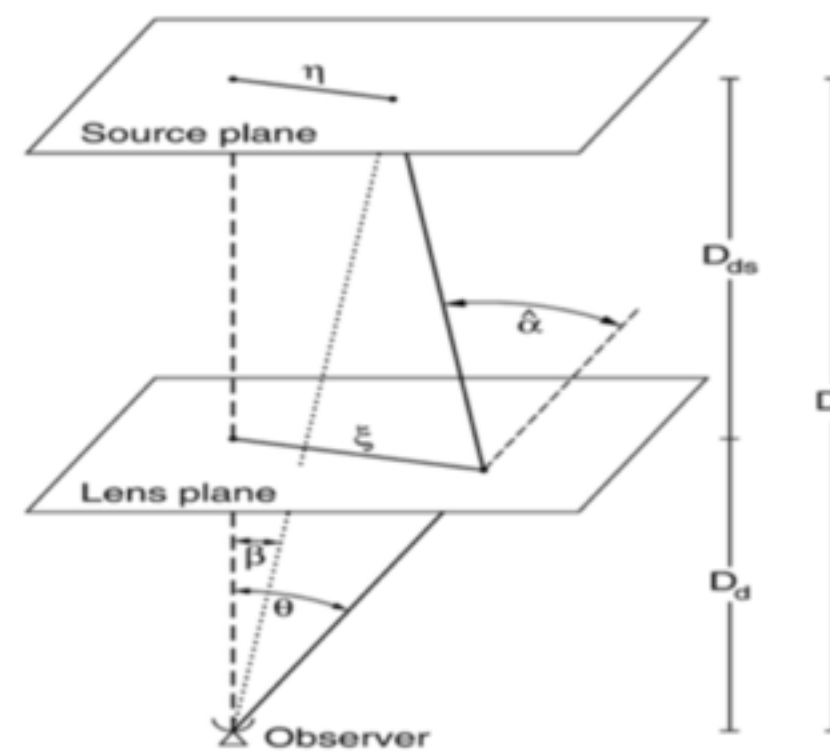
Observables on the sky

Studying galaxy clustering
in redshift space reveals
information about galactic velocities



$$s(\mathbf{r}) = \mathbf{r} + v(\mathbf{r}) \cdot \hat{\mathbf{r}}, \qquad \nabla v(r) \sim \nabla \Psi(r)$$

Light reaching us from distant sources
gets distorted on its way
due to large matter inhomogeneities



$$\frac{d^2 x^i}{dr^2} = \frac{\partial}{\partial x^i}\left(\Phi + \Psi\right) \equiv \frac{\partial}{\partial x^i}\Phi_{lens}$$

The gravitational slip is a model-independent observable

$$\eta(z,k) \equiv \frac{\Phi(z,k)}{\Psi(z,k)} = \frac{3(1+z)^3}{2E^2\left(\mathcal{O}'_\theta/\mathcal{O}_\theta + E'/E + 2\right)} \frac{\Phi_{lens}}{\mathcal{O}_\theta} - 1$$

$$E(z) \equiv H(z)/H_0 \qquad \mathcal{O}_\theta \equiv -\nabla v_{gal}/H$$

$\eta = 1$	$\eta \neq 1$
ΛCDM	f(R), f(Gauss-Bonnet), …, Horndeski
GR + minimally-coupled to curvature scalar: Quintessence, k-essence, KGB	Einstein-Aether
	Bi-metric (massive) gravity

L. Amendola, M. Kunz, IDS, I. Sawicki PRD 87, 023501 (2013)/M. Motta, I.Sawicki, M. Kunz, IDS, PRD 88, 124035 (2013)

Non-trivial gravitational slip leads to modified propagation of gravitational waves

For general modified gravity models introducing an extra scalar (*Horndeski*), vector (*Einstein-Aether*) and a tensor field (*massive/bi-metric*), *the theory parameters controlling the linear anisotropic stress match those modifying the evolution of gravitational waves.*

$$h''_{ij} + (2 + \alpha_M)Hh'_{ij} + c_T^2 k^2 h_{ij} + a^2 \mu^2 h_{ij} = a^2 \Gamma\gamma_{ij}$$

$$\Phi - \Psi = \mathbf{A}(\alpha_M, c_T^2, \mu^2, \Gamma) \cdot \mathbf{X}(t, k) \qquad \eta \equiv \frac{\Phi}{\Psi} \neq 1$$

IDS, I. Sawicki, L. Amendola, M. Kunz, PRL 113 191101 (2013)

