VILA DO CONDE,
PORTUGAL,
29-31 MARCH, 2016

11th Iberian Cosmology Meeting

IBERICOS 2016

SOC **ANA ACHÚCARRO** (LEIDEN/BILBAO), **FERNANDO ATRIO-BARANDELA** (SALAMANCA), **MAR BASTERO-GIL** (GRANADA), **JUAN GARCIA-BELLIDO** (MADRID), **RUTH LAZKOZ** (BILBAO), **CARLOS MARTINS** (PORTO), **JOSÉ PEDRO MIMOSO** (LISBON), **DAVID MOTA** (OSLO)

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SERIES OF MEETINGS WHICH AIM TO ENCOURAGE INTERACTIONS AND COLLABORATIONS BETWEEN RESEARCHERS WORKING IN COSMOLOGY AND RELATED AREAS IN PORTUGAL AND SPAIN.

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Status of multi-scale theory

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Instituto de Estructura de la Materia (IEM) – CSIC

March 31st, 2016

Motivations

- Dimensional flow: Changing behaviour of correlation functions, spacetime with scale-dependent 'dimension' ($d_{\rm H}$, $d_{\rm S}$). $d < 4$ in the UV. Universal feature in QG ['t Hooft 1993; Carlip 2009; G.C. PRL 2010] (perturbative QG, asymptotic safety, CDT, HL gravity, non-commutative spacetimes, LQG, spin foams, GFT).
- Dim. flow and UV finiteness? Power-counting renormalizability (gravity)?
- Phenomenology (from particle physics to cosmology)?

Proposal and results

- Dimensional flow at structural level via a change of integro-differential structure ("irregular" geometries).
- Captures the effective dynamics and the dim. flow of some non-commutative, QG, and VSL models [G.C. (et al.) PRD 2011, IJMPA 2013; PRD 2014].
- Power-counting renormalizability (gravity)?
- Independent approach with a lot of exotic physics (cosmology, particle physics, discrete geometry,. . .) and easily falsifiable predictions (many experimental constraints), much more easily than QG.

Multi-scale theories in a nutshell

G.C. 2012–2016

$$\int \mathrm{d}^D x\, \mathcal{L}[\partial_x, \phi^i] \to \int \mathrm{d}^D q(x)\, \mathcal{L}[\mathcal{D}_x, \phi^i]$$

$$q(x) = \left(x + \frac{\ell_*}{\alpha}\left|\frac{x}{\ell_*}\right|^\alpha\right) F_\omega\left(\ln\left|\frac{x}{\ell_{\rm Pl}}\right|\right), \qquad \ell_*^0 = t_*,\ \ell_*^i = \ell_*$$

Different choices of symmetries:

1. Ordinary derivatives: $\mathcal{D}_x = \partial_x$.
2. Weighted derivatives: $\mathcal{D}_x = (\partial_x q)^{-1/2}\partial_x[(\partial_x q)^{1/2}\,\cdot\,]$.
3. q-derivatives (multifractal): $\mathcal{D}_x = \partial_q = (\partial_x q)^{-1}\partial_x$.
4. Fractional derivatives (multifractal): $\mathcal{D}_x = \partial_x^\alpha$.

Status

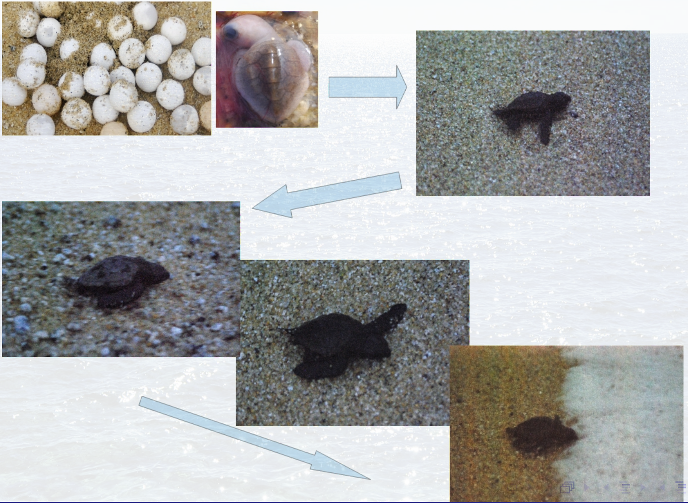
G.C. et al. 2012–2016

	$\Box$, $\Box^\dagger$	$\mathcal{D}^2$	$\Box_q$	$\partial^{2\alpha}$
Momentum transform	✗?	✓	✓	?
Relativistic mechanics	✓	✓	✓	?
Perturbative field theory	✓	✓	✓	✓?
QFT and SM	?	✓	✓	?
Perturbative renormalizability	?	✗	✗	✓?
Phenomenology (obs. constraints)	?	✓	✓	?
Gravity and cosmology	✓	✓	✓	?

5/7–

How do bodies move in fractal spacetimes?

G.C. EPJC 2016 (arXiv:1602.01470)



Observational constraints ($\alpha = 1/2$)

WEIGHTED DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
$\Delta\alpha/\alpha$ quasars	$< \mathbf{10^6}$	$< 10^{15}$	$> 10^{-23}$	G.C., Magueijo, Rodríguez PRD 2014
CMB black-body	$< 10^{-21}$	$< 10^{-12}$	$> \mathbf{10^3}$	G.C., Kuroyanagi, Tsujikawa to appear
Lamb shift	$< \mathbf{10^{-29}}$	$< 10^{-20}$	$> 10^{13}$	G.C., Nardelli, Rodríguez, arXiv:1512.06858
GW and GRB	—	—	—	G.C., arXiv:1603.03046

q-DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
CMB primordial	—	—	—	G.C., Kuroyanagi, Tsujikawa to appear
CMB black-body	$< 10^{-26}$	$< 10^{-18}$	$> \mathbf{10^{10}}$	G.C., Kuroyanagi, Tsujikawa to appear
muon lifetime	$< \mathbf{10^{-18}}$	$< 10^{-9}$	$> 10^2$	G.C., Nardelli, Rodríguez, PRD 2016
Lamb shift	$< 10^{-27}$	$< 10^{-19}$	$> \mathbf{10^{11}}$	G.C., Nardelli, Rodríguez, PRD 2016
GW*	$< 10^{-39}$	$< 10^{-30}$	$> \mathbf{10^{23}}$	G.C., arXiv:1603.03046
GRB $\sim$	$< 10^{-50}$	$< 10^{-42}$	$> \mathbf{10^{44}}$	G.C., arXiv:1603.03046

Multi-scale spacetimes vs. QG: dispersion relations and GWs

QG and string theory (IR limit $k \ll M$):

$$E^2 \simeq k^2 \left[1 + b\left(\frac{k}{M}\right)^n\right], \qquad \Delta v = \frac{\mathrm{d}E}{\mathrm{d}k} - 1 \sim \left(\frac{E}{M}\right)^n, \qquad n = 1, 2$$

GW150914 event: $|\Delta v| < 1.7 \times 10^{-18}$, $M(n=1) > 4 \times 10^4\,\mathrm{eV}$, $M(n=2) > 10^{-4}\,\mathrm{eV}$ [Arzano, G.C. to appear].

Viable fundamental mass $10\,\mathrm{TeV} < M < M_{\mathrm{Pl}}$ only if $0.44 < n < 0.68$ [Arzano, G.C. to appear].

Multi-scale theory with q-derivatives can be constrained by GW alone!

$$E^2 \simeq k^2 \left[1 \pm O(1)\left(\frac{k}{E_*}\right)^{1-\alpha}\right], \qquad 0 < 1 - \alpha < 1.$$

どうもありがとうございました！

Thank you!

Muito obrigado!

¡Muchas gracias!

Grazie!

Danke schön!

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IberiCos 2016, Vila do Conde
March 31, 2016

Effects of **canonical quantum gravity** on inflationary perturbations

joint work with: **David Brizuela** (Bilbao) and **Claus Kiefer** (Cologne)
arXiv:1511.05545

Manuel Krämer
Institute of Physics
University of Szczecin [ˈʃtʃɛ.tʃʲin]

Motivation

- in order to decide which of the Quantum Gravity candidate theories is the correct one, we need testable predictions
- best chances to find sizeable QG effects ➔ **Cosmic Microwave Background**
- *How do these effects look like **qualitatively** and **quantitatively** for a **conservative approach** to Quantum Gravity?*

➡ set up a model of an inflationary universe with perturbations

➡ apply canonical quantization ➔ **Wheeler–DeWitt equation**

➡ perform a **semiclassical approximation**

➡ calculate the **power spectra** of the perturbations and **quantum-gravitational corrections** to them

- using scalar-field perturbations: `1103.4967` (PRL), `1303.0531` (PRD)

➡ ***here:*** gauge-invariant scalar and tensor perturbations ➔ `1511.05545`

Wheeler–DeWitt equation and its semiclassical approx.

- de Sitter universe with scale factor a, $\mathrm{d}s^2 = a^2(\eta)\left(-\,\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right)$, constant scalar field leading to constant Hubble parameter H_0, with perturbations v_k

$\hbar = c = 1$

$$\left[\frac{1}{2a}\,\frac{1}{m_\mathrm{P}^2}\,\frac{\partial}{\partial a}\left(a\,\frac{\partial}{\partial a}\right) + \frac{a^4}{2}\,m_\mathrm{P}^2 H_0^2 + \sum_k \mathcal{H}_k^{\mathrm{pert}}(v_k)\right]\Psi(a, v_k) = 0$$

$$m_\mathrm{P}^2 := \frac{3}{4\pi G}$$

Wheeler–DeWitt equation and its semiclassical approx.

- de Sitter universe with scale factor a, $\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right)$, constant scalar field leading to constant Hubble parameter H_0, with perturbations v_k

$\hbar = c = 1$

$$\left[\frac{1}{2a}\frac{1}{m_\mathrm{P}^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) + \frac{a^4}{2} m_\mathrm{P}^2 H_0^2 + \sum_k \mathcal{H}_k^\mathrm{pert}(v_k)\right]\Psi(a, v_k) = 0$$

$m_\mathrm{P}^2 := \frac{3}{4\pi G}$

semiclassical approximation

$\mathcal{O}(m_\mathrm{P}^2)$: classical dynamics

$$\frac{\partial}{\partial\eta} = a^2 H_0^2 \frac{\partial}{\partial a}$$

$\mathcal{O}(m_\mathrm{P}^0)$: QM/QFT on curved spacetime

$$\mathrm{i}\frac{\partial}{\partial\eta}\psi_k^{(0)} = \mathcal{H}_k\psi_k^{(0)}$$

Kiefer and Singh, Phys. Rev. D **44**, 1067 (1991).