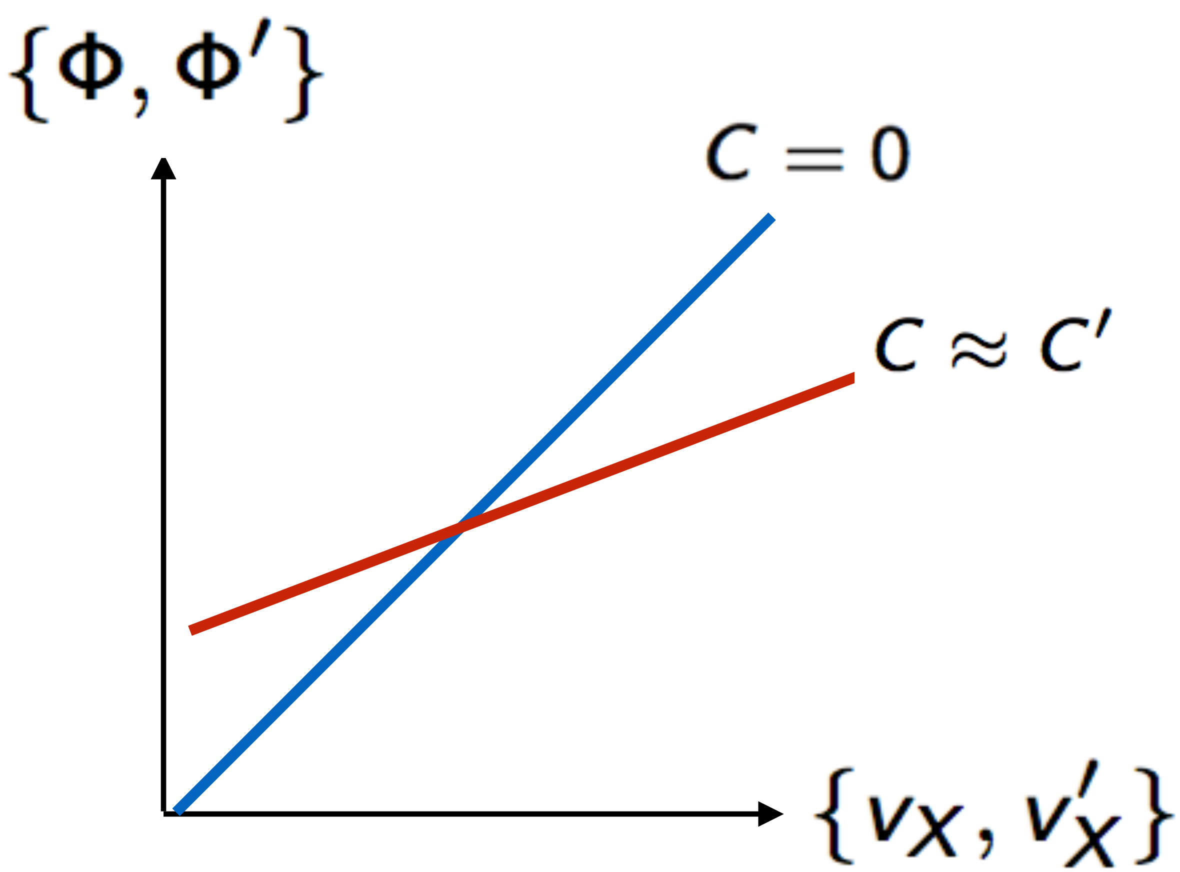
Modified propagation of gravitational waves implies the existence of shear at large scales

Can the large-scale effects of modified gravity be dynamically screened?

$$C \equiv \Phi - \Psi = \mathbf{A}\left[\sigma_i\right] \cdot \mathbf{X}(t, k)$$

model parameters

$\{\Phi, \Phi', v_X, v_X'\}$



If at some initial time and scale C = 0, preservation of the condition requires that on-shell

$$C \approx C' \qquad C \approx C'' \qquad C \approx C'''$$

Modified propagation of gravitational waves implies the existence of shear at large scales

Bi-metric gravity
1 free relevant coupling

μ^2

→

Einstein-Aether
3 free constant couplings

$\beta_1, \beta_2, \beta_3$

→

No go theorem:

No dynamical screening exists at large scales

Horndeski
4 free relevant functions of time

$\alpha_M, \alpha_T, \alpha_K, \alpha_B$

→

Numerical analysis:
No screening was found unless possible very fine tuned situations

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To appear very soon, together with L. Amendola, M. Kunz, M. Motta, I. Sawicki

Summary

The existence of gravitational slip modifies the propagation of gravitational waves:

The theory parameters controlling the gravitational slip, are exactly those modifying the propagation of gravitational waves.

A modified propagation of gravitational waves implies the existence of gravitational slip:

It is (almost) impossible to screen modifications of gravity at large scales within broad classes of models.


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Screenings with three-form fields

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Screening scalar fields

Scalar fields coupled to gravity

Chameleon fields, Vashtein mechanism, Symmetron fields, ...