$\mathcal{O}(m_\mathrm{P}^{-2})$: QG corrections

$$\mathrm{i}\frac{\partial}{\partial\eta}\psi_k^{(1)} = \mathcal{H}_k\psi_k^{(1)} - \frac{\psi_k^{(1)}}{2\,m_\mathrm{P}^2\,\psi_k^{(0)}}\left[\frac{\left(\mathcal{H}_k\right)^2}{V}\psi_k^{(0)} + \mathrm{i}\frac{\partial}{\partial\eta}\left(\frac{\mathcal{H}_k}{V}\right)\psi_k^{(0)}\right]$$

$V(\eta) = H_0^{-2}\eta^{-4}$

Wheeler–DeWitt equation for an inflationary universe

- inflation modelled using a scalar field ϕ with potential $\mathcal{V}(\phi)$

  ➔ slow roll: $\dot{\phi}^2 \ll |\mathcal{V}(\phi)|$ ➔ slow-roll parameters:

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \delta = \epsilon - \frac{\dot{\epsilon}}{2H\epsilon} = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

- for a flat Friedmann–Lemaître universe with minimally coupled scalar field

$$\alpha := \ln(a/a_0)$$

➡ Wheeler–DeWitt equation:

$$\mathcal{H}_0 \Psi(\alpha, \phi) = \frac{1}{2}\,\mathrm{e}^{-2\alpha}\left[\frac{1}{m_{\mathrm{P}}^2}\,\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} + 2\,\mathrm{e}^{6\alpha}\,\mathcal{V}(\phi)\right]\Psi(\alpha,\phi) = 0$$

- de Sitter background: neglect ϕ-kinetic term and set $\mathcal{V} = \frac{1}{2}\,m_{\mathrm{P}}^2\,H_0^2$
- slow-roll background: rescale $\tilde{\phi} = m_{\mathrm{P}}^{-2}\,\phi$

  ➔ during semiclassical approximation: $\mathcal{V} = \frac{1}{2}\,m_{\mathrm{P}}^2\,H^2\left(1 - \frac{\epsilon}{3}\right)$

Adding scalar and tensor perturbations

- origin of CMB anisotropies: quantum fluctuations "amplified" by inflation
  - gauge-invariant ***scalar*** perturbations to the metric

$$\mathrm{d}s^2 = a^2(\eta)\left\{-\left(1-2A\right)\mathrm{d}\eta^2 + 2\left(\partial_i B\right)\mathrm{d}x^i\mathrm{d}\eta + \left[\left(1-2\psi\right)\delta_{ij} + 2\partial_i\partial_j E\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

combined with perturbations of the scalar field ϕ

$$\delta\phi^{(\mathrm{gi})}(\eta,\mathbf{x}) = \delta\phi + \phi'\left(B - E'\right)$$

- *additionally:* ***tensor*** perturbations ➔ primordial gravitational waves

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

➡ both expressed by a gauge-invar. perturbation variable $v_k \propto a\,\delta x^{(\mathrm{gi})}_{\mathrm{S,T}}$

➡ for each mode (for both scalars and tensors), we get a WDW equation

$$\left[\mathcal{H}_0 + \not\sum_{\mathrm{S,T};k} {}^{\mathrm{S,T}}\mathcal{H}_k\right]\Psi_k(\alpha, v_k) = 0$$

$${}^{\mathrm{S,T}}\mathcal{H}_k = \frac{1}{2}\left[-\frac{\partial^2}{\partial v_k^2} + {}^{\mathrm{S,T}}\omega_k^2(\eta)\, v_k^2\right]$$

The *de Sitter* case: derivation of the power spectra

- Gaussian ansatz $\psi_k^{(0)}(\eta, v_k) = \mathcal{N}_k^{(0)}(\eta)\, \mathrm{e}^{-\frac{1}{2}\,\Omega_k^{(0)}(\eta)\, v_k^2}$ ➔ Schrödinger eq.
- we have to solve: $\mathrm{i}\,\Omega_k^{\prime(0)}(\eta) = \left(\Omega_k^{(0)}(\eta)\right)^2 - {}^{\mathrm{S,T}}\omega_k^2(\eta)$

$$^{\mathrm{S,T}}\omega_k^2(\eta) = k^2 - \frac{2}{\eta^2}$$

➡ solution: $\Omega_k^{(0)}(\eta) = \dfrac{k^2\eta}{\mathrm{i} + k\eta} + \dfrac{\mathrm{i}}{\eta}$

- power spectrum for scalar perturbations can be obtained via

$$\mathcal{P}_{\mathrm{S}}^{(0)}(k) = \frac{G\,H_0^2}{\pi\,\epsilon}\,\frac{k^3\eta^2}{\Re\mathfrak{e}\Omega_k^{(0)}}$$

- superhorizon limit $-k\eta \to 0$ ➔ $\Re\mathfrak{e}\Omega_k^{(0)}(\eta) = \dfrac{k^3\eta^2}{k^2\eta^2+1} \longrightarrow k^3\eta^2$

➡ **power spectrum for scalar pert.:**

$$\mathcal{P}_{\mathrm{S}}^{(0)}(k) = \left.\frac{G\,H_0^2}{\pi\,\epsilon}\right|_{k=H_0 a}$$

$$r^{(0)} = \frac{\mathcal{P}_{\mathrm{T}}^{(0)}(k)}{\mathcal{P}_{\mathrm{S}}^{(0)}(k)} = 16\,\epsilon$$

➡ **tensor perturb.:**

$$\mathcal{P}_{\mathrm{T}}^{(0)}(k) = \frac{16\,G\,H_0^2}{\pi}\,\frac{k^3\eta^2}{\Re\mathfrak{e}\Omega_k^{(0)}} = \frac{16\,G\,H_0^2}{\pi}$$

The *de Sitter* case: quantum-gravitational corrections

$$\mathrm{i}\,\frac{\partial}{\partial\eta}\,\psi_k^{(1)} = \mathcal{H}_k\psi_k^{(1)} - \frac{\psi_k^{(1)}}{2\,m_\mathrm{P}^2\,\psi_k^{(0)}}\left[\frac{\left(\mathcal{H}_k\right)^2}{V}\,\psi_k^{(0)} + \mathrm{i}\,\frac{\partial}{\partial\eta}\left(\frac{\mathcal{H}_k}{V}\right)\psi_k^{(0)}\right]$$

- also assume Gaussianity for corrected Schrödinger equation:

$$\psi_k^{(1)}(\eta, v_k) = \mathcal{N}_k^{(1)}(\eta)\,\mathrm{e}^{-\frac{1}{2}\,\Omega_k^{(1)}(\eta)\,v_k^2}$$

- we have to solve: $\mathrm{i}\,\Omega_k'^{(1)}(\eta) = \left(\Omega_k^{(1)}(\eta)\right)^2 - \widetilde{\omega}_k^2(\eta)$

  with $\widetilde{\omega}_k^2 := \omega_k^2 - \dfrac{1}{2m_\mathrm{P}^2 V}\left[\left(3\Omega_k^{(0)} - \mathrm{i}\,(\ln V)'\right)\left(\omega_k^2 - (\Omega_k^{(0)})^2\right) + 2\,\omega_k\,\omega_k'\right]$

- imaginary terms appear; note that $\Omega_k^{(0)}(\eta) = \dfrac{k^2\eta}{\mathrm{i} + k\eta} + \dfrac{\mathrm{i}}{\eta}$

➡ problem with unitarity

- additionally, numerical analysis of full equation with imaginary terms reveals that the solution oscillates heavily for early times

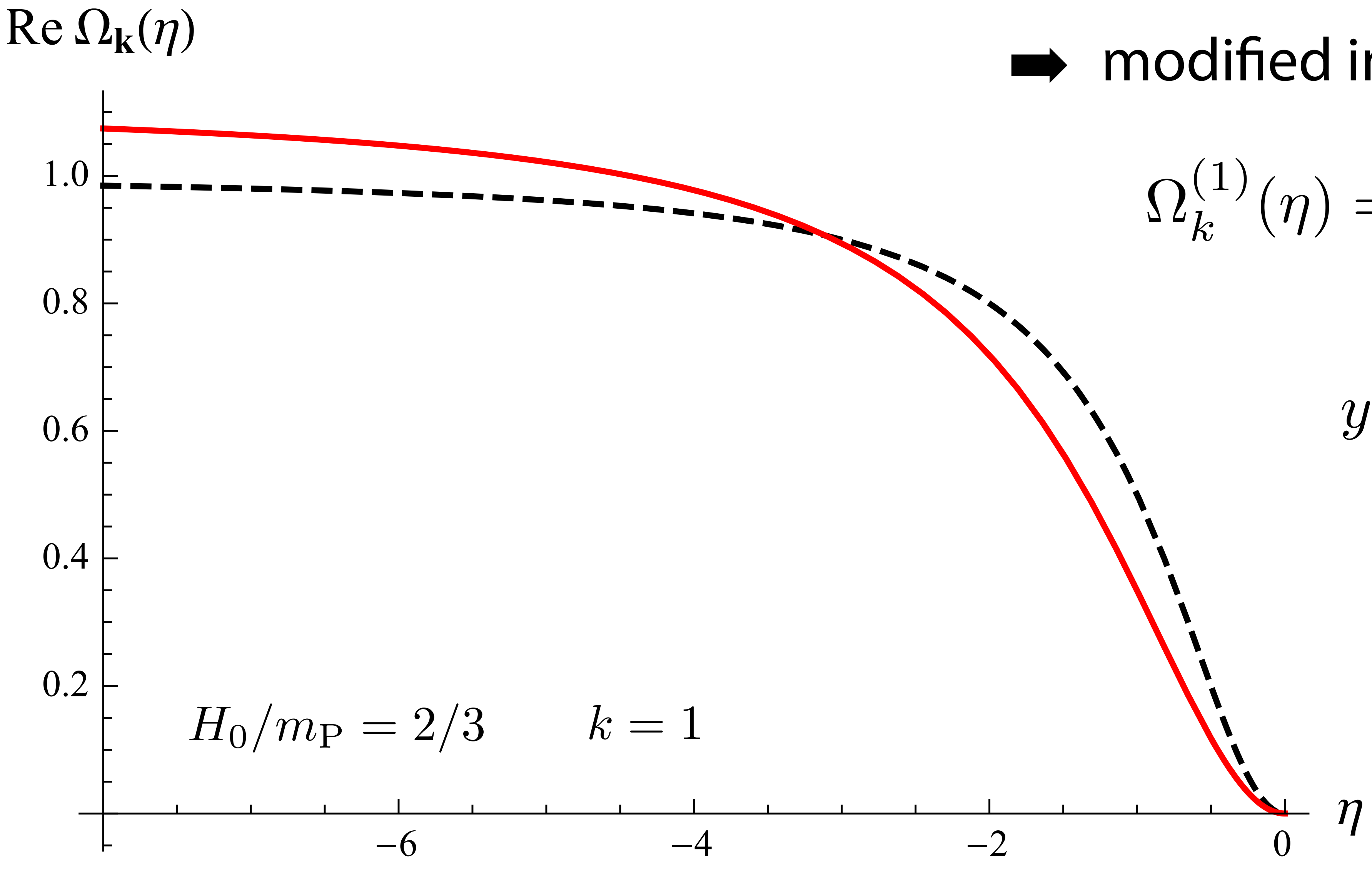
➡ no way to implement initial conditions ➔ neglect the imaginary terms