The couplings depend on the local mass density:

Low mass density (out there)	High mass density (sun, earth,...)
large coupling $\sim$ altered gravity	small coupling $\sim$ GR

Screening vector fields

Action for a vector field B^μ ← Jimenez, Froes & Mota (2013)

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}F^2 - \frac{1}{2}(\nabla_\mu B^\mu)^2 - V(B^2)\right] + \int d^4x\sqrt{-\tilde{g}}\, L_m$$

with $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$

Matter couples to a conformal metric

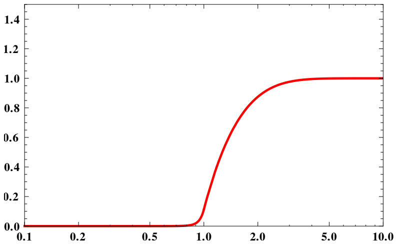
$$\tilde{g}_{\mu\nu} = \Omega^2(B^2) g_{\mu\nu}$$

Field equations in Minkowski spacetime

$$\Box B_\mu = \frac{\partial V_{\text{eff}}}{\partial B^2} B_\mu$$

$B_{z,in}(r)$ $\quad$ $B_{z,out}(r)$

$\frac{r_c}{r}\sinh(\alpha r/r_c)$ $\quad$ $\frac{r_c}{r}e^{-mr}+B_{z,0}$



Screening three-form fields

Action for a three-form field $A_{\alpha\beta\gamma}$

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{48}F^2 - V(A^2)\right] + \int d^4x\sqrt{-\tilde{g}}\,L_m$$

with $F_{\alpha\beta\gamma\delta} = 4\nabla_{[\alpha}A_{\beta\gamma\delta]}$.

again, matter couples to a conformal metric $\tilde{g}_{\mu\nu} = \Omega^2(A^2)g_{\mu\nu}$

Germani & Kehagias (2009), Koivisto, Mota & Pitrou (2010), Nunes&Koivisto (2010), ...

Dual vector field

It is more convenient to use the dual forms of A and F that are

- a vector field $B_\alpha = \frac{1}{3!}\varepsilon_{\alpha\beta\gamma\delta}A^{\beta\gamma\delta}$
- a scalar field $\Phi = \frac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta\gamma\delta}$

where $\varepsilon_{\alpha\beta\gamma\delta} = \sqrt{-g}\,\epsilon_{\alpha\beta\gamma\delta}$

in particular,

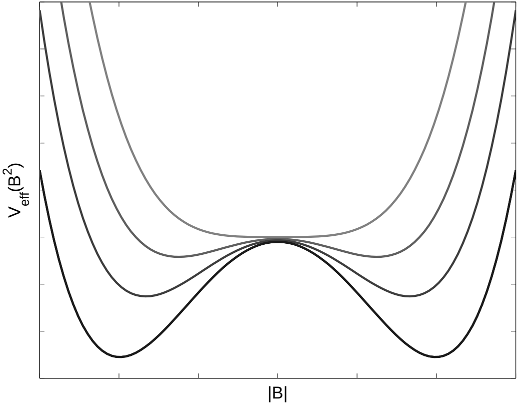
$$A^2 = -6B^2 \qquad F^2 = -24\Phi^2$$

Field equations

The field equations are

$$\underbrace{\nabla_\alpha(\nabla^\mu B_\mu)}_{\text{compare to } \Box B_\alpha} = \underbrace{-2\left(\frac{\partial V}{\partial B^2} + \rho\frac{\partial \Omega}{\partial B^2}\right)}_{\frac{\partial V_{\text{eff}}}{\partial B^2}} B_\alpha$$

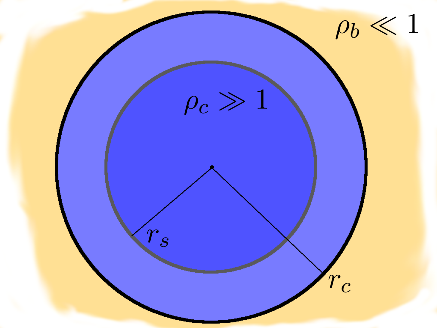
with the scalar field being $\Phi = \nabla^\mu B_\mu$



we use $V(B^2) = -\frac{1}{2}m^2B^2 - \frac{1}{4}B^4$ and $\Omega(B^2) = 1 + \frac{1}{2\mu^2}B^2$

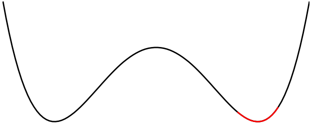
Spherical solutions

Spherical body: radius r_c, density ρ_c



"core" (inside)

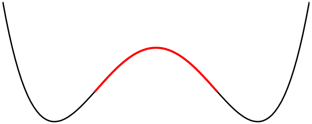
$r < r_s \qquad \rho_c \gg 1$



$m^2_{\text{eff}} \approx 2\rho_c/\mu^2$

"shell" (inside)