The *de Sitter* case: QG corrections – numerics

- equation we have to solve after removal of imaginary terms:

$$\mathrm{i}\,\Omega_k'^{(1)} = \left(\Omega_k^{(1)}\right)^2 - \omega_k^2 + \frac{H_0^2\eta^4}{2m_\mathrm{P}^2}\,\frac{k^3(11-k^2\eta^2)}{(1+k^2\eta^2)^3}$$

- numerical solution with Bunch–Davies initial conditions
  ➔ oscillation with constant amplitude around mean value $k + \dfrac{H_0^2}{4k\,m_\mathrm{P}^2}$



➡ modified initial conditions:

$$\Omega_k^{(1)}(\eta) = -\,\mathrm{i}\,\frac{y_k^{(1)\prime}(\eta)}{y_k^{(1)}(\eta)}$$

$$y_k^{(1)} \propto \mathrm{e}^{\mathrm{i}\beta_k\eta}$$

$$\beta_k \approx k + \frac{H_0^2}{4k\,m_\mathrm{P}^2}$$

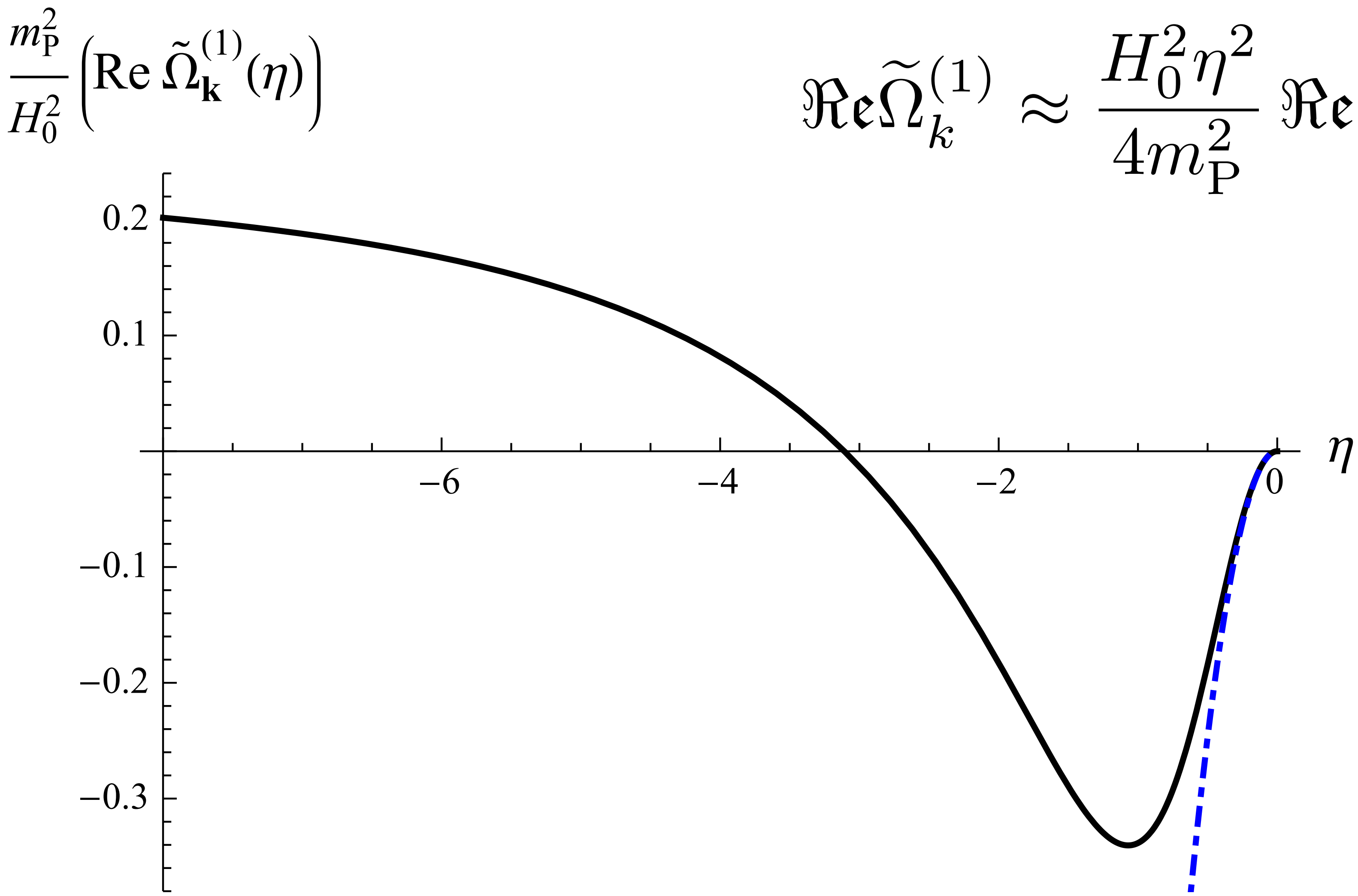
The *de Sitter* case: QG corrections – linearization