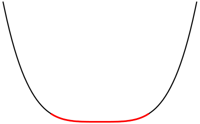
$r_s < r < r_c \qquad \rho_c \gg 1$



$m^2_{\text{eff}} \approx -\rho_c/\mu^2$

outside

$r > r_c \qquad \rho_b \ll 1$



$m^2_{\text{eff}} \approx m^2$

Spherical solutions

$\vec{B} = \vec{B}_0 + \vec{b}$

The field equations are $\vec{\nabla}(\vec{\nabla} \cdot \vec{b}) = m_{\text{eff}}^2 \vec{b}$

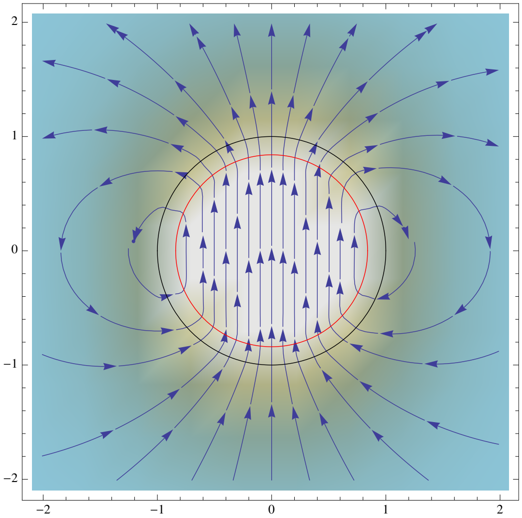
Defining ϕ so that $\vec{b} = \vec{\nabla}\phi$ they become

$$\nabla^2 \phi + m_{\text{eff}}^2 \phi = 0$$

Solutions are $\phi = \sum_l f_l(r) P_l(\cos\theta)$, where f_l are spherical Bessel functions.
We need the $l = 1$ solution:

inside: $f(r) = B_c r$
shell: $f(r) = a j_1(m_s r) + b y_1(m_s r) \approx \alpha \sin(m_s r + \beta)/(m_s r)$
outside: $f(r) = c k_1(m_0 r) = (1 + m_0 r) e^{-m_0 r}/(m_0 r)^2$

Spherical solutions



$$\Delta r \approx \frac{3\mu^2}{\rho_c r_c}$$

Observational constraints

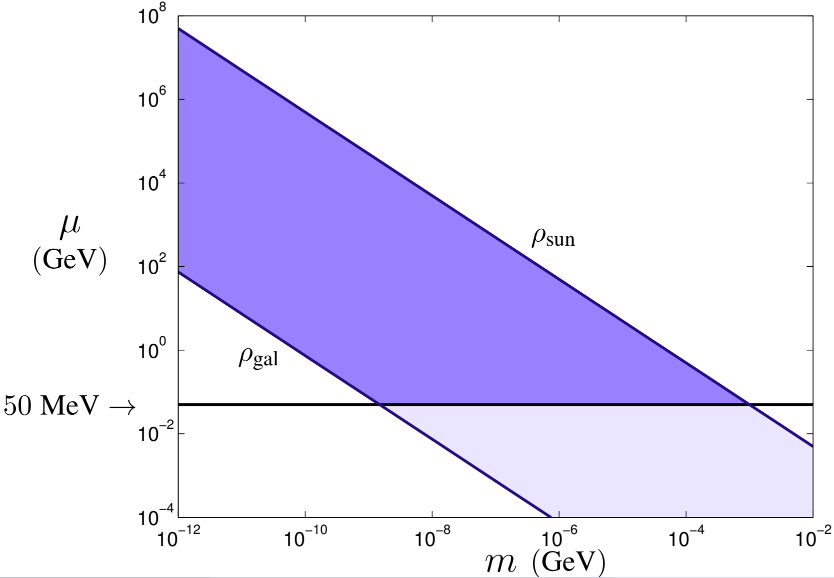
- Time-delay experiments
- Eddington parameter γ in the Jordan frame $\left| \begin{array}{l} \tilde{g}_{00} = -1 - 2\Psi \\ \tilde{g}_{ij} = 1 - 2\gamma\Psi \end{array} \right.$

$$\gamma = 1 - \frac{2B^2}{\mu^2 \Psi_E} \qquad B^2_{\text{max}} = \left(\frac{r_{\text{sun}}}{r}\right)^6 \frac{\rho_{\text{sun}}}{\mu^2}$$

- From the Cassini bound $|\gamma - 1| \lesssim 10^{-5}$ we get

$$\mu \gtrsim 50\,\text{MeV}$$

observational constraints



Summary

- We get a “thin shell” effect in this three-form scenario, the field is constant inside a massive body.
- Different behaviour to previous vector or scalar field solutions
- Angular dependence in modification of general relativity.

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