- find analytical solution at late times (superhorizon limit) $-k\eta \to 0$

➡ linearization around $\Omega_k^{(0)}$: $\quad \Omega_k^{(1)} = \Omega_k^{(0)} + \widetilde{\Omega}_k^{(1)}$

- we have to solve: $\quad \mathrm{i}\,\widetilde{\Omega}_k'^{(1)} = 2\,\Omega_k^{(0)}\widetilde{\Omega}_k^{(1)} - \left(\widetilde{\omega}_k^2 - \omega_k^2\right)$

➡ behavior of the solution at $-k\eta \to 0$



$$\Re\mathfrak{e}\,\widetilde{\Omega}_k^{(1)} \approx \frac{H_0^2\eta^2}{4m_P^2}\,\Re\mathfrak{e}\left(1 + 3\mathrm{e}^2\Gamma(0,2) + \frac{9\Gamma(0,-2)}{\mathrm{e}^2}\right)$$

$$\approx -0.988\,\frac{H_0^2\eta^2}{m_P^2}$$

➔ agrees very well with numerical solution

The *de Sitter* case: QG corrections – power spectra

- QG corrected power spectrum:

$$\mathcal{P}_{\rm S}^{(1)}(k) = \frac{4\pi G}{a^2\,\epsilon}\,\frac{k^3}{4\pi^2}\,\left(\Re\mathfrak{e}\,\Omega_k^{(0)} + \Re\mathfrak{e}\,\widetilde{\Omega}_k^{(1)}\right)^{-1}$$

$$= \mathcal{P}_{\rm S}^{(0)}(k)\left[1 - \frac{\Re\mathfrak{e}\,\widetilde{\Omega}_k^{(1)}}{\Re\mathfrak{e}\,\Omega_k^{(0)}} + \mathcal{O}\left(\frac{H_0^4}{m_{\rm P}^4}\right)\right]$$

➡ **we get for both scalars and tensors:**

$$\mathcal{P}_{\rm S,T}^{(1)}(k) = \mathcal{P}_{\rm S,T}^{(0)}(k)\left[1 + \frac{H_0^2}{m_{\rm P}^2}\,\frac{0.988}{k^3} + \mathcal{O}\left(\frac{H_0^4}{m_{\rm P}^4}\right)\right]$$

- QG corrections lead to an ***enhancement*** of power on large scales
- upper bound on $H_0^2/m_{\rm P}^2$ from tensor-to-scalar ratio $r \lesssim 0.11$

$$\frac{H_0^2}{m_{\rm P}^2} = \frac{2\,\mathcal{V}}{m_{\rm P}^4} \sim \frac{2\,r}{0.01}\left(\frac{10^{16}\,{\rm GeV}}{m_{\rm P}}\right)^4 \lesssim 1.74\times 10^{-10}$$

➡ upper limit:

$$\left|\frac{\mathcal{P}_{\rm S,T}^{(1)}(k) - \mathcal{P}_{\rm S,T}^{(0)}(k)}{\mathcal{P}_{\rm S,T}^{(0)}(k)}\right| \lesssim 1.72\times 10^{-10}\left(\frac{k_0}{k}\right)^3$$

Summary

- calculating quantum-gravitational corrections
  to the power spectra of inflationary scalar and tensor perturbations
  by performing a semiclassical approximation to the Wheeler–DeWitt eq.
  leads to a ***tiny enhancement*** of power on large scales

- other QG approaches also lead to modification of power on large scales

  - different method to realize the semiclassical approx. to the WDW eq.,
    Kamenshchik et al., `1305.6138` (PLB), `1403.2961` (PLB), `1501.06404` (JCAP).

  ➡ *de Sitter:* same behavior $\propto \frac{1}{k^3}\frac{H^2}{m_{\mathrm{P}}^2}$, same sign, slightly differ. prefactor

- ***Outlook:*** slow-roll case

  ➡ main contribution from *de Sitter* remains,
  additional terms with slow-roll parameters

- D. Brizuela, C. Kiefer, M. K.,
  Quantum-gravitational effects on scalar and tensor perturbations during inflation:
  The de Sitter case, `1511.05545`; *The slow-roll approximation,* in preparation.

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Quantisation of the holographic Ricci dark energy model

Imanol Albarran

Centro de Matemática e Aplicações. Universidade da Beira Interior (Portugal).

Talk based on: JCAP 1508 (2015) 051 [arXiv:1505.01353] and done in collaboration with Mariam Bouhmadi-López

IberiCos 2016

March 31, 2016

Outline

1. Introduction
2. Holographic dark energy
3. Holographic Ricci dark energy model (HRDE)
4. Wheeler-DeWitt equation
5. Quantisation of HRDE
6. Results
7. Conclusions

1 Introduction

The current Universe:

- Small deviations in the temperature of CMB $\Rightarrow$ Isotropy.
- Copernican Principle $\Rightarrow$ Homogeneity at large scales.
- Homogeneity + Isotropy $\Rightarrow$ FLRW space-time metric:

$$g_{\mu\nu} = -N^2(t)dt^2 + a(t)^2 \left[\left(\frac{1}{1 - kr^2} \right) dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\varphi^2 \right],$$

where $a(t)$ is the scale factor, $N(t)$ the lapse function and $k = -1, 0, 1$ for an open, flat and closed Universes.

- Components of the observable Universe $\Rightarrow$ radiation and matter.
- Those factors alone does not explain the current acceleration.
- Dark energy, which energy density can be described as:

$$\rho_d = \rho_{d0} a^{-3(1+\omega_d)}, \quad \text{where} \quad \omega_d \approx -1. \tag{1}$$

2 Holographic dark energy-1

The Holographic principle:

• Bekenstein proposal ⇒ The entropy of a given closed system, has an upper bound which is proportional to its surface area, L^2 (J.D. Bekenstein '81).

• For an effective quantum field theory ⇒ The following inequality was proposed (A.G. Cohen *et al* '99):

$$L^3 M_{\rm UV}^4 \lesssim L M_p^2. \quad (2)$$

• The saturation state of the inequality defines an energy density.

$$\rho_d \propto \frac{3}{8\pi G} L^{-2}. \quad (3)$$

Applying these ideas on a cosmological framework the holographic models arise (M. Li '04).

2 Holographic dark energy-2

Some examples of holographic dark energy models:

- $\rho_d \propto H^2$: , Do not provide acceleration, $\omega_d > -1/3$.
- $\rho_d \propto L_{PH}^{-2}$: Do not provide acceleration, $\omega_d > -1/3$.
- $\rho_d \propto L_{EH}^{-2}$: Drawback with the causality.
- $\rho_d \propto R$: Can describe an accelerated universe,

$$\rho_d(a) = 6\tilde{\beta}\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]. \qquad (4)$$

This suggestion is called the holographic Ricci dark energy model.

3 Holographic Ricci dark energy model (HRDE)

Classical description:

• We can be get the fundamental expression for the HRDE density. (C. Gao *et al* '09):

$$\rho_d(a) = \left(\frac{\beta}{2-\beta}\right)\rho_{m0}\left(\frac{a}{a_0}\right)^{-3} + \rho_{p0}\left(\frac{a}{a_0}\right)^{-2\left(2-\frac{1}{\beta}\right)} . \qquad (5)$$

• The effective energy density has two parts:

⇒ One imitates the behaviour of matter

⇒ A second one, whose EoS depends on the parameter β, can give rise to an accelerated Universe.

• If $0 < \beta < 1$ the Universe is accelerated. But also if $0 < \beta < 1/2$, a future doomsday called Big Rip will be reached.

4 Wheeler-DeWitt equation-1

Quantum version:

• Close to singularities quantum effects are expected, therefore the classical description must be corrected.

• The cosmological analogous to Schrödinger equation is the Wheeler-DeWitt equation.

• Try to find a wave function which fulfils the De-Witt boundary condition, i.e. the wave function vanishes close to singularities.

Wheeler-DeWitt equation:

• From the gravitational action: $S = S_{HE}$ where.

$$S_{HE} = \frac{1}{16\pi G}\left\{\int d^4x\sqrt{-g}\left[R - 2\Lambda\left(a\right)\right] - 2\int d^3x\sqrt{-h}K\right\}, \quad (6)$$

4 Wheeler-DeWitt equation-2

- For an isotropic and homogeneous Universe the classical Hamiltonian is (C. Kiefer '07, P. Vargas Moniz '10):

$$H = N\left[V(a) + \frac{1}{2}\tilde{G}^{AB} p_A p_B\right], \quad (7)$$

where the effective potential and and the Laplace-Beltrami operator read

$$V(a) \equiv \frac{3\pi}{4G}\left(ka - \frac{\Lambda(a)a^3}{3}\right), \tilde{G}^{AB} p_A p_B \equiv \left[\frac{4G}{3\pi}\frac{p_a^2}{a}\right]. \quad (8)$$

- The generalised momentum reads

$$\frac{p_a^2}{a} = -\hbar^2\left[a^{-\frac{1}{2}}\frac{d}{da}\right]\left[a^{-\frac{1}{2}}\frac{d}{da}\right] = -\frac{9}{4}\frac{\hbar^2}{a_0^3}\frac{d^2}{dx^2}, \quad (9)$$

where we apply the change $x = (a/a_0)^{\frac{3}{2}}$.

5 Quantisation of HRDE

Quantisation of HRDE:

- We will describe the HRDE as a perfect fluid
- The single degree of freedom is the scale factor.
- Wheeler-DeWitt equation: $\hat{\mathcal{H}}\Psi(x) = 0$.
- We will take the case of a spatially flat universe for simplicity and in accordance with the current observations (Planck '15).
- The WDW equation reduces to

$$\left\{\partial_x^2 + \gamma\left[\Omega_{r0}x^{-\frac{2}{3}} + \left(\frac{2}{2-\beta}\right)\Omega_{m0} + \Omega_{p0}x^{-\frac{2}{3}\left(1-\frac{2}{\beta}\right)}\right]\right\}\Psi(x) = 0, \tag{10}$$

- We perform some approximations to solve the equation:

$\Rightarrow$ Separate the effective potential in three domination epochs.

$\Rightarrow$ Then, apply the WKB approximation method where is necessary.

6 Results-1

Solutions for wave functions:

- For the radiation dominated era $\Rightarrow$ Exact solution.

$$\Psi_1(x) = \sqrt{x}\left[C_1 J_{\frac{3}{4}}\left(\frac{3}{2}\sqrt{\Omega_{r0}\gamma}x^{\frac{2}{3}}\right) + C_2 Y_{\frac{3}{4}}\left(\frac{3}{2}\sqrt{\Omega_{r0}\gamma}x^{\frac{2}{3}}\right)\right], \quad (11)$$

where C_1 and C_2 are constants. We choose $C_2 = 0$ to ensure that the wave function vanishes when $a \rightarrow 0$.

- For the later epochs $\Rightarrow$ First order WKB, which can be written as

$$\Psi_j(x) \approx \left[-\gamma g_j(x)\right]^{-\frac{1}{4}}\left[\alpha_j e^{ih_j(x)} + \delta_j e^{-ih_j(x)}\right], \quad (12)$$

where α_j and δ_j are constants and the $g_j(x)$'s can be written as

$$g_2(x) = \left[\Omega_{r0}x^{-\frac{2}{3}} + \left(\frac{2}{2-\beta}\right)\Omega_{m0}\right], g_3(x) = \left[\left(\frac{2}{2-\beta}\right)\Omega_{m0} + \Omega_{p0}x^{s}\right]$$

6 Results-2

where $s \equiv -2/3(1-2/\beta)$ and the functions $h_j(x)$ are written as

$$h_2(x) = \sqrt{\gamma\left(\frac{2}{2-\beta}\right)\Omega_{m0}}\left[x^{\frac{2}{3}} + \left(\frac{2-\beta}{2}\right)\frac{\Omega_{r0}}{\Omega_{m0}}\right]^{\frac{3}{2}}, \quad (14)$$

$$h_3(x) = \frac{\sqrt{\gamma}}{2+s}x\left\{2\sqrt{g_3(x)} + s\sqrt{\frac{2\Omega_{m0}}{2-\beta}}\;{}_2F_1\left[\tfrac{1}{2},\tfrac{1}{s};1+\tfrac{1}{s};\left(\tfrac{\beta-2}{2}\right)\tfrac{\Omega_{p0}}{\Omega_{m0}}x^{s}\right]\right\}.$$

- To build a smooth wave function $\Rightarrow$ we must impose:

$$\Psi_1(x_1) = \Psi_2(x_1), \Psi_{1'}(x_1) = \Psi_{2'}(x_1), \quad (15)$$

$$\Psi_2(x_2) = \Psi_3(x_2), \Psi_{2'}(x_2) = \Psi_{3'}(x_2). \quad (16)$$

- Where the selected x_1 point is when matter is sub dominant respect to the radiation content. The the point x_2 is when the presence of DM is dominant with respect to DE and radiation.

Conclusions

- The HRDE model can describe the late-time acceleration of the Universe. The best observationally constrained value of ρ_0 in this model provides $0 < \beta < 1/2$ (L.Xu, Y.Wang '10). Therefore, the Universe will reach a future singularity called the Big Rip.
- We apply a quantum approach to avoid the classical singularities.
- The solutions satisfy the DeWitt boundary condition.
- Therefore, we conclude that the Big bang singularity and Big Rip singularity (in HRDE) are avoidable in the quantum regime (Please see also several works by Kiefer and Collaborators).
- We have also analysed recently the Little Sibling of the Big Rip (I.A, M.Bouhmadi-López, P.Martín-Moruno and F.Cabral).JCAP 1511 (2015) 044

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